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Pentapartitioned Neutrosophic Almost Resolvable and Irresolvable Spaces

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Abstract

The aim of this paper is to develop many characterizations of Pentapartitioned Neutrosophic (PN) almost resolvable and irresolvable spaces and also the condition under that a PN almost resolvable space becomes a PN baire space. The interrelations between PN almost resolvable spaces and other spaces also are mentioned.

Key Words: Pentapartitioned neutrosophic set, pentapartitioned neutrosophic almost resolvable space, pentapartitioned neutrosophic almost irresolvable spaces.

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1 Introduction

In order to cope with uncertainties, the thought of fuzzy sets and fuzzy set operations was introduced by Zadeh [17]. The speculation of fuzzy topological space was studied and developed by C.L. Chang [3]. The paper of Chang sealed the approach for the following tremendous growth of the various fuzzy topological ideas. Since then a lot of attention has been paid to generalize the fundamental ideas of general topology in fuzzy setting and therefore a contemporary theory of fuzzy topology has been developed. Atanassov and plenty of researchers [1] worked on intuitionistic fuzzy sets within the literature. Florentin Smarandache [14] introduced the idea of Neutrosophic set in 1995 that provides the information of neutral thought by introducing the new issue referred to as uncertainty within the set. thus neutrosophic set was framed and it includes the parts of truth membership function(T), indeterminacy membership function(I), and falsity membership function(F) severally.

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Neutrosophic sets deals with non normal interval of]−0 1+[. Pentapartitioned Neutrosophic set and its properties were introduced by Rama Malik and Surpati Pramanik [13]. In this case, indeterminacy is divided into three components: contradiction, ignorance, and an unknown membership function.

The concept of Pentapartitioned neutrosophic pythagorean sets was initiated by R. Radha and A. Stanis Arul Mary[7]. Many authors have been discussed about the concept of Pythagorean Sets[6-12]. The concept of intuitionistic fuzzy almost resolvable spaces and irresolvable spaces was introduced by Sharmila s [15].R. Radha and A.Stanis Arul Mary introduced Pentapartitioned neutrosophic pythagorean resolvable and irresolvable spaces.

Now we extend the concepts to pentapartitioned neutrosophic sets. In this paper, we discussed about PN almost resolvable and irresolvable spaces in third section, the inter-relations with PN almost resolvable spaces with other spaces have been investigated in fourth section and the levels of Irresolvability can be studied in last section.

2 Preliminaries

Definition 2.1 [14] Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$$

Where $T_A\ , I_A, F_A: U \to [0,1]\ and\ 0 \leq T_A(x) + I_A(x) + F_A\ (x) \leq 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of inderminancy and $F_A(x)$ is the degree of non-membership.

Definition 2.2 [13] Let P be a non-empty set. A Pentapartitioned neutrosophic set A over P characterizes each element p in P a truth -membership function T_A , a contradiction membership function C_A, an ignorance membership function G_A, unknown membership function U_A and a false membership function F_A , such that for each p in P

$$T_A + C_A + G_A + U_A + F_A \leq 5$$

Definition 2.3 [7] The complement of a pentapartitioned neutrosophic set A on R is denoted by A^C or A^* and is defined as

$A^C = \{< x, F_A(x), U_A\ (x), 1 - G_A(x), C_A(x), T_A(x) > : x \in X\}$

Definition 2.4 [7] Let A = $< x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x) >$ and

B = $< x, T_B(x), C_B(x), G_B(x), U_B(x), F_B(x) >$ are Pentapartitioned Neutrosophic sets. Then

$A \cup B = < x, max(T_A(x), T_B(x)), max(C_A(x), C_B(x)), \min(G_A(x), G_B(x)),$

$\min(U_A(x), U_B(x)), \min(F_A(x), F_B(x)), >$

$A \cap B = < x, min(T_A(x), T_B(x)), min(C_A(x), C_B(x)), \max(G_A(x), G_B(x))$

$$,max\big(U_A(x),U_B(x)\big),max(F_A(x),F_{\ B}(x))$$

Definition 2.5 [12] A PN topology τ on a nonempty set R is a family of a PN sets in R satisfying the following axioms

1) 0,1$\in\tau$
2) $R_1\cap R_2\in\tau$ for any $R_1,R_2\in\tau$
3) $\cup R_i\ \in\tau$ for any R_i: $i\in I$

The complement R* of PN open set (PNOS, in short) in PN topological space [PNTS] (R,$\boldsymbol{\tau}$), is called a PN closed set [PNCS].

