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AN OVERVIEW OF $\mu_I g$-LOCALLY CLOSED SETS IN GITS

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Abstract

Our scope of this article is to construct a new concept of $\mu_I g$-locally closed sets in Generalized intuitionistic topological space(GITS). Some characterizations are to be evaluated. Also introduced $\wedge_{LC}^{\star}$- set in GITS which is derived from $\mu_I g$-locally closed set and discuss the natures and their behaviours.

Keywords: $LC(\mu_I)$, $LO(\mu_I)$, $LC(\mu_I g)$, $LO(\mu_I g)$, $\wedge_{LC}^{\star}$- set and $\vee_{LC}^{\star}$- set.
AMS classifications: 54A10, 03E72

1.INTRODUCTION AND PRELIMINARIES:

The concept of intuitionistic set(IS) was first coined by Coker[6]. $\mu_I g$-CSGITS derived from generalized intuitionistic topological space (GITS) and their attributes are discussed by P.Sivagami et.al[15]. Various concepts of topological spaces is based on the generalization of closed set. In 1989, the locally closed set were putforth by Ganster and Reilly[12] and they used this concept to retrieve LC-continuity. The locally closed set is the intersection of an open and closed subsets of X which was brought out by Bourbaki[4] in 1966. In 1986, Maki[2] continued the work of Levine and Dunham on generalized closed sets and exposure operators by introducing the notion of $\wedge$-set in topological space. $\wedge$-set is a set A of X (= kernel set),ie to the intersection of all open sets containing A. Caldas and others introducing $\wedge_s$-sets and $\vee_s$-sets. In this paper we discuss the properties of μ_Ig-locally closed sets, $\wedge_{LC}^{\star}$- set and $\vee_{LC}^{\star}$-set as well as their relations are to be studied. Throughout this paper, we discussed the non-void set X and mentioned GITS (X, μ_I) as X and we call $\langle X, \phi, X\rangle$ as $\mathfrak{E}$, $\langle X, \phi, \phi\rangle$ as

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$\mathcal{O}$ and $\langle X, X, \phi\rangle$ as Ú. We recall the following definitions and results which are useful in the sequel.

Definition:1.1[15] Let μ_I be the collection of ISs of X. Then X is said to be GITS if $\phi_{\sim}\in\mu_I$ and $\cup_i \mathcal{U}_i, \mathcal{U}_i \in \mu_I$. Then the components of μ_I are named as μ_I-open and their inverses are termed as μ_I-closed sets. Also their closure and interior are as given below,$c_{\mu_I}(A) = \cap\{F: F \in {\mu_I}^c, A \subseteq F\}$ and $i_{\mu_I}(A) = \cup\{G: G \in \mu_I, G \subseteq A\}$.

Definition:1.2[15] If $c_{\mu_I}(A) \subseteq U$ whenever $A \subseteq U$ where $U \in \mu_I$ then $A \subseteq X$ is called $\mu_I g$-closed set (μ_Ig-CSGITS) and their closure and interior are defined as follows, $c^*_{\mu_I}(A) = \cap\{F: F$ is $\mu_I g$-CSGITS and $A \subseteq F\}$ and $i^*_{\mu_I}(A) = \cup\{G: G$ is $\mu_I g$-open set (μ_Ig-OSGITS), $G \subseteq A\}$.

Definition:1.3[10] An ISs A of X is said to be μ_Ig-NDGITS if the μ_Ig-closure of A contains no μ_Ig- interior points or $i^*_{\mu_I}(c^*_{\mu_I}(A)) = \mathfrak{E}$.

Definition:1.4[10] If$c^*_{\mu_I}(A) = X_{\sim}$in X then A is named as μ_Ig-Dense set (μ_Ig-DGITS).

Theorem:1.5[10] Let A be an ISs of X. If $A \in Nd^*(\mu_I)$ in$X$, then $i^*_{\mu_I}(A) = \mathfrak{E}$.

Theorem:1.6[15] Every μ_I-open set (μ_I-closed set) in X is $\mu_I g$-OSGITS ($\mu_I g$-CSGITS).

Theorem:1.7[15] Intersection of two μ_Ig-CSGITS need not be a μ_Ig-CSGITS.

Result:1.8[15] 1.Intersection of two μ_I-closed set is μ_I-closed set.

2.$\bar{A} = \langle X, A_2, A_1\rangle$, (in intuitionistic, $\bar{A} = A^c$).

3. $A - B = A \cap \bar{B}$.

4. $c^*_{\mu_I}(A \cap B) \subseteq c^*_{\mu_I}(A) \cap c^*_{\mu_I}(B)$.

5.$c_{\mu_I}(A) \cap c_{\mu_I}(B) \supseteq c_{\mu_I}(A \cap B)$.

6.$\bar{\bar{A}} = A$.

7.$A \cap \bar{A} \neq \phi_{\sim}$;$A \cup \bar{A} \neq X_{\sim}$.

8.$c^*_{\mu_I}(c^*_{\mu_I}(A)) = c^*_{\mu_I}(A)$; .

9. $\overline{c^*_{\mu_I}(\bar{A})} = i^*_{\mu_I}(A)$.

2.$\mu_I g$-LOCALLY CLOSED SETS IN GITS

Definition:2.1 An ISs A of X is said to be μ_I-locally closed set in GITS(μ_I-LCSGITS) if $A = U \cap F$, where $F \in \mu_I^c$ and $U \in \mu_I$. The complement of μ_I-LCSGITS is named as μ_I-locally open set in GITS (μ_I-LOSGITS). The collection of μ_I-LCSGITS and μ_I-LOSGITS is denoted as $LC(\mu_I)$ and $LO(\mu_I)$.

Definition:2.2 An ISs ξ_X of X is said to be $\mu_I g$-locally closed set in GITS($\mu_I g$-LCSGITS) if $\xi_X = \S_X \cap \wp_X$, where $\wp_X \in \mu_I g$-CSGITS and $\S_X \in \mu_I g$-OSGITS. The complement of μ_Ig-LCSGITS is termed as μ_Ig-locally open set in GITS (μ_Ig-LOSGITS). The collection of μ_Ig-LCSGITS and μ_Ig-LOSGITS is denoted as $LC(\mu_I g)$ and $LO(\mu_I g)$.

Example:2.3 1. Let $X = \mathcal{R}$ with $\mu_I = \{\mathfrak{E}, A, B, A \cup B\}$, where $A = \langle X, A_T, A_F\rangle$, $A_T =$ Set of all multiples of 4, $A_F =$ set of all prime number and $B = \langle X, B_T, B_F\rangle$, $B_T =$ set of all multiples of 8, $B_F = \{2\}$. Then $\mu_I^c = \{\text{Ứ}, \bar{A}, \bar{B}, \overline{A \cup B}\}$. $LC(\mu_I g) = \{\mathfrak{E}, A, B, A \cup B, \langle X, \emptyset, B_T \cup A_F\rangle, \langle X, \emptyset, A_T \cup A_F\rangle, \langle X, \emptyset, A_T \cup B_F\rangle, \langle X, \emptyset, B_T \cup B_F\rangle\}$. Now $c_{\mu I}(\langle X, B_F, B_T\rangle) = \langle X, B_F, B_T\rangle$, $\bar{B} = \langle X, B_F, B_T\rangle$ is $\mu_I g$-CSGITS and $\langle X, B_T, B_F\rangle$ is $\mu_I g$-OSGITS $\Longrightarrow \langle X, B_F \cap B_T, B_T \cup B_F\rangle = \langle X, \emptyset, B_T \cup B_F\rangle$ is $\mu_I g$-LCSGITS.

