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Heat Equation Obtained By q-Difference Operator with Two Variable

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Abstract

The authors discuss the q heat equation model by partial difference operator. In this chapter simple hotness condition got by q difference operator with two variable based on their answer we get numerous theorem and corollaries.

Key words: Partial difference operator, q-difference operator, Difference Equation and Heat Equation

AMS classification: 39A70, 39A10, 47B39, 80A20.

1 Introduction

In 1984, [5] Jerzy popenda presented the distinctive administrator ∇_α characterized on $h(z)$ as $\nabla_\alpha h(z) = h(z+1) - h(z)$. In 1989 [8] miller and Rose presented the dicrete simple of the Riemam. Liouvilla partial derivatie the demonstrated since properties of the opposite fragmentary difference operator $\nabla_h^{-\nu}$ [3,4].In 2014 G.Britto Antony Xavier [2] I have presented q-difference operator characterized as $\nabla_q h(z) = h(qz) - h(z)$ [6] for the genuine esteemed capacity $h(z), q \in (0, \infty)$ and got limited series answer for the comparing generalize q-difference equation $\nabla_q h(z) = h(z)$ consider the appropriation of hotness through this composed of a homogenous material. Let $r_1, r_2 \cdots r_z$ be z equidistant focuses on the bar. In John Borg.s [3] Let $R_i(n)$ be the temperature at time $r_m = (r)_m$ at the point $z_i, 1 \leq i \leq z$. Mean the temperatures at the left and the right finish of the bar at time $R_0(m), R_{k+1}(m)$.

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2 Solution of Heat Equation

Definition 2.1 [2] 'Consider the temperature of an extremely long bor $h(z,r)$. Accept the pole is entirely long to the point that it can laid on top of the set R of genuine number. Let $h(z,r)$ be the temperature at the on going time r_1 and position r_2 of the rod.'

Theorem 2.2 Let $\alpha \in R, z, r$ are parameter and q_1, q_2 are difference operator. Then we have

$$h(z,r) = \frac{1}{1-2\alpha}\left[h(z,rq_2) - \alpha\left(\frac{z}{q_1},r\right) - \alpha h(zq_1,r)\right]$$

proof:

$$\underset{1\wedge q_2}{\nabla} h(z,r) = h(z,rq_2) - h(z,r)$$

$$\underset{q_1^{-1}\wedge 1}{\nabla} h(z,r) = h\left(\frac{z}{q_1},r\right) - h(z,r)$$

$$\underset{q_1^{-1}\wedge 1}{\nabla} h(z,r) = h(zq_2,r) - h(z,r)$$

$$\underset{1\wedge q_2}{\nabla} h(z,r) = \alpha\left[\underset{q_1^{-1}\wedge 1}{\nabla} h(z,r) + \underset{q_1\wedge 1}{\nabla} h(z,r)\right]$$

The values are given by.

$$h(z,rq_2) - h(z,r) = \alpha\left(h\left(\frac{z}{q_1},r\right) - h(z,r)\right) + \alpha(h(zq_1,r) - h(z,r)) \tag{1}$$

$$h(z,rq_2) - h(z,r) = \alpha h\left(\frac{z}{q_1},r\right) - \alpha h(z,r) + \alpha h(zq_1,r) - \alpha h(z,r) \tag{2}$$

$$h(z,r) - 2\alpha h(z,r) = h(z,rq_2) - \alpha h\left(\frac{z}{q_1},r\right) - \alpha h(zq_1,r) \tag{3}$$

$$h(z,r) = \frac{1}{1-2\alpha}\left[h(z,rq_2) - \alpha\left(\frac{z}{q_1},t\right) - \alpha h(zq_1,r)\right] \tag{4}$$

Corollary 2.3 Let the equation is $h(z,r) = \frac{1}{1-2\alpha}\left[h(z,rq_2) - \alpha\left(\frac{z}{q_1},r\right) - \alpha h(zq_1,r)\right)$

$, \alpha \neq \left(\frac{1}{2}\right)$, then we have $\alpha = \dfrac{h(z,rq_2)-h(z,r)}{h\left(\frac{z}{q_1},r\right)-2h(z,r)+h(zq_1,r)}$

proof:

$$h(z,rq_2)-h(z,r)=\alpha h\left(\frac{z}{q_1},r\right)-\alpha h(z,r)+\alpha h(zq_1,r)-\alpha h(z,r)$$
$$h(z,rq_2)-h(z,r)=\alpha h\left(\frac{z}{q_1},r\right)-2\alpha h(z,r)+\alpha h(z,r)$$
$$\alpha=\frac{h(z,rq_2)-h(z,r)}{h\left(\frac{z}{q_1},r\right)-2h(z,r)+h(zq_1,r)}$$