Definition 2.6 [7] Let (R,$\boldsymbol{\tau}$) be a PNTS and L be a PNTS in R. Then the PN interior and PN Closure of R denoted by

PNCl(L) = $\cap\{\boldsymbol{K}$: K is a PNPCS in R and L$\subseteq$ K}.

PNInt(L) = $\cup${G: G is a PNPOS in R and G$\subseteq$ L}.

Definition 2.7 [11] Let (R,$\boldsymbol{\tau}$) be a PNTS and K be a PN set in (R,$\boldsymbol{\tau}$). Then the PN closure operator satisfy the following properties.

1-PNPCl(K) = PNPInt(1-K)

1-PNPInt(K) = PNPCl(1-K)

Definition 2.8 [11] A PNP A in PNTS (R,$\boldsymbol{\tau}$) is called PN dense if there exists no PNCS L in (R,$\boldsymbol{\tau}$) such that K $\subseteq$ $\boldsymbol{L}\subseteq\mathbf{1}$. That is PNCl(K) = 1.

Definition 2.9 [11] A PN A in PNPTS (R,$\boldsymbol{\tau}$) is called PN nowhere dense if there exists no nonzero PNPOS L in (R,$\boldsymbol{\tau}$) such that $\boldsymbol{L}\subseteq$ PNPCl(K). That is PNPInt (PNPCl(K)) = 0.

Definition 2.10 [11] A PNTS (R,$\boldsymbol{\tau}$) is called PN resolvable if there exists a PN dense set K in (R,$\boldsymbol{\tau}$) such that PNCl (1-K) = 1. Otherwise (R,$\boldsymbol{\tau}$) is called PN irresolvable.

Definition 2.11 [11] A PNTS (R,$\boldsymbol{\tau}$) is called PN submaximal if PNCl(K) = 1 for any non-zero PN set K in (R,$\boldsymbol{\tau}$)

Definition 2.12 [11] A PNTS (R,$\boldsymbol{\tau}$) is called a PN open hereditarily resolvable if PNInt (PNPCl(K)) $\neq\mathbf{0}$ for any PN set K in (R,$\boldsymbol{\tau}$).

Definition 2.13 [11] APNTS (R,$\boldsymbol{\tau}$) is called PN first category if $\cup_{i=1}^{\infty}\boldsymbol{K_i}$, where $\boldsymbol{K_i's}$ are PN nowhere dense sets in (R,$\boldsymbol{\tau}$). A PNTS which is not first category is said to be PN second category.

Definition 2.14 [11] A PNTS (R,$\boldsymbol{\tau}$) is called a PN baire space if PNInt $(\cup_{i=1}^{\infty}\boldsymbol{K_i})=\mathbf{0}$, where $\boldsymbol{K_i's}$ are PN nowhere dense sets in (R,$\boldsymbol{\tau}$).

3 Pentapartitioned Neutrosophic (PN) Almost Resolvable and Irresolvable Spaces

Definition 3.1 A PNTS is called a PN almost resolvable space if $\bigcup_{i=1}^{\infty} K_i$ =1, where K_i's are PNS s in (R,τ) are such that PNInt (K_i) = 0. Otherwise (R,τ) is called PN almost irresolvable space.