2. Let $X = \{\text{ɕ}_X, \text{ɗ}_X, \text{ɚ}_X, \text{ɤ}_X\}$ with $\mu_I = \{\mathfrak{E}, \langle X, \{\text{ɕ}_X, \text{ɗ}_X, \text{ɚ}_X\}, \phi\rangle, \langle X, \phi, \{\text{ɕ}_X, \text{ɚ}_X\}\rangle, \langle X, \{\text{ɕ}_X\}, \{\text{ɗ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɕ}_X\}, \phi\rangle, \langle X, \{\text{ɗ}_X, \text{ɚ}_X\}, \{\text{ɤ}_X\}\rangle, \langle X, \{\text{ɗ}_X, \text{ɚ}_X\}, \phi\rangle, \langle X, \{\text{ɕ}_X, \text{ɗ}_X, \text{ɚ}_X\}, \{\text{ɤ}_X\}\rangle\}$.
Then $LC(\mu_I) = \{\langle X, \phi, \{\text{ɕ}_X, \text{ɗ}_X, \text{ɚ}_X\}\rangle, \langle X, \phi, \{\text{ɕ}_X\}\rangle, \langle X, \{\text{ɗ}_X\}, \{\text{ɕ}_X\}\rangle, \langle X, \phi, \{\text{ɗ}_X, \text{ɚ}_X\}\rangle,$
$\langle X, \phi, \{\text{ɕ}_X, \text{ɗ}_X, \text{ɤ}_X\}\rangle, \langle X, \phi, \{\text{ɚ}_X, \text{ɗ}_X, \text{ɤ}_X\}\rangle, \langle X, \phi, \{\text{ɕ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɗ}_X\}, \{\text{ɕ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɚ}_X\}, \{\text{ɤ}_X\}\rangle,$
$\langle X, \phi, \{\text{ɚ}_X, \text{ɗ}_X\}\rangle, \langle X, \{\text{ɚ}_X\}, \phi\rangle, \langle X, \{\text{ɕ}_X, \text{ɚ}_X\}, \{\text{ɤ}_X\}\rangle, \mathfrak{E}, \langle X, \{\text{ɕ}_X, \text{ɗ}_X, \text{ɚ}_X\}, \phi\rangle, \langle X, \phi, \{\text{ɕ}_X, \text{ɚ}_X\}\rangle,$
$\langle X, \{\text{ɕ}_X\}, \{\text{ɗ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɕ}_X\}, \phi\rangle, \langle X, \{\text{ɗ}_X, \text{ɚ}_X\}, \{\text{ɤ}_X\}\rangle, \langle X, \{\text{ɗ}_X, \text{ɚ}_X\}, \phi\rangle, \langle X, \{\text{ɕ}_X, \text{ɗ}_X, \text{ɚ}_X\}, \{\text{ɤ}_X\}\rangle\}$ and
$LC(\mu_I g) = \{\mathfrak{E}, \langle X, \{\text{ɕ}_X, \text{ɗ}_X, \text{ɚ}_X\}, \phi\rangle, \langle X, \phi, \{\text{ɕ}_X, \text{ɚ}_X\}\rangle, \langle X, \{\text{ɗ}_X, \text{ɕ}_X\}, \phi\rangle, \langle X, \phi, \{\text{ɤ}_X\}\rangle, \mathcal{O},$
$\langle X, \{\text{ɕ}_X\}, \{\text{ɗ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɕ}_X\}, \phi\rangle, \langle X, \{\text{ɗ}_X, \text{ɚ}_X\}, \{\text{ɤ}_X\}\rangle, \langle X, \{\text{ɗ}_X, \text{ɚ}_X\}, \phi\rangle, \langle X, \{\text{ɕ}_X, \text{ɗ}_X, \text{ɚ}_X\}, \{\text{ɤ}_X\}\rangle,$
$\langle X, \{\text{ɕ}_X\}, \{\text{ɤ}_X\}\rangle, \langle X, \{\text{ɗ}_X\}, \{\text{ɤ}_X\}\rangle, \langle X, \{\text{ɚ}_X\}, \{\text{ɤ}_X\}\rangle, \langle X, \{\text{ɗ}_X, \text{ɕ}_X\}, \{\text{ɤ}_X\}\rangle, \langle X, \{\text{ɚ}_X, \text{ɕ}_X\}, \{\text{ɤ}_X\}\rangle,$
$\langle X, \phi, \{\text{ɚ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɕ}_X\}, \{\text{ɚ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɗ}_X\}, \{\text{ɚ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɕ}_X, \text{ɗ}_X\}, \{\text{ɚ}_X, \text{ɤ}_X\}\rangle, \langle X, \phi, \{\text{ɕ}_X, \text{ɤ}_X\}\rangle,$
$\langle X, \{\text{ɗ}_X\}, \{\text{ɕ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɚ}_X\}, \{\text{ɕ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɗ}_X, \text{ɚ}_X\}, \{\text{ɕ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɗ}_X\}, \{\text{ɕ}_X, \text{ɚ}_X\}\rangle,$
$\langle X, \phi, \{\text{ɕ}_X, \text{ɚ}_X, \text{ɤ}_X\}\rangle, \langle X, \phi, \{\text{ɗ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɚ}_X\}, \{\text{ɗ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɕ}_X, \text{ɚ}_X\}, \{\text{ɗ}_X, \text{ɤ}_X\}\rangle,$
$\langle X, \phi, \{\text{ɗ}_X, \text{ɚ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɕ}_X\}, \{\text{ɗ}_X, \text{ɚ}_X, \text{ɤ}_X\}\rangle, \langle X, \{\text{ɗ}_X\}, \{\text{ɕ}_X, \text{ɚ}_X, \text{ɤ}_X\}\rangle, \langle X, \phi, \{\text{ɕ}_X, \text{ɗ}_X, \text{ɤ}_X\}\rangle,$
$\langle X, \{\text{ɚ}_X\}, \{\text{ɕ}_X, \text{ɗ}_X, \text{ɤ}_X\}\rangle$, $\langle X, \phi, \{\text{ɕ}_X\}\rangle, \langle X, \phi, \{\text{ɕ}_X, \text{ɗ}_X\}\rangle, \langle X, \phi, \{\text{ɚ}_X, \text{ɗ}_X\}\rangle,$
$\langle X, \phi, \{\text{ɗ}_X\}\rangle, \langle X, \phi, \{\text{ɕ}_X, \text{ɗ}_X, \text{ɚ}_X\}\rangle, \langle X, \{\text{ɕ}_X\}, \{\text{ɗ}_X\}\rangle, \langle X, \{\text{ɕ}_X\}, \{\text{ɚ}_X\}\rangle, \langle X, \{\text{ɕ}_X\}, \{\text{ɗ}_X, \text{ɚ}_X\}\rangle, \langle X, \{\text{ɗ}_X\}, \phi\rangle,$
$\langle X, \{\text{ɗ}_X\}, \{\text{ɕ}_X\}\rangle, \langle X, \{\text{ɗ}_X\}, \{\text{ɚ}_X\}\rangle, \langle X, \{\text{ɚ}_X\}, \phi\rangle, \langle X, \{\text{ɚ}_X\}, \{\text{ɕ}_X\}\rangle, \langle X, \{\text{ɚ}_X\}, \{\text{ɗ}_X\}\rangle, \langle X, \{\text{ɚ}_X\}, \{\text{ɤ}_X\}\rangle,$
$\langle X, \{\text{ɚ}_X\}, \{\text{ɕ}_X, \text{ɗ}_X\}\rangle, \langle X, \{\text{ɗ}_X, \text{ɕ}_X\}, \{\text{ɚ}_X\}\rangle, \langle X, \{\text{ɗ}_X, \text{ɚ}_X\}, \{\text{ɕ}_X\}\rangle, \langle X, \{\text{ɕ}_X, \text{ɚ}_X\}, \phi\rangle, \langle X, \{\text{ɕ}_X, \text{ɚ}_X\}, \{\text{ɗ}_X\}\rangle.$

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Theorem:2.4 1.An ISs A of X is μ_I-LCSGITS iff $\bar{A} = \mathfrak{U} \cup \mathfrak{V}$, where $\mathfrak{U} \in \mu_I$ and $\mathfrak{V} \in {\mu_I}^{c}$.
2. An ISs ξ_X of X is $\mu_I g$-LCSGITS iff $\overline{\xi_X}$ is the union of μ_Ig-OSGITS and $\mu_I g$-CSGITS.

