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On Pairwise Fuzzy Almost GP-spaces

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Abstract

In this paper, the concept of pairwise fuzzy almost GP-spaces is introduced by means of pairwise fuzzy dense and pairwise fuzzy G_δ-sets . It is shown that the pairwise fuzzy almost GP-spaces are pairwise fuzzy irresolvable spaces and pairwise fuzzy submaximal spaces are pairwise fuzzy almost GP-spaces. Also it is established that the pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec spaces are pairwise fuzzy almost GP-spaces. The conditions for the fuzzy bitopological spaces to become pairwise fuzzy σ-second category spaces and pairwise fuzzy weakly Volterra spaces are also obtained.

Key words: Pairwise fuzzy dense set, pairwise fuzzy G_δ-set, pairwise fuzzy F_σ-set, pairwise fuzzy submaximal space, pairwise fuzzy irresolvable space, pairwise fuzzy weakly Volterra space.

AMS classification: 54 A 40, 03 E 72.

1. Introduction

The concept of fuzzy sets as a new approach for modelling uncertainties was introduced by L.A.Zadeh [16] in his classic paper. C.L.Chang [3] introduced the notion of fuzzy topological spaces. Almost GP-spaces in classical topology was introduced by M.R.Ahmadi Zand [1]. A.Kandil [4] introduced the concept of fuzzy bitopological spaces as a generalization of fuzzy topological spaces. Since then, several bitopological notions are being generalised to the setting of fuzzy bitopological spaces.

The concept of pairwise fuzzy P-spaces was introduced by G.Thangaraj and V.Chandiran in [11]. Several characterizations of pairwise fuzzy P-spaces are established by the authors in [6]. In this paper, the concept of pairwise fuzzy almost GP-spaces is introduced by means of pairwise fuzzy dense and pairwise fuzzy G_δ-sets Several characterizations of pairwise fuzzy almost GP-spaces are obtained. It is shown that the pairwise fuzzy almost GP-spaces, are pairwise fuzzy irresolvable spaces and pairwise fuzzy submaximal spaces are pairwise fuzzy almost GP-spaces. Also it is

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established that the pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space, are pairwise fuzzy almost GP-spaces . The conditions for the pairwise fuzzy almost GP-spaces spaces to become pairwise fuzzy irresolvable spaces are obtained. The conditions for the fuzzy bitopological spaces to become pairwise fuzzy σ-second category spaces and pairwise fuzzy weakly Volterra spaces are also established.

2. Preliminaries

In order to make the exposition self- contained, some basic notions and results used in the sequel are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I, the unit interval [0,1]. A fuzzy set λ in X is a function from X into I. The null set 0_X is the function from X into I which assumes only the value 0 and the whole fuzzy set 1_X is the function from X into I which takes 1 only. By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X, T_1, T_2), where T_1 and T_2 are fuzzy topologies on the non-empty set X.

Definition 2.1 [3] Let λ and μ be fuzzy sets in X. Then, for all $x \in X$:

(i) $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$.
(ii) $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$.
(iii) $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = \max\{\lambda(x), \mu(x)\}$.
(iv) $\delta = \lambda \wedge \mu \Leftrightarrow \delta(x) = \min\{\lambda(x), \mu(x)\}$.
(v) $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i \mid i \in I\}$ of fuzzy sets in (X, T), the union $\psi = \vee_i(\lambda_i)$ and intersection $\delta = \wedge_i(\lambda_i)$ are defined respectively as

(vi) $\psi(x) = \sup_i\{\lambda_i(x) \mid x \in X\}$.
(vii) $\delta(x) = \inf_i\{\lambda_i(x) \mid x \in X\}$.

Definition 2.2 [2] Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The interior and the closure of λ are defined respectively as follows:

(i) $int(\lambda) = \vee\{\mu \mid \mu \leq \lambda,\ \mu \in T\}$ and
(ii) $cl(\lambda) = \wedge\{\mu \mid \lambda \leq \mu,\ 1 - \mu \in T\}$.

2456-8686, 5(2), 2021:025-040
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Lemma 2.3 [2] For a fuzzy set λ of a fuzzy topological space X:

(i) $1 - int(\lambda) = cl(1-\lambda)$,
(ii) $1 - cl(\lambda) = int(1-\lambda)$.

Definition 2.4 [11] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy open set if $\lambda \in T_i$ $(i = 1, 2)$. The complement of pairwise fuzzy open set in (X, T_1, T_2) is called a pairwise fuzzy closed set.

Definition 2.5 [5] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy dense set if $cl_{T_1} cl_{T_2}(\lambda) = cl_{T_2} cl_{T_1}(\lambda) = 1$ in (X, T_1, T_2).

Definition 2.6 [8] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nowhere dense set if $int_{T_1} cl_{T_2}(\lambda) = int_{T_2} cl_{T_1}(\lambda) = 0$ in (X, T_1, T_2).

Definition 2.7 [8] Let (X, T_1, T_2) be the fuzzy bitopological space. A fuzzy set λ defined on X is called a pairwise fuzzy first category set if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy second category set in (X, T_1, T_2).