Example 3.2 Let R = {p, q}. Let A3, A4, A5 and A6 be the PN sets defined on R as follows.
A1 = {[p,0.2,0.6,0.5,0.2,0.6], [q,0.4,0.5,0.7,0.4,0.5]}
A2 = {[p,0.5,0.6,0.1,0.2,0.4], [q,0.5,0.6,0.5,0.1,0.1]}.
Then, clearly τ = {0, A1, A2, 1} is a PN topology on R.
Now consider the PN sets defined on R as follows
A3 = {[p,0.3,1,0.6,0,0.5], [q,1,0.5,0,0.2,0.5]},
A4 = {[p,1,0.2,0.7,0.4,0], [q,0.2,0.3,0.5,0.3,0]},
A5 = {[p,0.4,0.3,0.4,0.5,0.2], [q,0.5,1,0.2,0,0.3]},
A6 = {[p,0.2,0.4,0,0.1,0.3], [q,0.2,0.3,0.7,0.4,0.5]}.
Then, PNInt (A3) = 0, PNInt (A4) = 0, PNInt (A5) = 0 and PNInt (A6) = 0 and
{(A3) ∪ (A4) ∪ (A5) ∪ (A6)} = 1.
Hence (R,τ) is a PN almost resolvable space.

Example 3.3 Let R = {p, q}. Let A3, A4, A5 and A6 be the PN sets defined on R as follows.
A1 = {[p,0.2,0.6,0.5,0.2,0.6], [q,0.4,0.5,0.7,0.4,0.5]}
A2 = {[p,0.5,0.6,0.1,0.2,0.4], [q,0.5,0.6,0.5,0.1,0.1]}.
Then, clearly τ = {0, A1, A2, 1} is a PN topology on R.
Now consider the PN sets defined on R as follows
A3 = {[p,0.2,1,0.5,0,0.4], [q,0.4,0.3,0.5,0.2,0.5]},
A4 = {[p,0.4,0.5,0.6,0.1,0], [q,0.2,0.3,0.5,0.3,0.2]},
A5 = {[p,0.2,0.4,0.5,0.2,0.1], [q,0.1,1,0.2,0,0.3]},
A6 = {[p,0.5,0.3,0.2,0.1,0.5], [q,0.2,0.3,0.7,0.4,0.5]}.
Then, PNInt (A3) = 0, PNInt (A4) = 0, PNInt (A5) = 0 and PNInt (A6) = 0 and
{(A3) ∪ (A4) ∪ (A5) ∪ (A6)} ≠1.
Hence (R,τ) is a PN almost irresolvable space.

Theorem 3.4 If $\bigcap_{i=1}^{\infty} K_i = 0$,where K_i's are PN dense sets in (R,τ), then (R,τ) is a PN almost resolvable space.
Proof: Suppose that $\bigcap_{i=1}^{\infty} K_i = 0$, where PNCl ($K_i$) = 1 in (R,$\tau$). Then we have 1 - $\bigcap_{i=1}^{\infty} K_i = 1 - 0 = 1$, where 1 - PNCl ($K_i$) = 0. This implies that $\bigcup_{i=1}^{\infty}(1 - K_i) = 1$, where PNInt (1 - K_i) $= 0. Let\ 1 - K_i = L_i$, then we have $\bigcup_{i=1}^{\infty} L_i = 1$, where PNInt ($L_i$)= 0 in (R,$\tau$). Hence (R,$\tau$) is PN almost resolvable space.

Definition 3.5 A PNTS (R,τ) is called a PN hyper- connected space if every PN open set is PN dense in (R,τ). That is PNPCl $(K_i) = 1$ for all $K_i \in \tau$.

Theorem 3.6 If $\bigcap_{i=1}^{\infty} K_i = 0$,where K_i's are PN open set in a PN hyper-connected space (R,τ), then (R,τ) is a PN almost resolvable space.
Proof: Suppose that $\bigcap_{i=1}^{\infty} K_i = 0$, where $K_i \in \tau$ .Since (R,τ) is a PN hyper – connected space, the PN open set K_i is a PN dense set in (R,τ) for each i. Hence we have $\bigcap_{i=1}^{\infty} K_i = 0$, where PNCl (K_i) = 1 in (R,τ). Then by theorem 3.2, (R, τ) is a PN almost resolvable space.

Definition 3.7 A PN K in a PNTS (R,τ) is called PNR_1 if K = $\bigcap_{1=1}^{\infty} K_i$ where each $K_i \in \tau$.