Remark:2.5 Every μ_I-open set in X is μ_I-LCSGITS but the reversal statement does not satisfied.For example:2.3(2),$\langle X,\phi,\{ɕ_X,ɗ_X,ɚ_X\}\rangle$,$\langle X,\phi,\{ɕ_X\}\rangle$,$\langle X,\{ɗ_X\},\{ɕ_X\}\rangle$,$\langle X,\phi,\{ɗ_X,ɚ_X\}\rangle$, $\langle X,\phi,\{ɕ_X,ɗ_X,ɤ_X\}\rangle$,$\langle X,\phi,\{ɚ_X,ɗ_X,ɤ_X\}\rangle$,$\langle X,\phi,\{ɕ_X,ɤ_X\}\rangle$,$\langle X,\{ɗ_X\},\{ɕ_X,ɤ_X\}\rangle$,$\langle X,\{ɚ_X\},\{ɤ_X\}\rangle$, $\langle X,\phi,\{ɚ_X,ɗ_X\}\rangle$,$\langle X,\{ɚ_X\},\phi\rangle$,$\langle X,\{ɕ_X,ɚ_X\},\{ɤ_X\}\rangle$ are μ_I-LCSGITS but not μ_I-open in X.

Remark:2.6 Every $\mu_I g$-OSGITS in X is $\mu_I g$-LCSGITS but contrary statement fails. For example:2.3(2),$\langle X,\{ɗ_X\},\{ɕ_X\}\rangle\langle X,\{ɕ_X,ɚ_X\},\{ɗ_X\}\rangle$ is $\mu_I g$-LCSGITS but which is not a $\mu_I g$-OSGITS.

Remark:2.7 Every μ_I-open set in X is $\mu_I g$-LCSGITS but the converse does not meet up the condition.

Proof: Since every μ_I-open set in X is $\mu_I g$-OSGITS and using above remark we get the result. But the converse does not satisfied. In example:2.3(1), $\mathcal{O}$and $\langle X,\phi,B_T \cup B_F\rangle$ are $\mu_I g$-LCSGITS but not a μ_I-open set.

Remark:2.8 1. The complement of a μ_I-LCSGITS need not be a μ_I-LCSGITS. For example 2.3(2), $\langle X,\{ɗ_X\},\{ɕ_X\}\rangle$ is μ_I-LCSGITS but the complement of $\langle X,\{ɗ_X\},\{ɕ_X\}\rangle$ is$\langle X,\{ɕ_X\},\{ɗ_X\}\rangle$, which is not a μ_I-LCSGITS.
2. The complement of a μ_Ig-LCSGITS need not be a μ_Ig-LCSGITS. For example 2.3(2), the complement of $\langle X,\phi,\{ɤ_X\}\rangle$ is $\langle X,\{ɤ_X\},\phi\rangle$ which is not a μ_Ig-LCSGITS.

Theorem:2.9 Every μ_I-LCSGITS in X is $\mu_I g$-LCSGITS but the converse does not hold.
Proof: Using theorem:1.6, we get a result. But the reverse fails. Now explained with the upcoming illustration.

Example:2.10 Let $X = \{\sigma_c,\mathscr{p}_c,\mathscr{q}_c\}$ with μ_I=$\{\mathfrak{E}$,$\langle X,\{\sigma_c\},\{\mathscr{p}_c\}\rangle$,$\langle X,\{\mathscr{p}_c\},\{\mathscr{q}_c\}\rangle$,$\langle X,\{\sigma_c,\mathscr{p}_c\},\phi\rangle$, $\langle X,\{\sigma_c\},\phi\rangle\}$ and $LC(\mu_I) = \{$ $\mathfrak{E}$,$\langle X,\{\sigma_c\},\{\mathscr{p}_c\}\rangle$,$\langle X,\{\mathscr{p}_c\},\{\mathscr{q}_c\}\rangle$,$\langle X,\{\sigma_c,\mathscr{p}_c\},\phi\rangle$, $\langle X,\{\sigma_c\},\phi\rangle$,$\langle X,\phi,\{\sigma_c\}\rangle$,$\langle X,\phi,\{\mathscr{p}_c\}\rangle$,$\langle X,\phi,\{\sigma_c,\mathscr{p}_c\}\rangle$,$\langle X,\phi,\{\mathscr{p}_c,\mathscr{q}_c\}\rangle$,$\langle X,\{\mathscr{p}_c\},\{\sigma_c,\mathscr{q}_c\}\rangle$, $\langle X,\{\mathscr{p}_c\},\{\sigma_c\}\rangle\}$. Then $LC(\mu_I g) = \{\mathfrak{E}$,$\langle X,\{\sigma_c\},\{\mathscr{p}_c\}\rangle$,$\langle X,\{\mathscr{p}_c\},\{\mathscr{q}_c\}\rangle$,$\langle X,\{\sigma_c,\mathscr{p}_c\},\phi\rangle$,$\mathcal{O}$, $\langle X,\{\sigma_c\},\phi\rangle$,$\langle X,\phi,\{\sigma_c\}\rangle$,$\langle X,\phi,\{\mathscr{p}_c\}\rangle$,$\langle X,\phi,\{\sigma_c,\mathscr{p}_c\}\rangle$,$\langle X,\phi,\{\mathscr{p}_c,\mathscr{q}_c\}\rangle$, $\langle X,\{\mathscr{p}_c\},\{\sigma_c,\mathscr{q}_c\}\rangle$,$\langle X,\{\mathscr{p}_c\},\phi\rangle$, $\langle X,\{\mathscr{p}_c\},\{\sigma_c\}\rangle$,$\langle X,\phi,\{\mathscr{q}_c\}\rangle$,$\langle X,\{\sigma_c\},\{\mathscr{q}_c\}\rangle$,$\langle X,\{\sigma_c,\mathscr{p}_c\},\{\mathscr{q}_c\}\rangle$, $\langle X,\{\sigma_c\},\{\mathscr{p}_c,\mathscr{q}_c\}\rangle$,$\langle X,\phi,\{\sigma_c,\mathscr{q}_c\}\rangle\}$. Here

$\langle X, \phi, \{q_c\}\rangle, \langle X, \{\sigma_c\}, \{q_c\}\rangle, \langle X, \{\sigma_c, p_c\}, \{q_c\}\rangle, \langle X, \{\sigma_c\}, \{p_c, q_c\}\rangle, \langle X, \phi, \{\sigma_c, q_c\}\rangle, \langle X, \{p_c\}, \phi\rangle, O$ are $\mu_I g$-LCSGITS but not a μ_I-LCSGITS

Theorem: 2.11 Let $\mathcal{Z}_c$ be an ISs of X. Then the subsequent statements are mean to be parallel in nature:

(i)$\mathcal{Z}_c \in LC(\mu_I)$.

