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# A Study on Stochastic Modelling of the Repairable system

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**Abstract**

All reliability models consisting of random time factors form stochastic processes. In this paper we recall the definitions of the most common point processes which are used for modelling of repairable systems. Particularly this paper presents stochastic processes as examples of reliability systems for the support of the maintenance related decisions. We consider the simplest one-unit system with a negligible repair or replacement time, i.e., the unit is operating and is repaired or replaced at failure, where the time required for repair and replacement is negligible. When the repair or replacement is completed, the unit becomes as good as new and resumes operation. The stochastic modelling of recoverable systems constitutes an excellent method of supporting maintenance related decision-making processes and enables their more rational use.

**Key words:** Repairable System, Point Process, Stochastic Modelling, Poisson Process.

**AMS classification:** 60H10, 60H30, 60H35.

## 1 Introduction

n maintenance modelling of a technical object, mathematics find applications in work sampling, inventory control analysis, failure data analysis, establishing optimum preventive maintenance policies, maintenance cost analysis, and project management control. Some of the areas of mathematics used in maintenance include set theory, probability, calculus, differential equations, stochastic processes, and Laplace transforms. A wide knowledge of probabilities, statistics, and stochastic processes is needed for learning the reliability theory mathematically. In the reliability theory, stochastic processes are the most powerful mathematical tools for analyzing reliability models. High system reliability can be achieved by maintenance. A classical problem is to determine how reliability can be improved by using mathematical models. For this reason this paper presents essential mathematical concepts of stochastic models

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for the repairable system. In order to minimize failures in engineering systems, the designer must understand "why" and "how" failures occur. This helps them prevent failures. In order to maximize system performance, it is also important to know how often such failures may occur. This involves predicting the occurrence of failures. The knowledge of stochastic processes and mathematical tools is indispensable for engineers, managers and researchers in reliability and maintenance.

## 2 Maintenance and failure taxonomies

This section presents some terms and definitions directly or indirectly used in engineering maintenance.

- repairable system a system which, after failing to perform one or more of its functions satisfactorily, can be restored to fully satisfactory performance by any method, other than replacement of the entire system;
- repair a restoration wherein a failed system (device) is returned to operable condition;
- perfect repair a repair under which a failed system is replaced with a new identical one;
- minimal repair a repair of limited effort wherein the device is returned to the operable state it was in just before failure;
- degraded failure the component is not in the state which the manufacturer intended for performing its function, but the system function is being fulfilled;
- critical failure the component is unable to perform its function due to critical-degradation of its state. Critically-degraded components are always repaired or renewed, and the repair typically begins as soon as possible;
- total failure a component not only fails to meet specifications, but fails to meet specifications to any degree. Non-total critical failure occurs when the component fails to meet specifications but still has some residual functionality;
- maintenance all actions appropriate for retaining an item/part/equipment in, or restoring it to a given condition;
- maintenance engineering the activity of equipment/item maintenance that develops concepts, criteria, and technical requirements in conceptual and acquisition phases to be used and maintained in a current status during the operating phase in order to assure effective maintenance support of equipment;
- preventive maintenance (PM) all actions carried out on a planned, periodic,

and specific schedule to keep an item/equipment in stated working condition through the process of checking and reconditioning. These actions are precautionary steps undertaken to forestall or lower the probability of failures or an unacceptable level of degradation in later service, rather than correcting them after they occur;

- corrective maintenance (CM) the unscheduled maintenance or repair in order to return items/equipment to a defined state, carried out because maintenance persons or users perceived deficiencies or failures;
- predictive maintenance the use of modern measurement and signal processing methods to accurately diagnose item/equipment condition during operation;
- maintenance concept a statement of the overall concept of the item/product specification or policy that controls the type of maintenance action to be employed for the item under consideration;
- maintenance plan a document that outlines the management and technical procedures to be employed in order to maintain an item; usually describes facilities, tools, schedules, and resources;
- reliability the probability that an item will perform its stated function satisfactorily for the desired period when used under specified conditions;
- maintainability the probability that a failed item will be restored to an adequate working condition;
- mean time to repair (MTTR) a figure of merit depending on item maintainability equal to the mean item repair time. In the case of exponentially distributed times to repair, MTTR is the reciprocal of the repair rate;
- overhaul a comprehensive inspection and restoration of an item or a piece of equipment to an acceptable level at a durability time or usage limit;
- quality the degree to which an item, function, or process satisfies requirements of the customer and user;
- maintenance person an individual who conducts preventive maintenance and responds to a user's service call to a repair facility, and performs corrective maintenance on an item. Also called custom engineer, service person, technician, field engineer, mechanic, repair person, etc.;
- inspection the qualitative observation of an item performance or condition.

Maintenance of units after failure may be costly, and sometimes requires a long time. Thus, the most important problem is to determine when and how to preventively maintain units before failure. However, it is not wise to maintain units with unnecessary frequency. From such viewpoints, the commonly considered maintenance policies are preventive replacement for units without repair, and preventive maintenance for units with repair on a specific schedule. Classifying into three large groups, planned maintenance of units is carried out at a certain age, after a certain period of time, or after a specified number of occurrences.

## 3 Poisson process as a model of a repairable system

If failures occur exponentially, i.e., the unit fails constantly during a time interval $(t, t+\mathrm{d}t]$ irrespective of time $t$, the system forms a Poisson process. Roughly speaking, failures occur randomly in $(t, t+\mathrm{d}t]$ with probability $\lambda \mathrm{d}t$ for constant $\lambda > 0$; and interarrival times between failures have an exponential distribution $\left(1-e^{-\lambda t}\right)$. Then, it is said that failures occur in a Poisson process with rate $\lambda$.