Definition 3.8 A PN K in a PNTS (R,τ) is called PNR_2 if K = $\bigcup_{i=1}^{\infty} K_i$ where each $K_i \in \tau$.

Definition 3.9 A PNTS (R,τ) is called PN R-space, if countable intersection of PNOS s in (R,τ) is PN open. That is, every nonzero PNR_1- set in (R,τ) PN open in (R,τ).

Theorem 3.10 If $\bigcap_{1=1}^{\infty} K_i = 0, K_i's$ are PNR_1- sets in an PN hyper-connected space and PNR-space (R,τ), then (R,τ) is an PN almost resolvable space.
Proof: Let $K_i's$ be PNR_1- sets in a PNR-space (R,τ). Then $K_i's$ are PN open sets in (R,τ). Hence, we have $\bigcap_{1=1}^{\infty} K_i = 0,\ where K_i's$ are PN open sets in an PN hyper-connected space (R,τ). Therefore, by theorem 3.4, (R,τ) is an PN almost resolvable space.

Theorem 3.11 If each PNS K_i is an PNR_2 – set in a PN almost resolvable space (R, τ), then $\bigcap_{1=1}^{\infty}(1 - K_i) = 0, where\ K_i's$ are PN dense sets in (R,τ).
Proof: Let (R,τ) be a PN almost resolvable space. Then $\bigcup_{i=1}^{\infty} K_i$ = 1, where $K_i's$ are such that PNInt (K_i) = 0. This implies that 1-$\bigcup_{i=1}^{\infty} K_i$ = 0 and 1-PNInt (K_i) = 1. Then $\bigcap_{1=1}^{\infty}(1 - K_i) =$ 0 and PNCl ($1 - K_i$) = 1. Since $K_i's$ are PNR_2-sets, (1-K_i)$'s$ are PNR_1-sets in (R,τ). Hence we have $\bigcap_{1=1}^{\infty}(1 - K_i) = 0$, where (1-$K_i$)'s are PN dense and PN$R_1$- sets in (R,$\tau$).

Definition 3.12 A PNTS (R,τ) is called Pentapartitioned Neutrosophic nodec space, if every non-zero PN nowhere dense set in (R,τ) is PN closed.
Theorem 3.13
IF the PNTS (R,τ) is a PN first category, then (R,τ) is a PN almost resolvable space.
Proof: Since (R,τ) is of PN first category, we have $\bigcup_{i=1}^{\infty} K_i$ = 1, where $K_i's$ are PN nowhere dense sets in (R,τ). Now K_i is a PN nowhere dense set implies that PNInt(K_i) = 0. Hence $\bigcup_{i=1}^{\infty} K_i$ = 1, where PNInt(K_i) = 0 and therefore (R,τ) is a PN almost resolvable space.

Theorem 3.14 If (R,τ) is a PN first category space and PN nodec space, then (R,τ) is a PN almost resolvable space.
Proof : Let (R,τ) be a first category space. Then we have $\bigcup_{i=1}^{\infty} K_i$ = 1, where $K_i's$ are PN nowhere dense subsets in (R,τ). Since (R,τ) is a PN nodec space, the PN nowhere dense sets are PN closed sets in (R,τ). Hence $K_i's$ are PNCS in (R,τ). That is, PNCl (K_i) = K_i. PNInt (PNCl (K_i)) = PNInt (K_i) = 0. Now PNInt (PNCl (K_i)) = 0 implies that PNInt (K_i) = 0. Hence we have $\bigcup_{i=1}^{\infty} K_i$ = 1, where $K_i's$ in (R,τ) are such that PNInt (K_i) = 0. Hence (R,τ) is a PN almost resolvable space.