(ii)$\mathcal{Z}_c = \mathcal{S}_c \cap c_{\mu I}(\mathcal{Z}_c)$ for some μ_I-open set $\mathcal{S}_c$.

Proof: (i) $\Longrightarrow$ (ii). Assume that $\mathcal{Z}_c \in LC(\mu_I)$. Then $\mathcal{Z}_c = \mathcal{S}_c \cap \mathcal{T}_c$, $\mathcal{S}_c \in \mu_I$ and $\mathcal{T}_c \in \mu_I^c \Rightarrow c_{\mu_I}(\mathcal{Z}_c) = c_{\mu_I}(\mathcal{S}_c \cap \mathcal{T}_c) \subseteq c_{\mu_I}(\mathcal{S}_c) \cap c_{\mu_I}(\mathcal{T}_c) \subseteq \mathcal{T}_c$. Now $\mathcal{S}_c \cap c_{\mu I}(\mathcal{Z}_c) \subseteq \mathcal{S}_c \cap \mathcal{T}_c = \mathcal{Z}_c$. Therefore $\mathcal{S}_c \cap c_{\mu I}(\mathcal{Z}_c) \subseteq \mathcal{Z}_c$ for some $\mathcal{S}_c \in \mu_I$. So we ETP $\mathcal{Z}_c \subseteq \mathcal{S}_c \cap c_{\mu I}(\mathcal{Z}_c)$. Now let we take $x \notin \mathcal{S}_c \cap c_{\mu I}(\mathcal{Z}_c)$. Then $x \notin \mathcal{S}_c$ or $x \notin c_{\mu I}(\mathcal{Z}_c) \Longrightarrow x \notin \mathcal{S}_c$ or $x \notin \cap F$, F is μ_I-closed set, $\mathcal{Z}_c \subseteq F \Longrightarrow x \notin \mathcal{S}_c$ or $x \notin F$, for some μ_I-closed set $\mathcal{Z}_c \subseteq F \Longrightarrow x \notin \mathcal{Z}_c \Longrightarrow \mathcal{Z}_c \subseteq \mathcal{S}_c \cap c_{\mu I}(\mathcal{Z}_c)$. Henceforth $\mathcal{Z}_c = \mathcal{S}_c \cap c_{\mu I}(\mathcal{Z}_c)$ for some μ_I-open set $\mathcal{S}_c$.

(ii) $\Longrightarrow$(i)Obvious.

Theorem:2.12 Let $\mathfrak{Q}_c$ be an ISs of X. Then the subsequent statements are Identical:

(i) $\mathfrak{Q}_c \in LC(\mu_I g)$.

(ii) $\mathfrak{Q}_c = \mathfrak{K}_c \cap c^*_{\mu_I}(\mathfrak{Q}_c)$ for some $\mu_I g$-OSGITS $\mathfrak{K}_c$.

Proof: (i) $\Longrightarrow$ (ii). Assume that $\mathfrak{Q}_c \in LC(\mu_I g)$. Then $\mathfrak{Q}_c = \mathfrak{K}_c \cap \mathfrak{P}_c$, $\mathfrak{K}_c \in \mu_I g$-OSGITS and $\mathfrak{P}_c \in \mu_I g$-CSGITS $\Longrightarrow c^*_{\mu_I}(\mathfrak{Q}_c) = c^*_{\mu_I}(\mathfrak{K}_c \cap \mathfrak{P}_c) \subseteq c^*_{\mu_I}(\mathfrak{K}_c) \cap c^*_{\mu_I}(\mathfrak{P}_c) = c^*_{\mu_I}(\mathfrak{K}_c) \cap \mathfrak{P}_c \subseteq \mathfrak{P}_c \Longrightarrow c^*_{\mu_I}(\mathfrak{Q}_c) \subseteq \mathfrak{P}_c$. Now $\mathfrak{K}_c \cap c^*_{\mu_I}(\mathfrak{Q}_c) \subseteq \mathfrak{K}_c \cap \mathfrak{P}_c = \mathfrak{Q}_c$. Therefore $\mathfrak{K}_c \cap c^*_{\mu_I}(\mathfrak{Q}_c) \subseteq \mathfrak{Q}_c$ for some $\mu_I g$-OSGITS $\mathfrak{K}_c$. Let $x \notin \mathfrak{K}_c \cap c^*_{\mu_I}(\mathfrak{Q}_c)$. Then $x \notin \mathfrak{K}_c$ or $x \notin c^*_{\mu_I}(\mathfrak{Q}_c) \Longrightarrow x \notin \mathfrak{K}_c$ or $x \notin \cap F$, F is $\mu_I g$-CSGITS containing $\mathfrak{Q}_c \Longrightarrow x \notin \mathfrak{K}_c$ or $x \notin F$, for some $\mu_I g$-CSGITS $\mathfrak{Q}_c \subseteq F \Longrightarrow x \notin \mathfrak{Q}_c \Longrightarrow \mathfrak{Q}_c \subseteq \mathfrak{K}_c \cap c^*_{\mu_I}(\mathfrak{Q}_c)$. Henceforth $\mathfrak{Q}_c = \mathfrak{K}_c \cap c^*_{\mu_I}(\mathfrak{Q}_c)$ for some $\mu_I g$-OSGITS $\mathfrak{K}_c$. (ii) $\Longrightarrow$(i) Obvious.

Note:2.13 In GTS, the following statements are equivalent. Let $A \subseteq X$.

(i) A in X is a locally closed set.

(ii) $A = B \cap cl(A)$ for some open set B.

(iii) $cl(A) - A$ is a closed set.

(iv) $A \cup \overline{cl(A)}$is a open set.

(v) $A \subseteq int(A \cup \overline{cl(A)})$. But in GITS, it does not holds because of Result:1.8(7), the intersection and union of $\mu_I g$-CSGITS need not be a $\mu_I g$-CSGITS and also the intersection and union of $\mu_I g$-OSGITS need not be a $\mu_I g$-OSGITS. So we have to modified the statements of this theorem as follows in GITS..

Theorem:2.14 Let $\mathfrak{Q}_c$ be an ISs of X. If $\mathfrak{Q}_c$ is $\mu_I g$-LCSGITS then $c^*_{\mu_I}(\mathfrak{Q}_c) - \mathfrak{Q}_c \supseteq c^*_{\mu_I}(\mathfrak{Q}_c) \cap \overline{\mathfrak{K}_c}$ for some $\mu_I g$-OSGITS $\mathfrak{K}_c$.

Proof: Suppose $\mathfrak{Q}_c$ is $\mu_I g$-LCSGITS. Now $c^*_{\mu_I}(\mathfrak{Q}_c) - \mathfrak{Q}_c = c^*_{\mu_I}(\mathfrak{Q}_c) - (\mathfrak{K}_c \cap c^*_{\mu_I}(\mathfrak{Q}_c))$, for some $\mu_I g$-OSGITS $\mathfrak{K}_c$. Then $c^*_{\mu_I}(\mathfrak{Q}_c) - \mathfrak{Q}_c = c^*_{\mu_I}(\mathfrak{Q}_c) \cap (\mathfrak{K}_c \cap \overline{c^*_{\mu_I}(\mathfrak{Q}_c)})^c = c^*_{\mu_I}(\mathfrak{Q}_c) \cap (\overline{\mathfrak{K}_c} \cup (c^*_{\mu_I}(\mathfrak{Q}_c))^c) = (c^*_{\mu_I}(\mathfrak{Q}_c) \cap \overline{\mathfrak{K}_c}) \cup (c^*_{\mu_I}(\mathfrak{Q}_c) \cap \overline{(c^*_{\mu_I}(\mathfrak{Q}_c))^c}) \supseteq c^*_{\mu_I}(\mathfrak{Q}_c) \cap \overline{\mathfrak{K}_c}$ because of $c^*_{\mu_I}(\mathfrak{Q}_c) \cap \overline{(c^*_{\mu_I}(\mathfrak{Q}_c))^c}$ need not be an empty in an IS. Henceforth $c^*_{\mu_I}(\mathfrak{Q}_c) - \mathfrak{Q}_c \supseteq c^*_{\mu_I}(\mathfrak{Q}_c) \cap \overline{\mathfrak{K}_c}$ for some $\mu_I g$-OSGITS $\mathfrak{K}_c$.