Example : Taking $z=2$, $r=3$, $q_1=7$, $q_2=9$

$$\alpha=\frac{h(z,rq_2)-h(z,r)}{h\left(\frac{z}{q_1},r\right)-2h(z,r)+h(zq_1,r)}$$
$$=\frac{2,3(9)-2,3}{\left(\frac{2}{7},3\right)}-2(2,3)+2(7),3$$
$$\alpha=\frac{336}{216}$$
$$h(z,r)=\frac{1}{1-2\alpha}\left[h(z,rq_2)-\alpha\left(\frac{z}{q_1},r\right)-\alpha h(zq_1,r)\right]$$
$$u(2,3)=\frac{1}{1-2\left(\frac{336}{216}\right)}\left[2,3(9)-\frac{336}{216}\left(\frac{2}{7},3\right)-\frac{336}{216}(2(7),3)\right]$$
$$6=\frac{1}{1-2\left(\frac{14}{9}\right)}\left[54-\frac{14}{9}\left(\frac{6}{7}\right)-\frac{14}{9}(42)\right]$$
$$6=6$$

Theorem 2.4 Let $\alpha \in R$, z , r are parameter and q_1 , q_2 are difference operator then we have.

$$h(z,r)=\left(\frac{1}{1-2\alpha}\right)^r h\left(x,rq_2^r\right)-\sum_{r=1}^{r}\left(\frac{1}{1-2\alpha}\right)^r \alpha h\left(\frac{z}{q_1},rq_2^{r-1}\right)$$
$$-\sum_{r=1}^{r}\left(\frac{1}{1-2\alpha}\right)^r \alpha h(zq_1,rq_2^{r-1})$$

proof:

Replace r by rq_2 in eqn (4)

$$h(z,rq_2)=\frac{1}{1-2\alpha}\left[h(z,rq_2^2)-\alpha h\left(\frac{z}{q_1},rq_2\right)-\alpha h(zq_1,rq_2)\right]$$

$$h(z,rq_2)=\left(\frac{1}{1-2\alpha}\right)h(z,rq_2^2)-\left(\frac{1}{1-2\alpha}\right)\alpha h\left(\frac{z}{q_1},rq_2\right)-\left(\frac{1}{1-2\alpha}\right)\alpha h(zq_1,rq_2) \tag{5}$$

Substitute equation (5) in equation (4)

$$h(z,r)=\left(\frac{1}{1-2\alpha}\right)^2 h(z,rq_2^2)-\left(\frac{1}{1-2\alpha}\right)^2\alpha h\left(\frac{z}{q_1},rq_2\right)$$
$$-\left(\frac{1}{1-2\alpha}\right)^2\alpha h(zq_1,tq_2)-\left(\frac{1}{1-2\alpha}\right)\alpha h\left(\frac{z}{q_1},r\right)-\left(\frac{1}{1-2\alpha}\right)\alpha h(zq_1,r) \tag{6}$$

Replacing rq_2 by rq_2^2 in eqn (5)

$$h(z,z_2q_2^2)=\left(\frac{1}{1-2\alpha}\right)h(z,rq_2^3)-\left(\frac{1}{1-2\alpha}\right)\alpha h\left(\frac{z}{q_1},rq_2^2\right)-\left(\frac{1}{1-2\alpha}\right)\alpha h(zq_1,rq_2^2) \tag{7}$$

Substitute equation (7) in equation (6)

$$h(z,r)=\left(\frac{1}{1-2\alpha}\right)^3 h(z,rq_2^3)-\left(\frac{1}{1-2\alpha}\right)^3\alpha h\left(\frac{z}{q_1},rq_2^2\right)$$
$$-\left(\frac{1}{1-2\alpha}\right)^3\alpha h\left(\frac{z}{q_1},rq_2^2\right)-\left(\frac{1}{1-2\alpha}\right)^2\alpha h(\frac{z}{q_1},rq_2)$$
$$-\left(\frac{1}{1-2\alpha}\right)^2\alpha h\,(zq_1,rq_2)-\left(\frac{1}{1-2\alpha}\right)\alpha h(\frac{z}{q_1},r)-\left(\frac{1}{1-2\alpha}\right)\alpha h(zq_1,r) \tag{8}$$