Definition 2.8 [8] If λ is a pairwise fuzzy first category set in the fuzzy bitopological space (X, T_1, T_2), then the fuzzy set $1 - \lambda$ is called a pairwise fuzzy residual set in (X, T_1, T_2).

Definition 2.9 [11] A fuzzy set λ defined on X in the fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy G_δ-set if $\lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy open sets in (X, T_1, T_2).

Definition 2.10 [11] A fuzzy set λ defined on X in the fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy F_σ-set if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy closed sets in (X, T_1, T_2).

Definition 2.11 [11] A fuzzy set λ defined on X in the fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy somewhere dense set in (X, T_1, T_2) if $int_{T_i} cl_{T_j}(\lambda) \neq 0$ $(i \neq j$ and $i, j = 1, 2)$. That is., λ is a pairwise fuzzy somewhere dense set in (X, T_1, T_2) if $int_{T_1} cl_{T_2}(\lambda) \neq 0$ and $int_{T_2} cl_{T_1}(\lambda) \neq 0$ in (X, T_1, T_2).

Definition 2.12 [11] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy P-space if every non-zero pairwise fuzzy G_δ-set in (X, T_1, T_2) is a pairwise fuzzy open set in (X, T_1, T_2). That is., if $\lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy open sets in (X, T_1, T_2), then λ is a pairwise fuzzy open set in (X, T_1, T_2).

Definition 2.13 [9] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy submaximal space if each pairwise fuzzy dense set in (X, T_1, T_2) is a pairwise fuzzy open set in (X, T_1, T_2). That is., if λ is a pairwise fuzzy dense set in the fuzzy bitopological space (X, T_1, T_2), then $\lambda \in T_i$ $(i = 1, 2)$.

Definition 2.14 [9] A fuzzy bitopological space (X, T_1, T_2) is said to be a pairwise fuzzy strongly irresolvable space if $cl_{T_1} int_{T_2}(\lambda) = 1 = cl_{T_2} int_{T_1}(\lambda)$ for each pairwise fuzzy dense set λ in (X, T_1, T_2).

Definition 2.15 [9] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nodec space if every non-zero pairwise fuzzy nowhere dense set in (X, T_1, T_2) is a pairwise fuzzy closed set in (X, T_1, T_2).

Definition 2.16 [14] A fuzzy set λ in the fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy σ-nowhere dense set if λ is a pairwise fuzzy F_σ-set in (X, T_1, T_2) such that $int_{T_1} int_{T_2}(\lambda) = int_{T_2} int_{T_1}(\lambda) = 0$, in (X, T_1, T_2).

Definition 2.17 [8] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy irresolvable space if for each pairwise fuzzy dense set λ in (X, T_1, T_2), $cl_{T_1} cl_{T_2}(1-\lambda) \neq 1$ and $cl_{T_2} cl_{T_1}(1 - \lambda) \neq 1$ in (X, T_1, T_2).

Definition 2.18 [8] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy Baire space if $int_{T_i}\left(\vee_{k=1}^{\infty}(\lambda_k)\right) = 0$ $(i = 1, 2)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2).

Definition 2.19 [8] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy first category space if the fuzzy set 1_X is a pairwise fuzzy first category set in (X, T_1, T_2). That is., $1_X = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Otherwise (X, T_1, T_2) will be called a pairwise fuzzy second category space.

2456-8686, 5(2), 2021:025-040
https://doi.org/10.26524/cm104

Definition 2.20 [14] Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy σ-first category set if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2). Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy σ-second category set in (X, T_1, T_2).

Definition 2.21 [14] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy σ-first category space if the fuzzy set 1_X is a pairwise fuzzy σ-first category set in (X, T_1, T_2). That is., $1_X = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2). Otherwise (X, T_1, T_2) will be called the pairwise fuzzy σ-second category space.

Definition 2.22 [11] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy weakly Volterra space if $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$, where (λ_k)'s are the pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2).

Definition 2.23 [7] A fuzzy bitopological space ((X, T_1, T_2) is said to be the pairwise fuzzy almost P-space if for each non-zero pairwise fuzzy G_δ-set λ in (X, T_1, T_2), $int_{T_i} int_{T_j}(\lambda) \neq 0$ ($i \neq j$ and $i, j = 1, 2$) in (X, T_1, T_2).

Theorem 2.24 [12] In the fuzzy bitopological space (X, T_1, T_2), the fuzzy set λ is a pairwise fuzzy σ-nowhere dense set in (X, T_1, T_2) if and only if $1 - \lambda$ is a pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2).

Theorem 2.25 [8] If λ is a pairwise fuzzy nowhere dense set in the fuzzy bitopological space (X, T_1, T_2), then $1-\lambda$ is the pairwise fuzzy dense set in (X, T_1, T_2).

Theorem 2.26 [9] If λ is the pairwise fuzzy dense and pairwise fuzzy G_δ-set in the pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then $1 - \lambda$ is the pairwise fuzzy first category set in (X, T_1, T_2).

Theorem 2.27 [10] If the fuzzy bitopological space (X, T_1, T_2) is the pairwise fuzzy submaximal space and λ is a pairwise fuzzy first category set in (X, T_1, T_2), then $1-\lambda$ is the pairwise fuzzy G_δ-set in (X, T_1, T_2).