Theorem 3.15 If PNCl(PNInt(K_i) = 1, for ach PN dense set K_i in a PN almost resolvable space (R,τ), then (R,τ) is a PN first category space.
Proof: Let (R,τ) be a PN almost resolvable space such that PNCl(PNInt(K_i) = 1, for each PN dense set K_i in (R,τ). Since (R,τ) is PN almost resolvable space, $\bigcup_{i=1}^{\infty} K_i$ = 1, where $K_i's$ in (R,τ) are such that PNInt(K_i) = 0.Now 1 – PNInt (K_i)= 1, which implies that PNCl(1 - K_i) = 1 , which implies that (1- K_i) is PN dense. Then by hypothesis, PNCl(PNInt(1- K_i)) = 1 for the PN dense set (1- K_i) in (R,τ). This implies that PNInt(PNCl(K_i)) = 0. So 1-PNCl(PNInt(1-K_i)) = 0. Hence $K_i's$ are PN nowhere dense set in (R,τ). Therefore $\bigcup_{i=1}^{\infty} K_i$ = 1, where $K_i's$ are PN nowhere dense set in (R,τ), implies that (R,τ) is a PN first category space.

Theorem 3.16 If PNCl(PNInt(K_i) = 1, for ach PN dense set K_i is a PN almost resolvable space (R,τ), then (R,τ) is not a PN Baire space.

Proof: Let (R,τ) be a PN almost resolvable space such that PNCl(PNInt(K_i) = 1, for each PN dense set K_i in (R,τ). Then by theorem 3.13, (R,τ) is a PN first category space. Since $\bigcup_{i=1}^{\infty} K_i$ = 1, where $K_i's$ are PN nowhere dense in (R,τ) This implies that $PNInt(\bigcup_{i=1}^{\infty} K_i)$ = 1 = PNInt(1) = 0 . Hence (R,τ) is not a PN Baire space.

Theorem 3.17 If (R,τ) is a PN second category space, then (R,τ) is a PN almost resolvable space.
Proof: Let (R,τ) be a PN second category space. Then $\bigcup_{i=1}^{\infty} K_i \neq 1$, where $K_i's$ are PN nowhere dense sets in (R,τ). That is $\bigcup_{i=1}^{\infty} K_i \neq 1$, where PNInt(PNCl($K_i$)) = 0. Now PNInt($K_i$) ⊆ PNInt(PNCl($K_i$)), implies that PNInt($K_i$) = 0. Hence $\bigcup_{i=1}^{\infty} K_i \neq 1$, where PNInt($K_i$) = 0 and therefore (R,$\tau$)is a PN almost resolvable space.
Definition 3.18 A PNTS (R,τ) is called PN Volterra space, if PNCl($\bigcap_{i=1}^{N} K_i$) = 1, where are PN dense and PNR_1 sets in (R,τ).

Definition 3.19 A PNTS (R,τ) is called PN weakly Volterra space, if PNCl ($\bigcap_{i=1}^{N} K_i$) ≠1, where are PN dense and PNR_1 sets in (R,τ).

Theorem 3.20 If PNTS (R,τ) is not a PN weakly Volterra space, then (R,τ) is a PN almost resolvable space.
Proof: Let (R,τ) be a non-weakly volterra space. Then we have PNCl ($\bigcap_{i=1}^{N} K_i) = 0$, where $K_i's$ are PN dense and PNR_1 sets in (R,τ).
Since $K_i's$ are PN dense , 1-PNCl(K_i) = 0. Now PNCl($\bigcap_{i=1}^{N} K_i$) = 0, implies that PNInt($\bigcup_{i=1}^{N}(1-K_i)$) = 1 and PNCl(K_i) = 1, implies that PNInt(1- K_i) = 0. Let $M_j's$ be such that PNInt($M_j's$) = 0 and take the first N $M_j's$ as (1-$M_j's$)'s. Now $\bigcup_{i=1}^{N}(1-K_i) \subseteq \bigcup_{j=1}^{\infty} M_j$, implies that $PNInt(\bigcup_{i=1}^{N}(1-K_i)) \subseteq PNInt(\bigcup_{j=1}^{\infty} M_j) \subseteq \bigcup_{j=1}^{\infty} M_j$. Then we have 1 ⊆ $\bigcup_{j=1}^{\infty} M_j$. That is $\bigcup_{j=1}^{\infty} M_j = 1$, where $M_j's$ in (R,τ) are such that PNInt(M_j) = 0. Hence (R,τ) is a PN almost resolvable space.