Theorem:2.15 Let $\mathfrak{Q}_c$ be an ISs of X. Then the upcoming statements are Identical:

(i) $\mathfrak{Q}_c \in LO(\mu_I g)$.

(ii) $\mathfrak{Q}_c = \mathfrak{K}_c \cup i^*_{\mu_I}(\mathfrak{Q}_c)$ for some $\mu_I g$-CSGITS $\mathfrak{K}_c$.

Proof: Let $\mathfrak{Q}_c \in LO(\mu_I g)$. Then $(\mathfrak{Q}_c)^c \in LC(\mu_I g)$. By theorem:2.12, $(\mathfrak{Q}_c)^c = \mathcal{U} \cap c^*_{\mu_I}(\mathfrak{Q}_c)^c$, for some $\mu_I g$-OSGITS $\mathcal{U}$. Now $((\mathfrak{Q}_c)^c)^c = \mathcal{U}^c \cup (c^*_{\mu_I}(\mathfrak{Q}_c)^c)^c \Longrightarrow \mathfrak{Q}_c = \mathfrak{K}_c \cup i^*_{\mu_I}(\mathfrak{Q}_c)$ for some $\mu_I g$-CSGITS $\mathcal{U}^c = \mathfrak{K}_c$.

Remark:2.16 The union of two μ_I-LCSGITS need not be a μ_I-LCSGITS.

Example:2.17 Let $X = \{\mathfrak{c}_1, \mathfrak{d}_1, \mathfrak{e}_1\}$ with $\mu_I = \{\mathfrak{E}, \langle X, \{\mathfrak{d}_1\}, \{\mathfrak{e}_1\}\rangle, \langle X, \{\mathfrak{c}_1\}, \{\mathfrak{d}_1\}\rangle, \langle X, \{\mathfrak{c}_1, \mathfrak{d}_1\}, \phi\rangle\}$. In this topology, $\langle X, \{\mathfrak{d}_1\}, \{\mathfrak{e}_1\}\rangle$ and $\langle X, \phi, \{\mathfrak{c}_1, \mathfrak{d}_1\}\rangle$ are μ_I-LCSGITS but their union $\langle X, \{\mathfrak{d}_1\}, \phi\rangle$ need not be a μ_I-LCSGITS.

Remark:2.18 The union of two $\mu_I g$-LCSGITS need not be a $\mu_I g$-LCSGITS.

Example:2.19 Let $X = \{\mathscr{r}_c, \mathfrak{v}_c, \mathscr{r}_c\}$ with $\mu_I = \{\mathfrak{E}, \langle X, \phi, \{\mathscr{r}_c\}\rangle, \langle X, \{\mathscr{s}_c\}, \phi\rangle, \langle X, \{\mathscr{s}_c\}, \{\mathfrak{v}_c\}\rangle, \langle X, \{\mathscr{s}_c, \mathscr{r}_c\}, \phi\rangle\}$. In this topology, $\langle X, \{\mathscr{s}_c\}, \{\mathfrak{v}_c\}\rangle$ and $\langle X, \{\mathfrak{v}_c\}, \{\mathscr{s}_c, \mathscr{r}_c\}\rangle$ are $\mu_I g$-LCSGITS but their union $\langle X, \{\mathscr{s}_c, \mathfrak{v}_c\}, \phi\rangle$ need not be a $\mu_I g$-LCSGITS.

Remark:2.20 The intersection of two $\mu_I g$-LOSGITS need not be a $\mu_I g$-LOSGITS. For example, let $\mu_I = \{\mathfrak{E}, \langle X, \phi, \{\mathscr{r}_c\}\rangle, \langle X, \{\mathscr{s}_c\}, \phi\rangle, \langle X, \{\mathscr{s}_c\}, \{\mathfrak{v}_c\}\rangle, \langle X, \{\mathscr{s}_c, \mathscr{r}_c\}, \phi\rangle\}$.Then the intersection of $\langle X, \{\mathfrak{v}_c\}, \{\mathscr{s}_c\}\rangle$ and $\langle X, \{\mathscr{s}_c, \mathscr{r}_c\}, \{\mathfrak{v}_c\}\rangle$ is $\langle X, \phi, \{\mathfrak{v}_c, \mathscr{s}_c\}\rangle$, which is not a $\mu_I g$-LOSGITS.

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Note:2.21 (i)$\mathfrak{E}$ is always in $LC(\mu_I)$ and $LC(\mu_I g)$.
(ii)Ú is always in $LO(\mu_I)$ and $LO(\mu_I g)$.

Remark:2.22 1.pre*μ_I-closed set and μ_Ig –LCSGITS doesn't depends on each other.
2.pre*μ_I-closed set and μ_Ig –LOSGITS are independent to each other.
For example, μ_I = {$\mathfrak{E}$,$\langle X,\phi,\{r_c\}\rangle$, $\langle X,\{s_c\},\phi\rangle$,$\langle X,\{s_c\},\{\mathfrak{v}_c\}\rangle$, $\langle X,\{s_c,r_c\},\phi\rangle$} be a GITS. The set of pre*μ_I-closed set is $\mathcal{O}$,Ú,$\mathfrak{E}$,$\langle X,\phi,\{s_c\}\rangle$,$\langle X,\phi,\{r_c\}\rangle$, $\langle X,\phi,\{\mathfrak{v}_c\}\rangle$,$\langle X,\phi,\{s_c,r_c\}\rangle$,$\langle X,\phi,\{\mathfrak{v}_c,r_c\}\rangle$,$\langle X,\phi,\{\mathfrak{v}_c,s_c\}\rangle$,$\langle X,\{s_c\},\{\mathfrak{v}_c\}\rangle$, $\langle X,\{s_c\},\{r_c,\mathfrak{v}_c\}\rangle$,$\langle X,\{r_c\},\phi\rangle$,$\langle X,\{r_c\},\{s_c\}\rangle$,$\langle X,\{r_c\},\{\mathfrak{v}_c\}\rangle$,$\langle X,\{r_c\},\{\mathfrak{v}_c,s_c\}\rangle$, $\langle X,\{\mathfrak{v}_c,r_c\},\phi\rangle$,$\langle X,\{\mathfrak{v}_c\},\phi\rangle$,$\langle X,\{\mathfrak{v}_c\},\{s_c\}\rangle$,$\langle X,\{\mathfrak{v}_c\},\{r_c\}\rangle$,$\langle X,\{\mathfrak{v}_c\},\{r_c,s_c\}\rangle$,$\langle X,\{\mathfrak{v}_c,r_c\},\{s_c\}\rangle$, $\langle X,\{\mathfrak{v}_c,s_c\},\phi\rangle$,$\langle X,\{\mathfrak{v}_c,s_c\},\{r_c\}\rangle$ and $LC(\mu_I g)$ = {$\mathcal{O}$,$\mathfrak{E}$,$\langle X,\phi,\{s_c\}\rangle$,$\langle X,\phi,\{r_c\}\rangle$, $\langle X,\phi,\{\mathfrak{v}_c\}\rangle$,$\langle X,\phi,\{s_c,r_c\}\rangle$,$\langle X,\phi,\{\mathfrak{v}_c,r_c\}\rangle$,$\langle X,\phi,\{\mathfrak{v}_c,s_c\}\rangle$,$\langle X,\{r_c\},\phi\rangle$,$\langle X,\{r_c\},\{s_c\}\rangle$, $\langle X,\{r_c\},\{\mathfrak{v}_c\}\rangle$,$\langle X,\{r_c\},\{\mathfrak{v}_c,s_c\}\rangle$,$\langle X,\{\mathfrak{v}_c\},\{r_c\}\rangle$,$\langle X,\{\mathfrak{v}_c\},\{r_c,s_c\}\rangle$,$\langle X,\{s_c\},\phi\rangle$,$\langle X,\{s_c\},\{\mathfrak{v}_c\}\rangle$, $\langle X,\{\mathfrak{v}_c,s_c\},\{r_c\}\rangle$,$\langle X,\{s_c,r_c\},\{\mathfrak{v}_c\}\rangle$,$\langle X,\{s_c,r_c\},\phi\rangle$}. Here $\langle X,\{\mathfrak{v}_c\},\phi\rangle$ and $\langle X,\{\mathfrak{v}_c\},\{s_c\}\rangle$ are pre*μ_I-closed set but not in $LC(\mu_I g)$. Similarly $\langle X,\{s_c\},\phi\rangle$,$\langle X,\{s_c\},\{\mathfrak{v}_c\}\rangle$ are $LC(\mu_I g)$ but not in a pre*μ_I-closed set. Therefore pre*μ_I-closed set and μ_Ig –LCSGITS are independent to each other. Also $\mathfrak{E}$ is a pre*μ_I-closed set but not a μ_Ig –LOSGITS and $\langle X,\{s_c\},\phi\rangle$ is a μ_Ig –LOSGITS but not a pre*μ_I-closed set.