Theorem 2.28 [10] If the fuzzy bitopological space (X, T_1, T_2) is the pairwise fuzzy submaximal space, then every pairwise fuzzy residual set is the pairwise fuzzy G_δ-set in (X, T_1, T_2).

Theorem 2.29 [10] If the fuzzy bitopological space (X, T_1, T_2) is the pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space, then (X, T_1, T_2) is the pairwise fuzzy submaximal space.

Theorem 2.30 [10] If every pairwise fuzzy G_δ-set is the pairwise fuzzy dense set in the pairwise fuzzy submaximal and pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then (X, T_1, T_2) is the pairwise fuzzy Baire space.

Theorem 2.31 [10] If every pairwise fuzzy G_δ-set is the pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space (X, T_1, T_2), then (X, T_1, T_2) is the pairwise fuzzy Baire space.

Theorem 2.32 [8] If the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy Baire space, then (X, T_1, T_2) is a pairwise fuzzy second category space.

Theorem 2.33 [15] If λ is the pairwise fuzzy σ-first category set in (X, T_1, T_2), then there is a pairwise fuzzy F_σ-set δ in (X, T_1, T_2) such that $\lambda \leq \delta$.

Theorem 2.34 [15] If $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$, where the fuzzy sets (λ_k)'s are pairwise fuzzy dense and pairwise fuzzy G_δ-sets in the fuzzy bitopological space (X, T_1, T_2), then (X, T_1, T_2) is the pairwise fuzzy σ-second category space.

Theorem 2.35 [13] If the fuzzy bitopological space (X, T_1, T_2) is the pairwise fuzzy σ-second category space, then (X, T_1, T_2) is the pairwise fuzzy weakly Volterra space.

3. Pairwise Fuzzy Almost GP-spaces

Definition 3.1 A fuzzy bitopological space (X, T_1, T_2) is said to be the pairwise fuzzy almost GP-space if for each non-zero pairwise fuzzy dense and pairwise fuzzy G_δ-set λ in (X, T_1, T_2), $int_{T_i} int_{T_j}(\lambda) \neq 0$ $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). That is., (X, T_1, T_2) is the pairwise fuzzy almost GP-space if for the pairwise fuzzy

G_δ-set λ with $cl_{T_1}cl_{T_2}(\lambda) = cl_{T_2}cl_{T_1}(\lambda) = 1$, in (X, T_1, T_2), $int_{T_1}int_{T_2}(\lambda) \neq 0$ and $int_{T_2}int_{T_1}(\lambda) \neq 0$, in (X, T_1, T_2).

Example 3.2 Let $X = \{a, b, c\}$. Consider the fuzzy sets λ, μ, γ, α and β defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.6$; $\lambda(b) = 0.4$; $\lambda(c) = 0.5$,

$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.4$; $\mu(b) = 0.7$; $\mu(c) = 0.6$,

$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.5$; $\gamma(b) = 0.3$; $\gamma(c) = 0.7$,

$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.5$; $\beta(b) = 0.2$; $\beta(c) = 0.7$.

$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.5$; $\alpha(b) = 0.4$; $\alpha(c) = 0.6$

Then, $T_1 = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \vee \gamma, \mu \vee \gamma, \lambda \wedge \mu, \lambda \wedge \gamma, \mu \wedge \gamma, \gamma \wedge (\lambda \vee \mu), \mu \wedge (\lambda \vee \gamma), \lambda \wedge (\mu \vee \gamma), \gamma \vee (\lambda \wedge \mu), \mu \vee (\lambda \wedge \gamma), \lambda \vee (\mu \wedge \gamma), \lambda \wedge \mu \wedge \gamma, \lambda \vee \mu \vee \gamma, 1\}$ and $T_2 = \{0, \lambda, \mu, \beta, \lambda \vee \mu, \lambda \vee \beta, \mu \vee \beta, \lambda \wedge \mu, \lambda \wedge \beta, \mu \wedge \beta, \beta \wedge (\lambda \vee \mu), \mu \wedge (\lambda \vee \beta), \lambda \wedge (\mu \vee \beta), \beta \vee (\lambda \wedge \mu), \mu \vee (\lambda \wedge \beta), \lambda \vee (\mu \wedge \beta), \lambda \wedge \mu \wedge \beta, \lambda \vee \mu \vee \beta, 1\}$ are fuzzy topologies on X. On computation, $\lambda, \mu, \lambda \vee \mu, \lambda \vee \gamma, \mu \vee \gamma, \lambda \wedge \mu, \mu \wedge (\lambda \vee \gamma), \lambda \wedge (\mu \vee \gamma), \gamma \vee (\lambda \wedge \mu), \mu \vee (\lambda \wedge \gamma), \lambda \vee (\mu \wedge \gamma), \lambda \vee \mu \vee \gamma, \lambda \vee \beta, \mu \vee \beta, \mu \wedge (\lambda \vee \beta), \lambda \wedge (\mu \vee \beta), \beta \vee (\lambda \wedge \mu), \mu \vee (\lambda \wedge \beta), \lambda \vee (\mu \wedge \beta), \lambda \vee \mu \vee \beta$ are pairwise fuzzy open sets in (X, T_1, T_2).