Replace rq_2^2 by rq_2^3 in eqn (7)

$$h(z,rq_2^3)=\left(\frac{1}{1-2\alpha}\right)h(z,rq_2^4)-\left(\frac{1}{1-2\alpha}\right)\alpha h\left(\frac{z}{q_1},rq_2^3\right)-\left(\frac{1}{1-2\alpha}\right)\alpha h(zq_1,rq_2^3) \tag{9}$$

Substitute equation (9) in equation (8)

$$h(z,r)=\left(\frac{1}{1-2\alpha}\right)^4 h(z,rq_2^4)-\left(\frac{1}{1-2\alpha}\right)^4\alpha h\left(\frac{z}{q_1},rq_2^3\right)$$
$$-\left(\frac{1}{1-2\alpha}\right)^4\alpha h(zq_1,rq_2^3)-\left(\frac{1}{1-2\alpha}\right)^3\alpha h\left(\frac{z}{q_1},rq_2^2\right)$$

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$$-\left(\frac{1}{1-2\alpha}\right)^3\alpha h(zq_1,rq_2^2)-\left(\frac{1}{1-2\alpha}\right)^2\alpha h\left(\frac{z}{q_1},rq_2\right)$$

$$-\left(\frac{1}{1-2\alpha}\right)^2\alpha h(zq_1,rq_2)-\left(\frac{1}{1-2\alpha}\right)^2\alpha h\left(\frac{z}{q_1},r\right)-\left(\frac{1}{1-2\alpha}\right)\alpha h(zq_1,r) \quad (10)$$

Replace rq_2^3 by rq_2^4 in eqn (9)

$$h(z,rq_2^4)=\left(\frac{1}{1-2\alpha}\right)h(z,rq_2^5)-\left(\frac{1}{1-2\alpha}\right)\alpha h\left(\frac{z}{q_1},rq_2^4\right)-\left(\frac{1}{1-2\alpha}\right)\alpha h(zq_1,rq_2^4) \quad (11)$$

Substitute equation (11) in equation (10)

$$\begin{aligned}h(z,r)=&\left(\frac{1}{1-2\alpha}\right)^5 h(z,rq_2^5)-\left(\frac{1}{1-2\alpha}\right)^5\alpha h\left(\frac{z}{q_1},rq_2^4\right)\\
&-\left(\frac{1}{1-2\alpha}\right)^5\alpha h(zq_1,rq_2^4)-\left(\frac{1}{1-2\alpha}\right)^4\alpha h\left(\frac{z}{q_1},rq_2^3\right)\\
&-\left(\frac{1}{1-2\alpha}\right)^4\alpha h(zq_1,rq_2^3)-\left(\frac{1}{1-2\alpha}\right)^3\alpha h\left(\frac{z}{q_1},rq_2^2\right)\\
&-\left(\frac{1}{1-2\alpha}\right)^2\alpha h\left(\frac{z}{q_1},rq_2\right)-\left(\frac{1}{1-2\alpha}\right)^2\alpha h(zq_1,rq_2)\\
&-\left(\frac{1}{1-2\alpha}\right)\alpha h(zq_1,r)-\left(\frac{1}{1-2\alpha}\right)\alpha h(zq_1,r)\end{aligned} \quad (12)$$

Replace rq_2^4 by rq_2^5 in eqn (11)

$$h(z,rq_2^5)=\left(\frac{1}{1-2\alpha}\right)h(z,rq_2^6)-\left(\frac{1}{1-2\alpha}\right)-\alpha h\left(\frac{z}{q_1},rq_2^5\right)-\left(\frac{1}{1-2\alpha}\right)\alpha h(zq_1,rq_2^4 \quad (13)$$