4. Inter -Relations between PN almost resolvable spaces and Irresolvable spaces with other spaces.

Theorem 4.1 If the PN almost resolvable space (R,τ) is a PN submaximal space, then (R,τ) is a PN first category space.
Proof: Let (R,τ) be a PN almost resolvable space. Then $\bigcup_{i=1}^{\infty} K_i = 1$, where $K_i's$ in (R,τ) are such that PNInt(K_i) = 0. Then we have $\bigcap_{i=1}^{\infty}(1-K_i) = 0 = 0$, where PNCl(1-$K_i$) = 1. Since the space (R,τ) is a PN submaximal space, the PN dense sets (1-K_i)'s are PNOS in (R,τ). Then $K_i's$ are PN closed sets in (R,τ) and hence PNCl(K_i) = K_i. Now PNInt(PNCl(K_i)) = PNInt(K_i) = 0. Then $K_i' sare$ PN nowhere dense sets in (R,τ).Hence $\bigcup_{i=1}^{\infty} K_i = 1$, where $K_i's$ are PN nowhere dense subsets in (R,τ) implies that (R,τ) is a PN first category space.

Theorem 4.2 If the PN almost irresolvable space (R,τ) is a PN submaximal space, then (R,τ) is a PN second category space.
Proof: Let (R,τ) be a PN almost irresolvable space. Then $\bigcup_{i=1}^{\infty} K_i \neq 1$, where $K_i's$in (R,τ) are such that PNInt(K_i) = 0. Now PNInt(K_i) = 0, implies that PNCl(1-K_i) = 1.Then $K_i's$ are PNCS s in (R,τ) and hence PNCl(K_i) = K_i. Now PNInt(K_i) = 0 implies that PNInt (PNCl(K_i)) = PNInt(K_i) = 0. Then $K_i's$ are PN nowhere dense sets in (R,τ). Hence $\bigcup_{i=1}^{\infty} K_i \neq 1$, where $K_i's$ are PN nowhere dense sets in (R,τ). Then (R,τ) is a PN second category space.

Theorem 4.3 If the PN almost irresolvable space (R,τ) is a PN submaximal space, then (R,τ) is not a PN Baire space.

Proof: Let the PN almost irresolvable space (R,τ) be a PN submaximal space. Then, by theorem 3.19 , (R,τ) is a PN first category space and hence $\bigcup_{i=1}^{\infty} K_i = 1$, where $K_i's$ are PN nowhere dense sets in (R,τ). Now PNInt($\bigcup_{i=1}^{\infty} K_i$) = PNInt(1) = 1 ≠0. Hence (R,τ) is not a PN baire space.

Theorem 4.4 If the PN almost irresolvable space (R,τ) is a PN open hereditarily irresolvable space, then (R,τ) is a PN second category space.

Proof: Let (R,τ) be a PN almost irresolvable space. Then $\bigcup_{i=1}^{\infty} K_i \neq 1$, where $K_i's$ in (R,τ) are such that PNInt(K_i) = 0.Since (R,τ) is a PN open hereditarily irresolvable space, PNInt(K_i) = 0, implies that PNInt(PNCl(K_i)) = 0. Then $K_i's$ are PN nowhere dense subsets in (R,τ). Hence $\bigcup_{i=1}^{\infty} K_i \neq 1$, where $K_i's$ are PN nowhere dense subsets in (R,τ) implies that (R,τ) is a PN second category space.

Theorem 4.5 If the PNTS (R,τ) is a PN baire space, then (R,τ) is a PN almost irresolvable space.

Proof: Let (R,τ) be a PN baire space, PNInt($\bigcup_{i=1}^{\infty} K_i$) = 0, where $K_i's$ are PN nowhere dense sets in (R,τ). Now K_i is a PN nowhere dense set implies that PNPInt(PNPCl(K_i)) = 0. Since PNInt(K_i) ⊆ PNInt(PNCl(K_i)), we have PNInt($\bigcup_{i=1}^{\infty} K_i$) = 0, where PNInt($\bigcup_{i=1}^{\infty} K_i$) = 0. Hence PNInt($K_i$) = 0. Suppose that (R,$\tau$) is a PN almost resolvable space. Then $\bigcup_{i=1}^{\infty} K_i$ = 1, where PNInt(K_i) = 0.