On computation, $\alpha = [\lambda \vee (\mu \wedge \gamma)] \wedge [\mu \vee (\lambda \wedge \gamma)] \wedge [\gamma \vee (\lambda \wedge \mu)]$ implies that α is the pairwise fuzzy G_δ-set in (X, T_1, T_2). Also $cl_{T_1}cl_{T_2}(\alpha) = 1$ and $cl_{T_2}cl_{T_1}(\alpha) = 1$, in (X, T_1, T_2). Hence α is the pairwise fuzzy dense and pairwise G_δ-set in (X, T_1, T_2). Also $int_{T_2}int_{T_1}(\alpha) = int_{T_2}[\lambda \wedge (\mu \vee \gamma)] = \lambda \wedge (\mu \vee \beta) \neq 0$ and $int_{T_1}int_{T_2}(\alpha) = int_{T_1}[\lambda \wedge (\mu \vee \beta)] = \lambda \wedge (\mu \vee \gamma) \neq 0$.

Now $\lambda \wedge \mu = (\lambda \vee \mu) \wedge (\lambda \vee \gamma) \wedge (\mu \vee \gamma) \wedge \lambda \wedge (\mu \vee \gamma) \wedge [\mu \wedge (\lambda \vee \beta)]$ and then $\lambda \wedge \mu$ is the pairwise fuzzy G_δ-set in (X, T_1, T_2). On computation one can find that $cl_{T_2}cl_{T_1}(\lambda \wedge \mu) = cl_{T_2}(1 - \lambda) = 1 - \lambda \neq 1$ and $cl_{T_1}cl_{T_2}(\lambda \wedge \mu) = cl_{T_1}(1 - \lambda) = 1 - \lambda \neq 1$. Hence $\lambda \wedge \mu$ is the pairwise fuzzy G_δ-set but not pairwise fuzzy dense set in (X, T_1, T_2).

Hence, for the pairwise fuzzy G_δ-set and pairwise fuzzy dense set α in (X, T_1, T_2), $int_{T_i}int_{T_j}(\alpha) \neq 0$ $(i \neq j$ and $i, j = 1, 2)$ and this shows that (X, T_1, T_2) is the pairwise fuzzy almost GP-space.

Remark 3.3 It is to be noted that pairwise fuzzy almost GP-spaces need not be pairwise fuzzy P-spaces. For, in example 3.2, α is the pairwise fuzzy G_δ-set in (X, T_1, T_2). But α is not the pairwise fuzzy open set in (X, T_1, T_2) and hence (X, T_1, T_2) is not the pairwise fuzzy P-space.

Proposition 3.4 If λ is the non-zero pairwise fuzzy dense and pairwise fuzzy G_δ-set in the pairwise fuzzy almost GP -space (X, T_1, T_2), then $1 - \lambda$ is not the pairwise fuzzy dense set in (X, T_1, T_2).

Proof: Let λ be the non-zero pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy almost GP-space, $int_{T_i} int_{T_j}(\lambda) \neq 0$ $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). Now $cl_{T_i} cl_{T_j}(1 - \lambda) = 1 - int_{T_i} int_{T_j}(\lambda) \neq 1$ and hence $1 - \lambda$ is not the pairwise fuzzy dense set in (X, T_1, T_2).

The following proposition shows that pairwise fuzzy nowhere dense and pairwise fuzzy F_σ-sets in pairwise fuzzy almost GP-spaces are not pairwise fuzzy dense sets.

Proposition 3.5 If λ is the non-zero pairwise fuzzy nowhere dense and pairwise fuzzy F_σ-set in the pairwise fuzzy almost GP-space (X, T_1, T_2), then λ is not the pairwise fuzzy dense set in (X, T_1, T_2).

Proof: Let λ be the non-zero pairwise fuzzy nowhere dense and pairwise fuzzy F_σ-set in (X, T_1, T_2). By the theorem 2.25, for the pairwise fuzzy nowhere dense set λ, $1 - \lambda$ is the pairwise fuzzy dense set in (X, T_1, T_2). Also $1 - \lambda$ is the pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy almost GP-space, $int_{T_i} int_{T_j}(1 - \lambda) \neq 0$ $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). Then, $1 - cl_{T_i} cl_{T_j}(\lambda) \neq 0$ and hence $cl_{T_i} cl_{T_j}(\lambda) \neq 1$. Thus, λ is not the pairwise fuzzy dense set in (X, T_1, T_2).

The following proposition shows that pairwise fuzzy σ-nowhere dense sets in pairwise fuzzy almost GP-spaces, are not pairwise fuzzy dense sets.

Proposition 3.6 If λ is the pairwise fuzzy σ-nowhere dense set in the pairwise fuzzy almost GP-space (X, T_1, T_2), then λ is not the pairwise fuzzy dense set in (X, T_1, T_2).

Proof: Let λ be the pairwise fuzzy σ-nowhere dense set in (X, T_1, T_2). Then, λ is the pairwise fuzzy F_σ-set such that $int_{T_1} int_{T_2}(\lambda) = 0 = int_{T_2} int_{T_1}(\lambda)$ in (X, T_1, T_2).