Theorem:2.23 Any $\mu_I g$-DGITS in X is $\mu_I g$-LCSGITS iff it is $\mu_I g$-OSGITS.
Proof: Assume that A is $\mu_I g$-DGITS in X. By given hypothesis, $A = U \cap c^*_{\mu_I}(A) =$ U which is $\mu_I g$-OSGITS and on contrary suppose every $\mu_I g$-DGITS is $\mu_I g$-OSGITS. By using remark:2.6, $\mu_I g$-DGITS is $\mu_I g$-LCSGITS.
Theorem:2.24 1.Any $\mu_I g$-rare set in X is $\mu_I g$-LOSGITS iff it is $\mu_I g$-CSGITS.
2. Any $\mu_I g$-NDGITS in X is $\mu_I g$-LOSGITS iff it is $\mu_I g$-CSGITS.
Proof: 1.By theorem:2.15 and using every $\mu_I g$-CSGITS is a $\mu_I g$-LOSGITS, we get a result.
2. It follows from $\mu_I g$-NDGITS in X is a $\mu_I g$-rare set.

Corollary:2.25 In a submaximal space, every dense subset of X is $\mu_I g$-LCSGITS.

The following figure:1 represents a relations between $\mu_I g$-OSGITS, $\mu_I g$-LCSGITS and $\mu_I g$- DGITS, $\mu_I g$-CSGITS, $\mu_I g$-LOSGITS and $\mu_I g$-NDGITS.

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μIg-LOSGITS

Figure:1

3.$\wedge^{\star}_{LC}$- SET AND $\vee^{\star}_{LC}$- SET IN GITS

Definition:3.1 Let A and B be two intuitionistic subsets of X. The $\wedge^{\star}_{LC}(A)$ and $\vee^{\star}_{LC}(B)$ defined as follows: $\wedge^{\star}_{LC}(A) = \cap\{U : U \supseteq A, U$ is $\mu_I g$-LOSGITS$\}$ and $\vee^{\star}_{LC}(B) = \cup\{F : B \supseteq F, F$ is $\mu_I g$-LCSGITS$\}$.

Definition:3.2 An IS A in X is called a $\wedge^{\star}_{LC}$- set (resp. $\vee^{\star}_{LC}$- set) if $\wedge^{\star}_{LC}(A) = A$ (resp. $\vee^{\star}_{LC}(A) = A$).

Example:3.3 Let $X = \{\mathfrak{u}_{k}, \mathfrak{v}_{k}, \mathfrak{d}_{k}\}$ with $\mu_I = \{\mathfrak{E}, \langle X, \{\mathfrak{d}_{k}\}, \{\mathfrak{u}_{k}, \mathfrak{v}_{k}\}\rangle, \langle X, \{\mathfrak{v}_{k}\}, \{\mathfrak{d}_{k}\}\rangle, \langle X, \{\mathfrak{v}_{k}, \mathfrak{d}_{k}\}, \phi\rangle, \langle X, \{\mathfrak{d}_{k}\}, \phi\rangle, \langle X, \phi, \{\mathfrak{d}_{k}\}\rangle, \langle X, \{\mathfrak{v}_{k}\}, \{\mathfrak{u}_{k}\}\rangle, \langle X, \{\mathfrak{v}_{k}\}, \phi\rangle, \langle X, \{\mathfrak{v}_{k}, \mathfrak{d}_{k}\}, \{\mathfrak{u}_{k}\}\rangle\}$.Then $\wedge^{\star}_{LC}(\langle X, \{\mathfrak{v}_{k}\}, \{\mathfrak{u}_{k}\}\rangle) = \langle X, \{\mathfrak{v}_{k}\}, \phi\rangle$ and $\vee^{\star}_{LC}(\langle X, \{\mathfrak{u}_{k}\}, \phi\rangle) = \mathcal{O}$.

$\wedge^{\star}_{LC}$- set $= \{\mathcal{O}, \langle X, \phi, \{\mathfrak{v}_{k}\}\rangle, \langle X, \phi, \{\mathfrak{d}_{k}\}\rangle, \langle X, \phi, \{\mathfrak{v}_{k}, \mathfrak{d}_{k}\}\rangle, \langle X, \{\mathfrak{u}_{k}\}, \phi\rangle, \langle X, \{\mathfrak{u}_{k}\}, \{\mathfrak{v}_{k}\}\rangle, \langle X, \{\mathfrak{u}_{k}\}, \{\mathfrak{d}_{k}\}\rangle, \langle X, \{\mathfrak{u}_{k}\}, \{\mathfrak{y}_{k}, \mathfrak{d}_{k}\}\rangle, \langle X, \{\mathfrak{v}_{k}\}, \phi\rangle, \langle X, \{\mathfrak{v}_{k}\}, \{\mathfrak{d}_{k}\}\rangle, \langle X, \{\mathfrak{d}_{k}\}, \phi\rangle, \langle X, \{\mathfrak{d}_{k}\}, \{\mathfrak{v}_{k}\}\rangle, \langle X, \{\mathfrak{v}_{k}, \mathfrak{u}_{k}\}, \phi\rangle, \langle X, \{\mathfrak{v}_{k}, \mathfrak{u}_{k}\}, \{\mathfrak{d}_{k}\}\rangle, \langle X, \{\mathfrak{v}_{k}, \mathfrak{d}_{k}\}, \phi\rangle, \langle X, \{\mathfrak{u}_{k}, \mathfrak{d}_{k}\}, \phi\rangle, \langle X, \{\mathfrak{u}_{k}, \mathfrak{d}_{k}\}, \{\mathfrak{v}_{k}\}\rangle, \acute{U}\}$.

