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Interval-Valued Pythagorean Fuzzy Perfectly Weakly Generalized Continuous Mappings

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Abstract

The purpose of this paper is to introduce and study the concepts of Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mappings and Interval-valued Pythagorean fuzzy perfectly weakly generalized open mappings in Interval-valued Pythagorean fuzzy topological space. Some of their properties are explored.

Key words: Interval-valued Pythagorean fuzzy topology, Interval-valued Pythagorean fuzzy weakly generalized closed set, Interval-valued Pythagorean fuzzy weakly generalized open set, Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mappings, Interval-valued Pythagorean fuzzy perfectly weakly generalized open mappings.

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1. Introduction

Fuzzy set(FS), proposed by Zadeh [18]in 1965, as a framework to encounter uncertainty, vagueness and partial truth , represents a degree of membership for each member of the universe of discourse to a subset of it. Later on, fuzzy topology was introduced by Chang [2]in 1967.By adding the degree of non-membership to Fuzzy Set, Atanassov[1]proposed Interval-valued fuzzy set (IVFS) in 1983 which looks more accurately to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. After this, there have been several generalizations of notions of fuzzy sets and fuzzy topology. In the last few years various concepts in fuzzy were extended to Interval-valued fuzzy sets. In 1997, Coker [3]introduced the concept of Interval-valued fuzzy topological space.

In this paper, we introduce the notion of Interval-valued fuzzy perfectly weakly generalized continuous mappings and Interval-valued fuzzy perfectly weakly

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generalized open mappings in Interval-valued fuzzy topological space and study some of their properties. We provide some characterizations of Interval-valued fuzzy perfectly weakly generalized continuous mappings and establish the relationships with other classes of early defined forms of Interval-valued fuzzy mappings.

2.Preliminaries:

Definition 2.1 [15, 16] Let X be non-empty set and I the unit interval [0,1]. A PF set S is an object having the form $P = \{x, \mu_p(x), \nu_p(x) : x \in X\}$ where the function $\mu_p : X \to [0,1]$ and $\nu_p : X \to [0,1]$ denote respectively the degree of membership and degree of non-membership of each element $x \in X$ to the set P, and $0 \leq (\mu_p(x))^2$, $(\nu_p(x))^2 \leq 1$, for each $x \in X$.

Definition 2.2 [1] Let A and B be IFS's of the forms $A = \{[x, \mu_A(x), \nu_A(x)]/x \in X\}$ and $B = \{[x, \mu_B(x), \nu_B(x)]/x \in X\}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for all $x \in X$.
(b) A = B if and only if $A \subseteq B$ and $B \subseteq A$.
(c) $A^c = \{[x, \nu_A(x), \mu_A(x)]/x \in X\}$.
(d) $A \cap B = \{[x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)]/x \in X\}$.
(e) $A \cup B = \{[x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)]/x \in X\}$.

Definition 2.3 [4] An Intuitionistic fuzzy topology (IFT in short) on a non empty X is a family τ of IFS in X satisfying the following axioms:

(a) $0, 1 \in \tau$,
(b) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
(c) $\bigcup G_i \in \tau$, for any arbitrary family $\{G_i/i \in J\} \subseteq \tau$.

The pair (X, τ) is called an Interval-valued fuzzy topological space (IFTS in short) and any IFS in τ is known as an Intuitionistic fuzzy open set (IFOS for short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an Intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.4 [3] Let (X, τ) be an IVPFTS and $A = \{x, \mu_1(x), v_1(x)/x \in X\}$ be an IVPFs in X. Then the interior and closure of A are defined as
IVPF int(A)=U{G/G is an IVPFOS in X and G⊆A.

IVPF cl(A)=n{K/K is an IVPFCS in X and A⊆K}.
For any IVPFs A in (X,τ) we have,
IVPF $cl(A^c) = (IVPF \ \ int(A))^c$ and IVPF $int(A^c) = (IVPF \ \ cl(A)^c)$.

Definition 2.5 An IFS $A = \{[x, \mu_A(x), \nu_A(x)]/x \in X\}$ in an IFTS (X,τ) is called an

(a) intuitionistic fuzzy semi closed set (IFSCS) if int(cl (A)) $\subseteq A$[4].
(b) intuitionistic fuzzy α-closed set (IF αCS) if cl(int(cl(A))) $\subseteq A$[4].
(c) intuitionistic fuzzy pre-closed set (IFPCS) if cl(int(A)) $\subseteq A$ [4].
(d) intuitionistic fuzzy regular closed set (IFRCS) if cl(int(A))=A[4].
(e) intuitionistic fuzzy generalized closed set (IFGCS) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS[4].
(f) intuitionistic fuzzy generalized semi closed set (IFGSCS) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS [13].
(g) intuitionistic fuzzy α generalized closed set (IFαGCS) if $\alpha cl(A) \subseteq U$,whenever $A \subseteq U$ and U is an IFOS[12].