2456-8686, 5(2), 2021:025-040
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Now $cl_{T_i}cl_{T_j}(1-\lambda) = 1-int_{T_i}int_{T_j}(\lambda) = 1-0 = 1$ ($i \neq j$ and $i, j = 1, 2$) in (X, T_1, T_2). This implies that $1-\lambda$ is the pairwise fuzzy dense and pairwise fuzzy G_δ-set in the pairwise fuzzy almost GP-space (X, T_1, T_2). Now, $int_{T_i}int_{T_j}(1-\lambda) \neq 0$, in (X, T_1, T_2). Then, $1 - cl_{T_i}cl_{T_j}(\lambda) \neq 0$ and hence $cl_{T_i}cl_{T_j}(\lambda) \neq 1$ in (X, T_1, T_2). Thus, λ is not the pairwise fuzzy dense set in (X, T_1, T_2).

Proposition 3.7 If $int_{T_i}int_{T_j}\left(\wedge_{k=1}^{\infty}(\lambda_k)\right) \neq 0$, ($i \neq j$ and $i, j = 1, 2$), where (λ_k)'s are the pairwise fuzzy dense and pairwise fuzzy G_δ-sets in the fuzzy bitopological space (X, T_1, T_2), then (X, T_1, T_2) is the pairwise fuzzy almost GP-space.

Proof: Let (λ_k)'s ($k = 1$ to ∞) be the pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2). By the hypothesis, $int_{T_i}int_{T_j}\left(\wedge_{k=1}^{\infty}(\lambda_k)\right) \neq 0$, ($i \neq j$ and $i, j = 1, 2$), $int_{T_i}int_{T_j}\left(\wedge_{k=1}^{\infty}(\lambda_k)\right) \leq \wedge_{k=1}^{\infty}\left(int_{T_i}int_{T_j}(\lambda_k)\right)$, implies that $\wedge_{k=1}^{\infty}\left(int_{T_i}int_{T_j}(\lambda_k)\right) \neq 0$. Then $int_{T_i}int_{T_j}(\lambda_k) \neq 0$ and hence (X, T_1, T_2) is the pairwise fuzzy almost GP-space.

Proposition 3.8 If λ is the pairwise fuzzy dense and pairwise fuzzy G_δ-set in the pairwise fuzzy almost GP-space (X, T_1, T_2), then λ is the pairwise fuzzy somewhere dense set in (X, T_1, T_2).

Proof: Let λ be the pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy almost GP-space, $int_{T_i}int_{T_j}(\lambda) \neq 0$, ($i \neq j$ and $i, j = 1, 2$), in (X, T_1, T_2). Now $int_{T_i}int_{T_j}(\lambda) \leq int_{T_i}cl_{T_j}(\lambda)$ implies that $int_{T_i}cl_{T_j}(\lambda) \neq 0$ and hence λ is the pairwise fuzzy somewhere dense set in (X, T_1, T_2).

Proposition 3.9 If λ is the pairwise fuzzy dense and pairwise fuzzy G_δ-set in the pairwise fuzzy almost GP-space (X, T_1, T_2), then $cl_{T_i}int_{T_j}(1-\lambda) \neq 1$, ($i \neq j$ and $i, j = 1, 2$), in (X, T_1, T_2).

Proof: Let λ be the pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy almost GP-space and by proposition 3.8, λ is the pairwise fuzzy somewhere dense set in (X, T_1, T_2). Hence $int_{T_i}cl_{T_j}(\lambda) \neq 0$, ($i \neq j$ and $i, j = 1, 2$), in (X, T_1, T_2) and then $1 - int_{T_i}cl_{T_j}(\lambda) \neq 1$ and this implies that $cl_{T_i}int_{T_j}(1-\lambda) \neq 1$, in (X, T_1, T_2).

4. Pairwise Fuzzy Almost GP-spaces and Other Fuzzy Bitopological Spaces

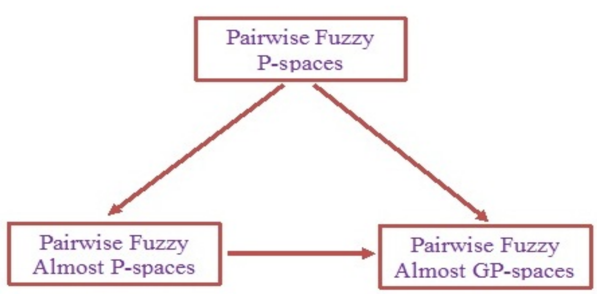
Proposition 4.1 If (X, T_1, T_2) is the pairwise fuzzy almost GP-space, then (X, T_1, T_2) is the pairwise fuzzy irresolvable space.

Proof: Let λ be the pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy almost GP-space and by the proposition 3.4, $1 - \lambda$ is not the pairwise fuzzy dense set in (X, T_1, T_2). Thus, for the pairwise fuzzy dense set λ in (X, T_1, T_2), $cl_{T_i} cl_{T_j}(1 - \lambda) \neq 1$, $(i \neq j$ and $i, j = 1, 2)$ implies that (X, T_1, T_2) is the pairwise fuzzy irresolvable space.

Proposition 4.2 If (X, T_1, T_2) is the pairwise fuzzy almost P-space, then (X, T_1, T_2) is the pairwise fuzzy almost GP-space.