Now PNInt($\bigcup_{i=1}^{\infty} K_i$) = PNInt(1) = 1, which implies that 0 = 1, a contradiction. Hence we must have $\bigcup_{i=1}^{\infty} K_i \neq 1$, where PNInt($K_i$) = 0. Therefore (R,$\tau$) is a PN almost resolvable space.

Theorem 4.6 If K is a PN dense and PNOS in a PNTS (R,τ), then 1-K is PN nowhere dense set in (R,τ).

Proof: Since K is PN dense set, PNCl(K) = 1 and let K be a PNOS, then 1-K be PNCS. Hence PNCl(1-K) = 1-K and 1-PNCl(K) =1-1 = 0, which implies that PNCl(1-K) = 1-K and PNInt(1-K) = 0. Therefore PNInt(PNCl(1-K)) = 0, which implies 1-K is a PN nowhere dense set in (R,τ).

Theorem 4.7 If each PN R_1- set is PN fuzzy open and PN dense set in a PNTS (R,τ), then (R,τ) is a PN almost irresolvable space.

Proof: Let K be a PNR_1-set in (R,τ) such that K is a PN dense and PN open set in (R,τ). Then K = $\bigcap_{i=1}^{\infty} K_i$,where K_i's in (R,τ) are PN open set in (R,τ). Now 1-K = 1- $\bigcap_{i=1}^{\infty} K_i$. Then PNCl(1-K) = PNCl(1 –($\bigcap_{i=1}^{\infty} K_i$)) = PNCl($\bigcap_{i=1}^{\infty}(1-K_i)$) and hence PNCl(1-K) = PNCl($\bigcup_{i=1}^{\infty}(1-K_i)$) ⊇ $\bigcup_{i=1}^{\infty} PN$Cl(1- K_i). Since $K_i's$ are PNOS s in (R,τ), (1-K)'s are PNCS in (R,τ) and hence PNCl(1- K_i) = 1- K_i. PNCl(1-K) ⊇ $\bigcup_{i=1}^{\infty}$(1- K_i), which implies that

PNInt(PNCl(1-K)) ⊇ PNInt($\bigcup_{i=1}^{\infty}$(1- K_i)) (1)

Since $\bigcup_{i=1}^{\infty} PN$Int(1- K_i) ⊆ PNInt($\bigcup_{i=1}^{\infty}$(1- K_i)), we have

PNInt(PNPCl(1-K)) ⊆PNInt(1- K_i) (2)

Since K is a PN dense set in (R,τ). By theorem 4.6, the PNS (1-K) is a PN nowhere sense set in (R,τ). Then , we have PNInt(PNCl(1-K)) = 0.

Hence from (2), 0 ⊇ $\cup_{i=1}^{\infty}$(PNInt(1- K_i)). *That is* $\cup_{i=1}^{\infty}$(PNInt(1- K_i)) = 0. Then PNInt(1- K_i) = 0, for each I = 1 to ∞. Hence PNInt(PNPCl(1- K_i)) = 0. [since PNCl(1- K_i) = 1- K_i].This implies that (1-K_i)'s are PN nowhere dense sets in (R,τ). From (1), we have 0 ⊇ PNInt($\cup_{i=1}^{\infty}$(1- K_i)).
That is, PNInt((1- K_i)) = 0. *Hence* (R, τ) *is a PN baire space*. Therefore , by theorem 4.5 is a PN almost irresolvable space.

Theorem 4.8 If $\cup_{i=1}^{\infty} K_i$ = 1, where K_i's are nonzero PN open sets in PNTS (R, τ), then (R, τ) is a PN almost irresolvable space.