An IFS A is called intuitionistic fuzzy semi open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy α generalized open set (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS and IFαGOS) if the complement of A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS and IFαGCS respectively.

Definition 2.6 Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be

(a) intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B)\square IFO(X)$ for every $B\square\sigma$[4],
(b) intuitionistic fuzzy α continuous(IFα continuous in short) if $f^{-1}(B)\square IF\alpha O(X)$ for every $B\square\sigma$[6],
(c) intuitionistic fuzzy pre continuous (IFP continuous in short) if $f^{-1}(B)\square IFPO(X)$ for every $B\square\sigma$[6],
(d) intuitionistic fuzzy generalized continuous (IFG continuous in short) if $f^{-1}(B)\square IFGO(X)$ for every $B\square\sigma$[14],
(e) intuitionistic fuzzyα generalized continuous (IFαG continuous in short) if $f^{-1}(B)\square IF\alpha GO(X)$ for every $B\square\sigma$[13],

(f) intuitionistic fuzzy weakly generalized continuous (IFWG continuous in short) if $f^{-1}(B)\square IFWGO(X)$ for every $B\square\sigma$[10],
(g) intuitionistic fuzzy almost continuous (IFA continuous in short) if $f^{-1}(B)\square IFO(X)$ for every IFROS$B\square\sigma$[17],
(h) intuitionistic fuzzy almost weakly generalized continuous (IFAWG continuous in short) if $f^{-1}(B)\square IFWGO(X)$for every IFROS $B\square\sigma$[11],
(i) intuitionistic fuzzy quasi weakly generalized continuous if $f^{-1}(B)\square IFO(X)$ for every IFWGOS $B\square\sigma$[13],
(j) intuitionistic fuzzy weakly generalized irresolute (IFWG irresolute in short) if $f^{-1}(B)\square IFWGO(X)$ for every IFWGOS $B\square\sigma$ [9],
(k) intuitionistic fuzzy totally continuous mapping if the inverse image of every IFCS in Y is an intuitionistic fuzzy clopen subset in X [7],
(l) intuitionistic fuzzy weakly generalized * open mapping if f(A) is an IFWGOS in Y for every IFWGOS A in X [11].

3. Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mappings:

In this section, we introduce Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mappings and study some of their properties.

Definition 3.1 A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous (IVPF perfectly WG continuous in short) mapping if the inverse image of every IVPFWGCS of Y is Interval-valued pythagorean fuzzy clopen in X.

Theorem 3.2 Every Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping is an Interval-valued pythagorean fuzzy continuous mapping but not conversely.

Proof: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping. Let A be an IVPFCS in Y. Since every IVPFCS is an IVPFWGCS, A is an IVPFWGCS in Y. By hypothesis, $f^{-1}(A)$ is Interval-valued Pythagorean fuzzy clopen in X. Thus $f^{-1}(A)$ is an IVPFCS in X. Therefore f is an Interval-valued Pythagorean fuzzy continuous mapping.

Theorem 3.3 Every Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping is an Interval-valued fuzzy αcontinuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping. Let A be an IVPFCS in Y. Since every IVPFCS is an IVPFWGCS, A is an IVPFWGCS in Y. By hypothesis, $f^{-1}(A)$ is Interval-valued fuzzy clopen in X. Thus $f^{-1}(A)$ is an IVPFCS in X. Since every IVPFCS is an IVPFαCS, $f^{-1}(A)$ is an IVPFαCS in X. Hence f is an Interval-valued Pythagorean fuzzy α continuous mapping.

Theorem 3.4 Every Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping is an Interval-valued Pythagorean fuzzy pre-continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping. Let A be an IVPFCS in Y. Since every IVPFCS is an IVPFWGCS, A is an IVPFWGCS in Y. By hypothesis, $f^{-1}(A)$ is Interval-valued Pythagorean fuzzy clopen in X. Thus $f^{-1}(A)$ is an IVPFCS in X. Since every IVPFCS is an IVPFPCS, $f^{-1}(A)$ is an IVPFPCS in X. Hence f is an Interval-valued fuzzy pre-continuous mapping.