Proof: Let (λ_k)'s $(k = 1$ to $\infty)$ be the pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2). Then by the theorem 2.24, $(1 - \lambda_k)$'s are pairwise fuzzy σ-nowhere dense sets in (X, T_1, T_2) and then $\vee_{k=1}^{\infty}(1 - \lambda_k)$ is the pairwise fuzzy σ-first category set in (X, T_1, T_2). By the theorem 2.33, there is a pairwise fuzzy F_σ-set δ in (X, T_1, T_2) such that $\vee_{k=1}^{\infty}(1 - \lambda_k) \leq \delta$. This implies that $1 - \wedge_{k=1}^{\infty}(\lambda_k) \leq \delta$ and $1 - \delta \leq \wedge_{k=1}^{\infty}(\lambda_k)$ and then $int_{T_i} int_{T_j}(1 - \delta) \leq int_{T_i} int_{T_j}(\wedge_{k=1}^{\infty}(\lambda_k))$. Since $1 - \delta$ is the pairwise fuzzy G_δ-set in the pairwise fuzzy almost P-space (X, T_1, T_2), $int_{T_i} int_{T_j}(1 - \delta) \neq 0$. This implies that $int_{T_i} int_{T_j}(\wedge_{k=1}^{\infty}(\lambda_k)) \neq 0$, in (X, T_1, T_2). Then by the proposition 3.7, (X, T_1, T_2) is the pairwise fuzzy almost GP-space.

Remark 4.3 The inter-relations between pairwise fuzzy P-spaces, pairwise fuzzy almost P-spaces and pairwise fuzzy almost GP-spaces can be summarized as follows:



Proposition 4.4 If the pairwise fuzzy G_δ-sets are pairwise fuzzy dense sets in the pairwise fuzzy irresolvable space (X, T_1, T_2), then (X, T_1, T_2) is the pairwise fuzzy almost GP-space.

Proof: Let λ be the pairwise fuzzy G_δ-set in (X, T_1, T_2). By the hypothesis, λ is the pairwise fuzzy dense set in (X, T_1, T_2). Then λ is the pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy irresolvable space, for the pairwise fuzzy dense set λ in (X, T_1, T_2), $1 - \lambda$ is not the pairwise fuzzy dense set in (X, T_1, T_2). Thus $cl_{T_i} cl_{T_j}(1 - \lambda) \neq 1$, $(i \neq j$ and $i, j = 1, 2)$ and this implies that $1 - int_{T_i} int_{T_j}(\lambda) \neq 1$ and hence $int_{T_i} int_{T_j}(\lambda) \neq 0$, for the pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). Hence (X, T_1, T_2) is the pairwise fuzzy almost GP-space.

Proposition 4.5 If the pairwise fuzzy G_δ-sets are pairwise fuzzy dense sets in the pairwise fuzzy almost GP-space, then (X, T_1, T_2) is the pairwise fuzzy almost P-space.

Proof: Let λ be the pairwise fuzzy G_δ-set in (X, T_1, T_2). By the hypothesis, λ is the pairwise fuzzy dense set in (X, T_1, T_2). Then λ is the pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy GP-space, $int_{T_i} int_{T_j}(\lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$, in (X, T_1, T_2). Thus, for the pairwise fuzzy G_δ-set λ, $int_{T_i} int_{T_j}(\lambda) \neq 0$, in (X, T_1, T_2), implies that (X, T_1, T_2) is the pairwise fuzzy almost P-space.

The following proposition shows that pairwise fuzzy submaximal spaces are pairwise fuzzy almost GP-spaces.

Proposition 4.6 If (X, T_1, T_2) is the pairwise fuzzy submaximal space, then (X, T_1, T_2) is the pair wise fuzzy almost GP-space.

Proof: Let λ be the non-zero pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy submaximal space, the pairwise fuzzy dense set λ is the pairwise fuzzy open set in (X, T_1, T_2) and then $int_{T_i}(\lambda) = \lambda$ $(i = 1, 2)$. This implies that $int_{T_i} int_{T_j}(\lambda) = int_{T_i}(\lambda) = \lambda \neq 0$, $(i \neq j$ and $i, j = 1, 2)$, Thus, for the pairwise fuzzy dense and pairwise fuzzy G_δ-set λ, $int_{T_i} int_{T_j}(\lambda) \neq 0$ in (X, T_1, T_2), implies that (X, T_1, T_2) is the pairwise fuzzy almost GP-space.

2456-8686, 5(2), 2021:025-040
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The following proposition shows that pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec spaces are pairwise fuzzy almost GP-spaces.

Proposition 4.7 If (X, T_1, T_2) is the pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space, then (X, T_1, T_2) is the pairwise fuzzy almost GP-space.

Proof: Since (X, T_1, T_2) is the pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space and by the theorem 2.29, (X, T_1, T_2) is the pairwise fuzzy submaximal space. By the proposition 4.6, (X, T_1, T_2) is the pairwise fuzzy almost GP-space.

Proposition 4.8 If $int_{T_i} int_{T_j} (\wedge_{k=1}^{\infty}(\lambda_k)) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$, where (λ_k)'s are the pairwise fuzzy dense and pair wise fuzzy G_δ-sets in the fuzzy bitopological space (X, T_1, T_2), then (X, T_1, T_2) is the pairwise fuzzy almost GP-space and the pairwise fuzzy σ-second category space.