Proof: Suppose that $\cup_{i=1}^{\infty} K_i$ = 1, where the PN sets K_i's are non zero PNOS s in (R, τ). Since $K_i's$ are non zero PNOSs, PNInt(K_i) = K_i ≠ 0. Hence we have $\cup_{i=1}^{\infty} K_i$ = 1, where PNInt(K_i) ≠ 0 fo all (i= 1 to ∞). Therefore (R, τ) is a PN almost irresolvable space.

5 Levels of Pentapartitioned Neutrosophic Irresolvability

Theorem 5.1 Let (R, τ) be a PNTS. If (R, τ) is PN open hereditarily irresolvable then PNInt(K) = 0 for any nonzero PN dense sets K in (R, τ) ,implies that PNInt(PNCl(K)) = 0.
Proof: Let K be a PNTS in (R, τ) $\exists PNInt(K)$ = 0. To Prove PNInt(PNCl(K)) = 0. Suppose that PNInt(PNCl(K))≠ 0. Since (R, τ) is PN open hereditarily irresolvable, we have PNInt(K) ≠ 0,which is a contradiction to the assumption. Hence PNInt(PNCl(K)) = 0.

Theorem 5.2 For any PNTS (R, τ), Every PN submaximal space is PN open hereditarily space.
Proof: Let (R, τ) be a PN submaximal space. Then, PNCl(K) = 1 implies that K∈ τ. To Prove (R, τ) is PNO hereditarily irresolvable, suppose that PNInt(K) = 0 for any nonzero PN set K in (R, τ).
Then 1-PNInt(K) = 1-0 = 1 implies that PNCl(1-K) =1. Since (R, τ) is PN submaximal, (1-K) ∈ τ.Then K is PN closed set in (R, τ). Hence A = PNCl(K) = PNInt(K) = PNInt(PNCl(K), Then PNInt(K) = 0 implies that PNInt(PNCl(K)) = 0. By theorem 5.1, (R, τ) is PN open hereditarily space. Hence PN sub-maximality implies PN open hereditarily irresolvable space.

Theorem 5.3 A PN open hereditarily irresolvable space and PN dense set in a PNTS (R, τ) is PN almost irresolvable space.
Proof: Let K be a PN dense set in PNTS (R, τ). Then PNCl(K) = 1 implies that PNInt(1-K) = 0. Since (R, τ) is a PN open hereditarily irresolvable space, PNCl(1-K) = 0 implies that PNInt(PNCl(1-K)) = 0. Now we claim that PNCl(1-K) ≠ 1. Suppose PNCl(1-K) = 1. Then PNInt(PNCl(1-K)) = PNInt(1) = 1 implies that 0=1, a contradiction. Hence we must have PNCl(1-K)≠1. Therefore PNCl(K) = 1 implies that PNCl(1-K)≠ 1, which means that (R, τ) is PN irresolvable space. Now let (R, τ) be PN irresolvable space. Then PNCl(K_i) = 1 *implies that* $PNCl(1-K)$ ≠ 1.Then PNInt(K_i) ≠ 0.Now PNCl(K_i) = 1 implies that PNInt($1 - K_i$) = 0. We claim that $\cup_{i=1}^{\infty}(1 - K_i) \neq$ 1.Suppose that $\cup_{i=1}^{\infty}(1 - K_i) = 1$.Then 1-$\cap_{i=1}^{\infty}(1 - K_i) = 1$ implies that $\cap_{i=1}^{\infty}(K_i) = 0$ Hence there must be atleast two non zero disjoint PN sets K_i and K in (R, τ). Then $K_i + K \subseteq 1$, which implies that $K_i \subseteq 1 - K$.Hence PNCl(K_i) ⊆ PNCl(1-K). Then 1 ⊆ (1-K). That is PNCl(1−K_i) = 1. Then PNInt(K_i) = 0,a contradiction to PNInt(K_i) ≠ 0.Hence $\cup_{i=1}^{\infty}(1 - K_i) \neq$ 1,where PNInt(1-K_i) = 0.Therefore (R, τ) is PN almost irresolvable space.

6. Conclusions

In this paper, we discussed about PN almost resolvable and irresolvable spaces. In future, we extend our ideas to strongly PN resolvable and PN irresolvable spaces.

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