Theorem 3.5 Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IVPFTS (X, τ) into an IVPFTS(Y,σ). Then the following statements are equivalent.

(a) f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping,

(b) $f^{-1}(B)$ is Interval-valued Pythagorean fuzzy clopen in X for every IVPFWGOS B in Y.

Proof: (a)⇒ (b): Let B be an IVPFWGOS in Y. Then B^c is an IVPFWGCS in Y. Since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(B^c) = (f^{-1}(B))^c$ is Interval-valued Pythagorean fuzzy clopen in X. This implies $f^{-1}(B)$ is Interval-valued Pythagorean fuzzy clopen in X.

(b) ⇒ (a): Let B be an IVPFWGCS in Y. Then B^c is an IVPFWGOS in Y. By hypothesis, $f^{-1}(B^c) = (f^{-1}(B))^c$ is Interval-valued Pythagorean fuzzy clopen in X, which implies f^{-1}(B) is Interval-valued Pythagorean fuzzy clopen in X. Therefore f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.

Theorem 3.6 If $f : (X, \tau) \to (Y, \sigma)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping from an IVPFTS (X, τ) into an IVPFTS (Y, σ), then $f(cl(A))\Box wgcl(f(A))$ for every IVPFS A in X.

Proof: Let A be an IVPFS in X. Then wgcl(f(A)) is an IVPFWGCS in Y. Since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(wgcl(f(A)))$ is Interval-valued Pythagorean fuzzy clopen in X. Thus $f^{-1}(wgcl(f(A)))$ is an IVPFCS in X. Clearly $A\Box f^{-1}(wgcl(f(A)))$. Therefore, $cl(A)\Box cl(f^{-1}(wgcl(f(A)))) = f^{-1}(wgcl(f(A)))$.
Hence $f(cl(A))\Box wgcl(f(A))$ for every IVPFS A in X.

Theorem 3.7 If $f:(X,\tau)\rightarrow(Y,\sigma)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping from an IVPFTS (X, τ) into an IVPFTS (Y, σ), then $cl(f^{-1}(B))\Box f^{-1}(wgcl(B))$ for every IVPFS B in Y.

Proof: Let B be an IVPFS in Y. Then wgcl(B) is an IVPFWGCS in Y.
By hypothesis, $f^{-1}(wgcl(B))$ is Interval-valued Pythagorean fuzzy clopen in X. Thus $f^{-1}(wgcl(B))$ is an IVPFCS in X. Clearly $B\Box wgcl(B)$ implies $f^{-1}(B)\Box f^{-1}(wgcl(B))$.
Therefore $cl(f^{-1}(B))\Box cl(f^{-1}(wgcl(B))) = f^{-1}(wgcl(B))$.
Hence $cl(f^{-1}(B))\Box f^{-1}(wgcl(B))$ for every IVPFS B in Y.

Theorem 3.8 If $f:(X,\tau)\rightarrow(Y,\sigma)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping from an IVPFTS (X, τ) into an IVPFTS (Y,σ), then $f^{-1}(wgint(B))\Box int(f^{-1}(B))$ for every IVPFS B in Y.

Proof: Let B be an IVPFS in Y. Then wgint(B) is an IVPFWGOS in Y.
By hypothesis, $f^{-1}(wgint(B))$ is Interval-valued Pythagorean fuzzy clopen in X. Thus $f^{-1}(wgint(B))$ is an IVPFOS in X. Clearly $wgint(B)\Box B$ implies $f^{-1}(wgint(B))\Box f^{-1}(B)$. Therefore $int(f^{-1}(wgint(B)))$ $\Box int(f^{-1}(B))$. Hence, $f^{-1}(wgint(B))\Box int(f^{-1}(B))$ for every IVPFS B in Y.

Theorem 3.9 The composition of two Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping in general.

Proof: Let A be an IVPFWGCS in Z. By hypothesis, $g^{-1}(A)$ is Interval-valued Pythagorean fuzzy clopen in Y and hence an IVPFCS in Y. Since every IVPFCS is an IVPFWGCS, $g^{-1}(A)$ is an IVPFWGCS in Y.
Further, since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(g^{-1}(A)) = (gf)^{-1}(A)$ is Interval-valued Pythagorean fuzzy

clopen in X. Hence gof is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.