Proof: Suppose that (λ_k)'s $(k = 1$ to $\infty)$ are the pairwise fuzzy dense and pairwise fuzzy G_δ-sets in (X, T_1, T_2) such that $int_{T_i} int_{T_j} (\wedge_{k=1}^{\infty}(\lambda_k)) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$. Then by the proposition 3.7, (X, T_1, T_2) is the pairwise fuzzy almost GP-space. Since $int_{T_i} int_{T_j} (\wedge_{k=1}^{\infty}(\lambda_k)) \leq \wedge_{k=1}^{\infty}(\lambda_k)$, $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$, in (X, T_1, T_2). Then by the theorem 2.34, (X, T_1, T_2) is the pairwise fuzzy σ-second category space.

Proposition 4.9 If $int_{T_i} int_{T_j} (\wedge_{k=1}^{\infty}(\lambda_k)) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$, where (λ_k)'s are the pairwise fuzzy dense and pair wise fuzzy G_δ-sets in the fuzzy bitopological space (X, T_1, T_2), then (X, T_1, T_2) is the pairwise fuzzy almost GP-space and the pairwise fuzzy weakly Volterra space.

Proof: The proof follows from the proposition 4.8 and the theorem 2.35.

Proposition 4.10 If each pairwise fuzzy G_δ-set is the pairwise fuzzy dense set in the pair wise fuzzy submaximal and pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then (X, T_1, T_2) is the pairwise fuzzy Baire and pairwise fuzzy almost GP-space.

Proof: Let λ be the pairwise fuzzy G_δ-set in (X, T_1, T_2). By the hypothesis, λ is the pairwise fuzzy dense set in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy

2456-8686, 5(2), 2021:025-040
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strongly irresolvable and pairwise fuzzy submaximal space and by the theorem 2.30, (X, T_1, T_2) is the pairwise fuzzy Baire space. Also since (X, T_1, T_2) is the pairwise fuzzy submaximal space and by the proposition 4.6, (X, T_1, T_2) is the pairwise fuzzy almost GP-space. Hence (X, T_1, T_2) is the pairwise fuzzy Baire and pairwise fuzzy almost GP-space.

Proposition 4.11 If each pairwise fuzzy G_δ-set is the pairwise fuzzy dense set in the pairwise fuzzy submaximal and pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then (X, T_1, T_2) is the pairwise fuzzy second category and pairwise fuzzy almost GP-space.

Proof: The proof follows from the proposition 4.10 and the theorem 2.32.

Proposition 4.12 If every pairwise fuzzy G_δ-set is the fuzzy pairwise fuzzy dense set in a pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space (X, T_1, T_2), then (X, T_1, T_2) is the pairwise fuzzy Baire and pairwise fuzzy almost GP-space.

Proof: The proof follows from the proposition 4.7 and the theorem 2.31.

Proposition 4.13 If each pairwise fuzzy G_δ-set is the pairwise fuzzy dense set in the pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec space (X, T_1, T_2), then (X, T_1, T_2) is the pairwise fuzzy second category and pairwise fuzzy almost GP-space.

Proof: The proof follows from the proposition 4.12 and the theorem 2.32.

Proposition 4.14 If λ is the pairwise fuzzy dense and pairwise fuzzy G_δ-set in the pairwise fuzzy strongly irresolvable and pairwise fuzzy almost GP-space (X, T_1, T_2), then the pairwise fuzzy first category set $1 - \lambda$ is not the pairwise fuzzy dense set in (X, T_1, T_2).

Proof: Let λ be the pairwise fuzzy dense and pairwise fuzzy G_δ-set in ((X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy strongly irresolvable space and by the theorem 2.26, $1-\lambda$ is the pairwise fuzzy first category set in (X, T_1, T_2). Also since (X, T_1, T_2) is the pairwise fuzzy almost GP-space, $int_{T_i} int_{T_j}(\lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$. Now

$cl_{T_i}cl_{T_j}(1-\lambda) = 1 - int_{T_i}int_{T_j}(\lambda) \neq 1$ and this implies that $1-\lambda$ is not the pairwise fuzzy dense set in $((X, T_1, T_2)$.

Proposition 4.15 If λ is the pairwise fuzzy dense and pairwise fuzzy G_δ-set in the pairwise fuzzy strongly irresolvable and pairwise fuzzy almost GP-space (X, T_1, T_2), then λ is the pairwise fuzzy residual set in (X, T_1, T_2) such that $int_{T_i}int_{T_j}(\lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$.

Proof: Let λ be the pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy strongly irresolvable space and by the theorem 2.26, $1-\lambda$ is the pairwise fuzzy first category set in (X, T_1, T_2) and then λ is the pairwise fuzzy residual set in (X, T_1, T_2). Also since (X, T_1, T_2) is the pairwise fuzzy almost GP-space, $int_{T_i}int_{T_j}(\lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$. Thus, λ is the pairwise fuzzy residual set in (X, T_1, T_2) such that $int_{T_i}int_{T_j}(\lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$.