$\vee^{\star}_{LC}$- set $= \{\mathcal{O}, \langle X, \phi, \{\mathfrak{u}_{k}\}\rangle, \langle X, \phi, \{\mathfrak{v}_{k}\}\rangle, \langle X, \phi, \{\mathfrak{d}_{k}\}\rangle, \langle X, \phi, \{\mathfrak{u}_{k}, \mathfrak{v}_{k}\}\rangle, \langle X, \phi, \{\mathfrak{v}_{k}, \mathfrak{d}_{k}\}\rangle, \langle X, \phi, \{\mathfrak{u}_{k}, \mathfrak{d}_{k}\}\rangle, \langle X, \{\mathfrak{v}_{k}\}, \phi\rangle, \langle X, \{\mathfrak{v}_{k}\}, \{\mathfrak{u}_{k}\}\rangle, \langle X, \{\mathfrak{v}_{k}\}, \{\mathfrak{d}_{k}\}\rangle, \langle X, \{\mathfrak{v}_{k}\}, \{\mathfrak{u}_{k}, \mathfrak{d}_{k}\}\rangle, \langle X, \{\mathfrak{d}_{k}\}, \phi\rangle, \langle X, \{\mathfrak{d}_{k}\}, \{\mathfrak{u}_{k}\}\rangle, \mathfrak{E}, \langle X, \{\mathfrak{d}_{k}\}, \{\mathfrak{v}_{k}\}\rangle, \langle X, \{\mathfrak{d}_{k}\}, \{\mathfrak{u}_{k}, \mathfrak{v}_{k}\}\rangle, \mathfrak{E}, \langle X, \{\mathfrak{v}_{k}, \mathfrak{d}_{k}\}, \phi\rangle, \langle X, \{\mathfrak{v}_{k}, \mathfrak{d}_{k}\}, \{\mathfrak{u}_{k}\}\rangle\}$.

Note:3.4 (i)$\wedge^{\star}_{LC}(\mathfrak{E}) \neq \mathfrak{E}$; $\vee^{\star}_{LC}(\mathfrak{E}) = \mathfrak{E}$.

(ii)$\wedge^{\star}_{LC}(\acute{U}) = \acute{U}$; $\vee^{\star}_{LC}(\acute{U}) \neq \acute{U}$.

Theorem:3.5 Let $\mathcal{N}_{k}, \mathcal{O}_{k} \in X$. Then the following are meets up with the condition.

(i) $\mathcal{N}_{k} \subseteq \wedge_{LC}^{\star}(\mathcal{N}_{k})$.

(ii) If $\mathcal{N}_{k} \subseteq \mathcal{O}_{k}$ then $\wedge_{LC}^{\star}(\mathcal{N}_{k}) \subseteq \wedge_{LC}^{\star}(\mathcal{O}_{k})$.

(iii) If $\mathcal{N}_{k}$ is $\mu_I g$-LOSGITS then $\mathcal{N}_{k} = \wedge_{LC}^{\star}(\mathcal{N}_{k})$ (ie, every $\mu_I g$-LOSGITS is a $\wedge_{LC}^{\star}$- set).

(iv) $\wedge_{LC}^{\star}(\wedge_{LC}^{\star}(\mathcal{N}_{k})) = \wedge_{LC}^{\star}(\mathcal{N}_{k})$.

(v) $\wedge_{LC}^{\star}(\mathcal{N}_{k} \cup \mathcal{O}_{k}) = \wedge_{LC}^{\star}(\mathcal{N}_{k}) \cup \wedge_{LC}^{\star}(\mathcal{O}_{k})$.

(vi) $\wedge_{LC}^{\star}(\mathcal{N}_{k} \cap \mathcal{O}_{k}) \subseteq \wedge_{LC}^{\star}(\mathcal{N}_{k}) \cap \wedge_{LC}^{\star}(\mathcal{O}_{k})$.

(vii) $\wedge_{LC}^{\star}(\overline{\mathcal{N}_{k}}) = \overline{\vee_{LC}^{*}(\mathcal{N}_{k})}$.

Proof:(i) By the definition of $\wedge_{LC}^{\star}(\mathcal{N}_{k})$, we get $\mathcal{N}_{k} \subseteq \wedge_{LC}^{\star}(\mathcal{N}_{k})$.

(ii) Suppose $x \notin \wedge_{LC}^{\star}(\mathcal{O}_{k})$. Then there exits an $\mu_I g$-LOSGITS G such that $G \supseteq \mathcal{O}_{k}$ with $x \notin G$. Since $\mathcal{N}_{k} \subseteq \mathcal{O}_{k}$, $\mathcal{N}_{k} \subseteq G$ and so $x \notin \wedge_{LC}^{\star}(\mathcal{N}_{k})$. Hence $\wedge_{LC}^{\star}(\mathcal{N}_{k}) \subseteq \wedge_{LC}^{\star}(\mathcal{O}_{k})$.

(iii) From (i) and the definition:3.1, we have $\mathcal{N}_{k} = \wedge_{LC}^{\star}(\mathcal{N}_{k})$ if $\mathcal{N}_{k}$ is $\mu_I g$-LOSGITS.

(iv) Follows from (i) and (ii), $\wedge_{LC}^{\star}(\mathcal{N}_{k}) \subseteq \wedge_{LC}^{\star}(\wedge_{LC}^{\star}(\mathcal{N}_{k}))$.If $x \notin \wedge_{LC}^{\star}(\mathcal{N}_{k})$ then there exits an $\mu_I g$-LOSGITS G such that $G \supseteq \mathcal{N}_{k}$ and $x \notin G$. Hence $\wedge_{LC}^{\star}(\mathcal{N}_{k}) \subseteq G \Longrightarrow x \notin \wedge_{LC}^{\star}(\wedge_{LC}^{\star}(\mathcal{N}_{k}))$. Henceforth $\wedge_{LC}^{\star}(\wedge_{LC}^{\star}(\mathcal{N}_{k})) = \wedge_{LC}^{\star}(\mathcal{N}_{k})$.

(v) WKT $\mathcal{N}_{k} \subseteq \mathcal{N}_{k} \cup \mathcal{O}_{k}$ and $\mathcal{O}_{k} \subseteq \mathcal{N}_{k} \cup \mathcal{O}_{k}$, which gives $\wedge_{LC}^{\star}(\mathcal{N}_{k}) \subseteq \wedge_{LC}^{\star}(\mathcal{N}_{k} \cup \mathcal{O}_{k})$ and $\wedge_{LC}^{\star}(\mathcal{O}_{k}) \subseteq \wedge_{LC}^{\star}(\mathcal{N}_{k} \cup \mathcal{O}_{k}) \Longrightarrow \wedge_{LC}^{\star}(\mathcal{N}_{k}) \cup \wedge_{LC}^{\star}(\mathcal{O}_{k}) \subseteq \wedge_{LC}^{\star}(\mathcal{N}_{k} \cup \mathcal{O}_{k})$. Let $x \in \wedge_{LC}^{\star}(\mathcal{N}_{k} \cup \mathcal{O}_{k})$. Then $x \in \cap \{U: U \supseteq \mathcal{N}_{k} \cup \mathcal{O}_{k}, U$ is $\mu_I g$-LOSGITS$\} \Longrightarrow x \in \cap \{U: U \supseteq \mathcal{N}_{k}, U$ is $\mu_I g$-LOSGITS$\}$ or $x \in \cap \{U: U \supseteq \mathcal{O}_{k}, U$ is $\mu_I g$-LOSGITS$\} \Longrightarrow x \in \wedge_{LC}^{\star}(\mathcal{N}_{k})$ or $x \in \wedge_{LC}^{\star}(\mathcal{O}_{k}) \Longrightarrow x \in \wedge_{LC}^{\star}(\mathcal{N}_{k}) \cup \wedge_{LC}^{\star}(\mathcal{O}_{k}) \Longrightarrow \wedge_{LC}^{\star}(\mathcal{N}_{k} \cup \mathcal{O}_{k}) \subseteq \wedge_{LC}^{\star}(\mathcal{N}_{k}) \cup \wedge_{LC}^{\star}(\mathcal{O}_{k})$ and thus $\wedge_{LC}^{\star}(\mathcal{N}_{k} \cup B) = \wedge_{LC}^{\star}(\mathcal{N}_{k}) \cup \wedge_{LC}^{\star}(B)$.