Theorem 3.10 Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\delta)$ be any two mappings. Then the following statements hold.
(i) Let $f:(X,\tau)\to(Y,\sigma)$ be an Interval-valued Pythagorean fuzzy continuous mapping and $g:(Y,\sigma)\to(Z,\delta)$ an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping. Then their composition $g\circ f:(X,\tau)\to(Z,\delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.
(ii) Let $f:(X,\tau)\to(Y,\sigma)$ be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping and $g:(Y,\sigma)\to(Z,\delta)$ an Interval-valued Pythgorean fuzzy continuous mapping [respectively Interval-valued Pythagorean fuzzy α continuous mapping, Interval-valued Pythagorean fuzzy pre continuous mapping, Interval-valued Pythagorean fuzzy α generalized continuous mapping and Interval-valued Pythagorean fuzzy generalized continuous mapping]. Then their composition $g\circ f:(X,\tau)\to(Z,\delta)$ is an Interval-valued Pythagorean fuzzy continuous mapping.
(iii) Let $f:(X,\tau)\to(Y,\sigma)$ be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping and $g:(Y,\sigma)\to(Z,\delta)$ an Interval-valued Pythagorean fuzzy weakly generalized continuous mapping. Then their composition $g\circ f:(X,\tau)\to(Z,\delta)$ is an Interval-valued Pythagorean fuzzy continuous mapping.

Proof: (i) Let A be an IVPFWGCS in Z. By hypothesis, $g^{-1}(A)$ is Interval-valued Pythagorean fuzzy clopen in Y and hence an IVPFCS in Y. Since f is an Interval-valued Pythagorean fuzzy continuous mapping, $f^{-1}(g^{-1}(A))=(g\circ f)^{-1}(A)$ is an IVPFCS in X. Hence $g\circ f$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.

(ii) Let A be an IVPFCS in Z. By hypothesis, $g^{-1}(A)$ is an IVPFCS [respectively IVPFαCS, IVPFPCS, IVPF αGCS and IVPFGCS] in Y. Since every IVPFCS [respectively IVPFαCS, IVPFPCS, IVPFαGCS and IVPFGCS] is an IVPFWGCS, $g^{-1}(A)$ is an IVPFWGCS in Y. Then $f^{-1}(g^{-1}(A))=(g\circ f)^{-1}(A)$ is Interval-valued Pythagorean fuzzy clopen in X, by hypothesis. Thus $(g\circ f)^{-1}(A)$ is an IVPFCS in X. Hence $g\circ f$ is an Interval-valued Pythagorean fuzzy continuous mapping.

(iii) Let A be an IVPFCS in Z. By hypothesis, $g^{-1}(A)$ is an IVPFWGCS in Y. Since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(g^{-1}(A))=(g\circ f)^{-1}(A)$ is Interval-valued Pythagorean fuzzy clopen in

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X. Thus $(g \circ f)^{-1}(A)$ is an IVPFCS in X. Hence $g \circ f$ is an Interval-valued Pythagorean fuzzy continuous mapping.

Theorem 3.11 If $f : (X, \tau) \to (Y, \sigma)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ is an Interval-valued Pythagorean fuzzy weakly generalized irresolute mapping, then $g \circ f : (X, \tau) \to (Z, \delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.

Proof:Let A be an IVPFWGCS in Z. By hypothesis, $g^{-1}(A)$ is an IVPFWGCS in Y. Since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is Interval-valued Pythagorean fuzzy clopen in X. Hence $g \circ f$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.

Theorem 3.12 Let $f : (X, \tau) \to (Y, \sigma)$ be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ be any mapping. Then $g \circ f : (X, \tau) \to (Z, \delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping if and only if g is an Interval-valued Pythagorean fuzzy weakly generalized irresolute mapping.

Proof: Let $g : (Y, \sigma) \to (Z, \delta)$ be an Interval-valued Pythagorean fuzzy weakly generalized irresolute mapping.
Then the proof follows from the theorem 3.11,
Conversely, let $g \circ f : (X, \tau) \to (Z, \delta)$ be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping. Let A be an IVPFWGCS in Z. Since $g \circ f : (X, \tau) \to (Z, \delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping, $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is Interval-valued Pythagorean fuzzy clopen in X. Since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping,$g^{-1}(A)$ is an IVPFWGCS in Y. Thus the inverse image of each IVPFWGCS in Z is an IVPFWGCS in Y. Hence g is an Interval-valued Pythagorean fuzzy weakly generalized irresolute mapping.

4. Interval-valued Pythagorean fuzzy perfectly weakly generalized open mappings:

In this section, we introduce Interval-valued Pythagorean fuzzy perfectly weakly generalized open mappings and study some of their properties.

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Definition 4.1 A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping if the image of every IVPFWGOS in X is Interval-valued Pythagorean fuzzyclopen in Y.