Substitute equation (13) in equation (12)

$$\begin{aligned}h(z,r)=&\left(\frac{1}{1-2\alpha}\right)^6 h(z,rq_2^6)-\left(\frac{1}{1-2\alpha}\right)^6\alpha h\left(\frac{z}{q_1},rq_2^5\right)\\
&-\left(\frac{1}{1-2\alpha}\right)^6\alpha h(zq_1,rq_2^5)-\left(\frac{1}{1-2\alpha}\right)^5\alpha h\left(\frac{z}{q_1},rq_2^4\right)\\
&-\left(\frac{1}{1-2\alpha}\right)^5\alpha h(zq_1,rq_2^4)-\left(\frac{1}{1-2\alpha}\right)^4\alpha h\left(\frac{z}{q_1},rq_2^3\right)\\
&-\left(\frac{1}{1-2\alpha}\right)^4\alpha h\left(\frac{z}{q_1},rq_2^3\right)-\left(\frac{1}{1-2\alpha}\right)^3\alpha h(zq_1,rq_2^2)\end{aligned}$$

$$-\left(\frac{1}{1-2\alpha}\right)^{3}\alpha h(zq_1,rq_2^2)-\left(\frac{1}{1-2\alpha}\right)^{2}\alpha h\left(\frac{z}{q_1},rq_2\right)$$

$$-\left(\frac{1}{1-2\alpha}\right)^{2}\alpha h(zq_1,rq_2)-\left(\frac{1}{1-2\alpha}\right)\alpha h\left(\frac{z}{q_1},r\right)-\left(\frac{1}{1-2\alpha}\right)\alpha h(zq_1,r) \qquad (14)$$

Similarly the we have.

$$h(z,r)=\left(\frac{1}{1-2\alpha}\right)^{m}h(zq_1,rq_2^m)-\sum_{r=1}^{m}\left(\frac{1}{1-2\alpha}\right)^{r}\alpha h\left(\frac{z}{q_1},rq_2^{r-1}\right)$$
$$-\sum_{r=1}^{m}\left(\frac{1}{1-2\alpha}\right)^{r}\alpha h(zq_1,rq_2^{r-1})$$

Example : Taking $z=12$, $r=10$, $q_1=7$, $q_2=8$

$$\alpha=\frac{h(z,rq_2)-h(z,r)}{g\left(\frac{z}{q_1},r\right)-2h(z,r)+h(zq_1,r)}$$
$$=\frac{10,6(7))-(10,6)}{\left(\frac{10}{8},6\right)-2(10,6)+(10(8),6)}$$
$$\alpha=\frac{48}{49}$$
$$h(z,rq_2)=\left(\frac{1}{1-2\alpha}\right)-h(z,rq_2^2)-\alpha h\left(\frac{z}{q_1},rq_2\right)-\alpha h(zq_1,rq_2)$$
$$(10,6(7))=\frac{1}{1-2\left(\frac{48}{49}\right)}\left[10,6(7^2)-\frac{48}{49}\left(\frac{10}{8},6(7)\right)-\frac{48}{49}(10(8),6(7))\right]$$
$$=\frac{1}{-\frac{47}{49}}\left[2940-\frac{360}{7}-\frac{161280}{49}\right]$$
$$420=420$$

3 Conclusion

In the above concentrate on the two boundary heat equation. This equation gives the temperature and time. Likewise we inferred a few theorems and examples.

References

[1] Ablowitz MJ and Ladik JF, On the solution fo a class of nonlinear partial difference equations,Studies Appl. Math., 57, 1-12(1977).

[2] Britto Antony Xavier G, Gerly TG and Nasira Begum H, Fine series of Polynominals and Polynomial Factorials arising from Generalized q-Difference operator ,Far East Journal of Mathematical Sciences, 94(1),47-63(2014).

[3] Britto Antony Xavier G, John Borg S and Meganathan M, Discrete Heat Equation Model with Shift Values, Applied Mathematics,8,1343-1350(2017).

[4] Bastos NRO, Ferreira RAC and Torres DFM, Discrete-Time Fractional Variational Problems, Signal Processing, vol.91, no.3, pp. 513-524, 2011.

[5] Jerzy Proprnda and Blazej Szmanda, On the oscillation of solutions of certain Difference Equation, Demonstration Mathematica, XVII (1), 153-164(1984).

[6] Maria Susai Manuel M, Chandrasekar V and Britto Antony Xavier G, Solutions and Applications of Certain class of Difference Equation, International journal of Applied mathematics, 24(6),943-945(2011).

[7] Maria Susai Manuel M, Chandrasekar V, Britto Antony Xavier Gand Pugalarau R, Theory and application of the Generalized Difference operator of the 'n' kind (Part 1) Demonstration mathematics, 45(1), 95-106(2012).