(vi) WKT $\mathcal{N}_{k} \cap B \subseteq \mathcal{N}_{k}$ and $\mathcal{N}_{k} \cap \mathcal{O}_{k} \subseteq \mathcal{O}_{k} \Longrightarrow \wedge_{LC}^{\star}(\mathcal{N}_{k} \cap \mathcal{O}_{k}) \subseteq \wedge_{LC}^{\star}(\mathcal{N}_{k})$ and $\wedge_{LC}^{\star}(\mathcal{N}_{k} \cap \mathcal{O}_{k}) \subseteq \wedge_{LC}^{\star}(\mathcal{O}_{k}) \Longrightarrow \wedge_{LC}^{\star}(\mathcal{N}_{k} \cap \mathcal{O}_{k}) \subseteq \wedge_{LC}^{\star}(\mathcal{N}_{k}) \cap \wedge_{LC}^{\star}(\mathcal{O}_{k})$.

(vii) $\overline{\vee_{LC}^{*}(\mathcal{N}_{k})} = \cap \{\overline{U}: \overline{U} \supseteq \overline{\mathcal{N}_{k}}, \overline{U}$ is $\mu_I g$-LOSGITS$\} = \wedge_{LC}^{\star}(\overline{\mathcal{N}_{k}})$.

Theorem:3.6 Let $\mathcal{N}_{k}, \mathcal{O}_{k} \in X$. Then the ensuing statement meets up with the condition.

(i) $\vee_{LC}^{\star}(\mathcal{N}_{k}) \subseteq \mathcal{N}_{k}$.

(ii) If $\mathcal{N}_{k} \subseteq \mathcal{O}_{k}$ then $\vee_{LC}^{\star}(\mathcal{N}_{k}) \subseteq \vee_{LC}^{\star}(\mathcal{O}_{k})$.

(iii) If $\mathcal{N}_{k}$ is $\mu_I g$-LCSGITS then $\mathcal{N}_{k} = \vee_{LC}^{\star}(\mathcal{N}_{k})$ (ie, every $\mu_I g$-LCSGITS is a $\vee_{LC}^{\star}$-set).

(iv) $\vee_{LC}^{\star}(\vee_{LC}^{\star}(\mathcal{N}_{k})) = \vee_{LC}^{\star}(\mathcal{N}_{k})$.

(v) $\vee_{LC}^{\star}(\mathcal{N}_{k} \cup \mathcal{O}_{k}) \supseteq \vee_{LC}^{\star}(\mathcal{N}_{k}) \cup \vee_{LC}^{\star}(\mathcal{O}_{k})$.

(vi) $\vee_{LC}^{\star}(\mathcal{N}_{k} \cap \mathcal{O}_{k}) = \vee_{LC}^{\star}(\mathcal{N}_{k}) \cap \vee_{LC}^{\star}(\mathcal{O}_{k})$.

Proof: As theorem:3.5, (i),(ii),(iii),(iv),(v) and (vi) true.

Theorem:3.7 1. If A is $\mu_I g$-CSGITS then A is $\wedge^{\star}_{LC}$- set but converse fails.

2.If A is $\mu_I g$-OSGITS then A is$\vee^{\star}_{LC}$- set but converse fails.

Proof: 1.WKT every $\mu_I g$-CSGITS is a $\mu_I g$-LOSGITS and by theorem:3.5, we get every $\mu_I g$-CSGITS is a $\wedge^{\star}_{LC}$- set. But reverse fails, for example:3.3, $\langle X, \{\mathfrak{y}_1\}, \phi\rangle$ is a $\wedge^{\star}_{LC}$- set but which is not a $\mu_I g$-CSGITS.

2. Using remark:2.6 and theorem:3.6, we get every $\mu_I g$-OSGITS is a $\vee^{\star}_{LC}$-set. For example:3.3,$\langle X, \phi, \{\mathfrak{y}_1\}\rangle$ is a $\vee^{\star}_{LC}$-set but which is not a $\mu_I g$-OSGITS.

Theorem:3.8 In X, the following statements are hold:

(i)Ú is always in $\wedge^{\star}_{LC}$- set.

(ii) $\mathfrak{E}$ is always in $\vee^{\star}_{LC}$- set.

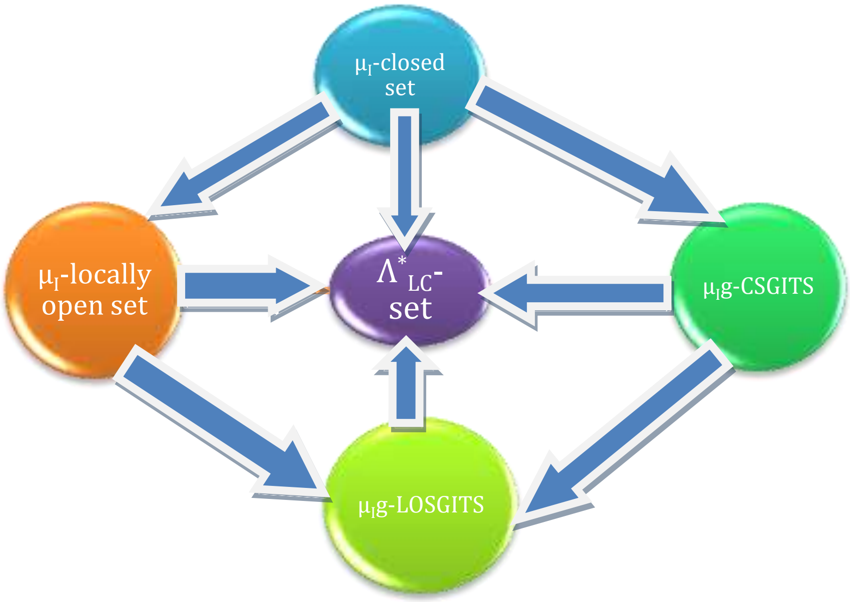

Figure:2

The above figure:2 indicates, the relations between $\mu_I g$-CSGITS,$\mu_I g$-LOSGITS,μ_I-closed set, μ_I-locally open set, and $\wedge^{\star}_{LC}$- set. The reverse of figure:3.9 need not be true.

CONCLUSION: For this article, we establish the new notions like $\mu_I g$-LOSGITS, $\mu_I g$-LCSGITS, $\wedge^{\star}_{LC}$- set, $\vee^{\star}_{LC}$- set and we have done with the generalizations of their characters. Also discussed how to correlate them with other sets and spaces .Also we explained them in detail with suitable examples. There is no relations between pre*μ_I-closed set and others (ie,$\mu_I g$-LOSGITS, $\mu_I g$-LCSGITS). Hence we conclude that, they are independent to each other.

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