Theorem 4.2 Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IVPFTS (X, τ) into an IVPFTS(Y,σ). Then the following statements are equivalent.
(a) f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping,
(b) f(B) is Interval-valued Pythagorean fuzzy clopen in Y for every IVPFWGCS B in X.

Proof: (a) ⇒ (b): Let B be an IVPFWGCS in X. Then B^c is an IVPFWGOS in X. Since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping, $f(B^c) = (f(B))^c$ is Interval-valued Pythagorean fuzzy clopen in Y. This implies f(B) is Interval-valued Pythagorean fuzzy clopen in Y.

(b) ⇒(a): Let B be an IVPFWGOS in X. Then B^c is an IVPFWGCS in X. By hypothesis, $f(B^c) = (f(B))^c$ is Interval-valued Pythagorean fuzzy clopen in Y, which implies that f(B) is Interval-valued Pythagorean fuzzy clopen in Y. Therefore f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

Theorem 4.3 Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping from an IVPFTS (X, τ) into an IVPFTS(Y,σ), then the following statements are equivalent.
(a) Inverse of f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.
(b) f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

Proof: (a) ⇒(b): Let A be an IVPFWGOS of X. By assumption, $(f^{-1})^{-1}(A) = f(A)$ is Interval-valued Pythagorean fuzzy clopen in Y. Hence f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

(b) ⇒(a): Let B be an IVPFWGOS in X. Then f(B) is Interval-valued Pythagorean fuzzy clopen in Y. That is $(f^{-1})^{-1}(B) = f(B)$ is Interval-valued Pythagorean fuzzy clopen in Y. Therefore f^{-1} is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.

Theorem 4.4 The composition of two Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping is again an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

Proof:Suppose $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\delta)$ are any two Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping. Let A be an IVPFWGOS in X. Since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping, f(A) is Interval-valued Pythagorean fuzzy clopen in Y. Hence it is an IVPFOS in Y. But every IVPFOS is an IVPFWGOS, which implies f(A) is an IVPFWGOS in Y. Since g is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping, $g(f(A))=(g\circ f)(A)$ is Interval-valued Pythagorean fuzzy clopen in Z. Thus the image of each IVPFWGOS in X is Interval-valued Pythagorean fuzzy clopen in Z. Therefore $g\circ f:(X,\tau)\to(Z,\delta)$ is Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

Theorem 4.5 If $f:(X,\tau)\to(Y,\sigma)$ is an Interval-valued Pythagorean fuzzy weakly generalized * open mapping and $g:(Y,\sigma)\to(Z,\delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping, then their composition $g\circ f:(X,\tau)\to(Z,\delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

Proof: Let A be an IVPFWGOS in X. Since f is an Interval-valued Pythagorean fuzzy weakly generalized * open mapping, f(A) is an IVPFWGOS in Y. Further, since g is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping, $g(f(A))=(g\circ f)(A)$ is Interval-valued Pythagorean fuzzy clopen in Z. Hence $g\circ f:(X,\tau)\to(Z,\delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

Theorem 4.6 Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\delta)$ be two mappings such that $g\circ f:(X,\tau)\to(Z,\delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping. Then the following statements hold.

(a) If f is an Interval-valued Pythagorean fuzzy weakly generalized irresolute mapping and surjective, then g is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

(b) If g is an Interval-valued Pythagorean fuzzy totally continuous mapping and injective, then f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

Proof: ***(a)*** Let A be an IVPFWGOS in Y. Then $f^{-1}(A)$ is an IVPFWGOS in

X, because f is an Interval-valued Pythagorean fuzzy weakly generalized irresolute mapping. Since $(g \circ f)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping, $(g \circ f)(f^{-1}(A)) = g(A)$ is Interval-valued Pythagorean fuzzy clopen in Z. This shows that g is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

(b)Since g is injective, we have, $f(A) = g^{-1}(g \circ f)(A)$ is true for every subset A of X. Let B be an IVPFWGOS in X. Therefore $(g \circ f)(B)$ is Interval-valued Pythagorean fuzzy clopen in Z and hence an IVPFOS in Z. Since g is Interval-valued Pythagorean fuzzy totally continuous, $g^{-1}(g \circ f)(A) = f(A)$ is Interval-valued Pythagorean fuzzy clopen in Y. This shows that f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

4. Conclusion

In this paper, We studied the concepts of Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mappings and Interval-valued Pythagorean fuzzy perfectly weakly generalized open mappings in Interval-valued Pythagorean fuzzy topological space. Some of their properties are explored.

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