Proposition 4.16 If λ is the pairwise fuzzy first category set such that $int_{T_i}int_{T_j}(\lambda) = 0$, $(i \neq j$ and $i, j = 1, 2)$ in the pairwise fuzzy submaximal space (X, T_1, T_2), then λ is not the pairwise fuzzy dense set in (X, T_1, T_2).

Proof: Let λ be the pairwise fuzzy first category set in (X, T_1, T_2). Then by the theorem 2.27, $1-\lambda$ is the pairwise fuzzy G_δ-set in (X, T_1, T_2). Since $int_{T_i}int_{T_j}(\lambda) = 0$, $(i \neq j$ and $i, j = 1, 2)$, $cl_{T_i}cl_{T_j}(1-\lambda) = 1 - int_{T_i}int_{T_j}(\lambda) = 1 - 0 = 1$ and then $1-\lambda$ is the pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). By the proposition 4.6, the pairwise fuzzy submaximal space (X, T_1, T_2) is the pairwise fuzzy almost GP-space and then $int_{T_i}int_{T_j}(1-\lambda) \neq 0$. This implies that $1 - cl_{T_i}cl_{T_j}(\lambda) \neq 0$ and thus $cl_{T_i}cl_{T_j}(\lambda) \neq 1$, in $(X, T_1, T_2))$. Hence λ is not the pairwise fuzzy dense set in $(X, T_1, T_2))$.

Proposition 4.17 If the pairwise fuzzy residual set λ is the pairwise fuzzy dense set in the pairwise fuzzy submaximal space (X, T_1, T_2), then $int_{T_i}int_{T_j}(\lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2).

Proof: Let λ be the pairwise fuzzy residual set in (X, T_1, T_2) such that $cl_{T_i}cl_{T_j}(\lambda) = 1$, $(i \neq j$ and $i, j = 1, 2)$. Since (X, T_1, T_2) is the pairwise fuzzy submaximal space and by the theorem 2.28, the pairwise fuzzy residual set λ is the pairwise fuzzy

G_δ-set in (X, T_1, T_2). Hence λ is the pairwise fuzzy dense and pairwise fuzzy G_δ-set in (X, T_1, T_2). By the proposition 4.6, the pairwise fuzzy submaximal space (X, T_1, T_2) is the pairwise fuzzy almost GP-space and then $int_{T_i} int_{T_j}(\lambda) \neq 0$, $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2).

Proposition 4.18 If $\lambda \leq 1-\eta$, where λ is the pairwise fuzzy dense and pairwise fuzzy G_δ-set and η is the pairwise fuzzy G_δ-set in the pairwise fuzzy strongly irresolvable space (X, T_1, T_2), then (X, T_1, T_2) is not the pairwise fuzzy almost P-space.

Proof: Let λ be the pairwise fuzzy dense and pairwise fuzzy G_δ-set and η is the pairwise fuzzy G_δ-set in (X, T_1, T_2). Since (X, T_1, T_2) is the pairwise fuzzy strongly irresolvable space, for the pairwise fuzzy dense set λ in (X, T_1, T_2), $cl_{T_i} int_{T_j}(\lambda) = 1$, $(i \neq j$ and $i, j = 1, 2)$. Now $\lambda \leq 1-\eta$ implies that $cl_{T_i} int_{T_j}(\lambda) \leq cl_{T_i} int_{T_j}(1-\eta)$ and then $1 \leq cl_{T_i} int_{T_j}(1-\eta)$. That is., $cl_{T_i} int_{T_j}(1-\eta) = 1$. This implies that $1 - int_{T_i} cl_{T_j}(\eta) = 1$. Then $int_{T_i} cl_{T_j}(\eta) = 0$. Since $int_{T_i} int_{T_j}(\eta) \leq int_{T_i} cl_{T_j}(\eta)$, $int_{T_i} int_{T_j}(\eta) = 0$ in (X, T_1, T_2). Thus, for the pairwise fuzzy G_δ-set η, $int_{T_i} int_{T_j}(\eta) = 0$ in (X, T_1, T_2) implies that (X, T_1, T_2) is not the pairwise fuzzy almost P-space.

5. Conclusion

In this paper, the concept of pairwise fuzzy almost GP-spaces is introduced by means of pairwise fuzzy dense and pairwise fuzzy G_δ-sets. It is established that pairwise fuzzy nowhere dense, pairwise fuzzy F_σ-sets, pairwise fuzzy σ-nowhere dense sets are not pairwise fuzzy dense sets in pairwise fuzzy almost GP-spaces. Also it is obtained that pairwise fuzzy dense and pairwise fuzzy G_δ-sets in pairwise fuzzy almost GP-spaces are pairwise fuzzy somewhere dense sets. It is established that pairwise fuzzy almost GP-spaces are pairwise fuzzy irresolvable spaces and pairwise fuzzy almost P-spaces are pairwise fuzzy almost GP-spaces and pairwise fuzzy submaximal spaces, pairwise fuzzy strongly irresolvable and pairwise fuzzy nodec spaces are pairwise fuzzy almost GP-spaces. The conditions for pairwise fuzzy irresolvable spaces to become pairwise fuzzy almost GP-spaces and the conditions for fuzzy bitopological spaces to become pairwise fuzzy σ-second category spaces and pairwise fuzzy weakly Volterra spaces are also established.

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