We consider a Poisson process with only one state in which the unit is operating. When we consider a one-unit system whose repair time is non-negligible, the process forms an alternating renewal process with two states that repeats up and down alternately.

Let $N(t)$ denote the number of failures (events) in the time interval $(0, t]$ and let $T_i$ be the time of the $i$th failure. The times $T_i$ are called failure times or event times. Define $T_0 = 0$ and denote $X_i = T_i - T_{i-1}, i = 1, 2, \ldots, -$ the time between failure number $i-1$ and failure number $i$. The times $X_i$ are called working times or waiting times and also inter-arrival times. The observed sequence $\{T_i, i = 1, 2, \ldots\}$ of failure times $T_1, T_2, \ldots$ forms a point process, and $\{N(t), t \geq 0\}$ is the corresponding counting process. In the context of failure-repair models it is assumed here that all repair times are equal to 0. In practice this corresponds to the situation, when repair actions are conducted immediately or the repair times can be neglected with comparison to the working times $X_i$.

The process $N(t), t \geq 0$ is said to be a (homogeneous) Poisson process with rate (or intensity) $\lambda > 0$ if

- **(i)** $N(0) = 0$; i.e. there are no events at time 0 ;
- **(ii)** The number of events $N(s_2) - N(s_1)$ and $N(t_2) - N(t_1)$ in disjoint time intervals $(s_1, s_2]$ and $(t_1, t_2]$ are independent random variables (independent increments);
- **(iii)** the distribution of the number of events in a certain interval depends only on the length of the interval and not on its position (stationary increments);

**(iv)** there exists a constant $\lambda$ such that $\lim_{\Delta t \to 0} \frac{\mathbb{P}(N(\Delta t)=1)}{\Delta t} = \lambda$;
**(v)** $\lim_{\Delta t \to 0} \frac{\mathbb{P}(N(\Delta t)\geq 2)}{\Delta t} = 0$.

The process defined above is denoted by HPP($\lambda$). Note that if the random variables $X_1, X_2, \ldots$ (waiting times) are independent and exponentially distributed Exp $\left(\frac{1}{\lambda}\right)$, the counting process $\{N(t), t \geq 0\}$ is the HPP($\lambda$). The corresponding sequence $\{T_i, i = 1, 2, \ldots\}$ is also called the HPP($\lambda$). Let us note that in the HPP($\lambda$) the number of events in an interval of length $t$ is a random variable having the Poisson distribution with the parameter $\lambda t$, and that the time of the $n$-th event is a random variable having Erlang distribution with the parameters $n$ and $1/\lambda$. Farther practical properties of the Poisson process are formulated.

## 4 Superposition of Poisson process.

Let $N_1(t)$ be the number of failed units from a certain population in $(0, t]$ and $N_2(t)$ be the number of failed units from the other. If the arrival times from two populations are independent and have the respective Poisson processes with rates $\lambda_1$ and $\lambda_2$, the total number $N(t)$ of failed units arriving at a repair shop has the probability

$$\mathbb{P}(N(t) = n) = \frac{((\lambda_1 + \lambda_2)\, t)^n}{n!} e^{-(\lambda_1+\lambda_2)t}$$

Thus, the process $\{N(t), t > 0\}$ is also a Poisson process with rate $\lambda_1 + \lambda_2$.

## 5 Decomposition of Poisson process.

Let $N(t)$ be the number of failed units occurring in $(0, t]$ and be a Poisson process with rate $\lambda$. Classifying into two large groups of failed units, the number $N_1(t)$ of minor ones occurs with probability $p$ and the number $N_2(t)$ of major ones occurs with probability $q = 1 - p$, independent of $N_1(t)$. Then, because the number $N_1(t)$ has a binomial distribution with parameter $p$, given that $n$ failures have occurred, the joint probability is

$$\mathbb{P}\left(N_1(t) = k, N_2(t) = n - k\right) = \frac{(p\lambda t)^k}{k!} e^{-p\lambda t} \frac{(pqt)^{n-k}}{(n-k)!} e^{-q\lambda t}$$

Thus, the two Poisson processes $(N_1(t), t > 0)$ and $(N_2(t), t > 0)$ are independent and have the Poisson processes with rates $p\lambda$ and $q\lambda$, respectively. In general, when the total number $N(t)$ of failed units with a Poisson process with rate $\lambda$ is classified into

$k$ groups of $N_j(t), j = 1, \ldots, k$ with probability $p_j$ where $\sum_{j=1}^{k} N_j(t) = N(t)$ and $\sum_{j=1}^{k} p_j = 1$, the joint probability is

$$\mathbb{P}\left(N_1(t) = n_1, \ldots, N_k(t) = n_k\right) = \prod_{j=1}^{k} \frac{(p_j \lambda t)^{n_j}}{n_j!} e^{-p_j \lambda t}$$

that is called a multi-Poisson process.

Next, when events occur in a Poisson process, we obtain the distribution of the inter-arrival time $X$; given that there was an event in $[0, t]$. This probability is given, for $u \leq t$

$$\mathbb{P}(X \leq u \mid N(t) = 1) = \frac{\mathbb{P}(N(u) = 1, N(t) - N(u) = 0)}{\mathbb{P}(N(t) = 1)} = \frac{\lambda u e^{-\lambda u} e^{-\lambda(t-u)}}{\lambda t e^{-\lambda t}} = \frac{u}{t}$$

that is a uniform distribution over $[0, t]$. That is, when the event was detected at time $t$; it occurs constantly over $[0, t]$.

## 6 Conclusion

In this paper a need of the stochastic modelling was justified as well as definitions applied in the maintenance engineering were quoted. Moreover the theoretical bases of a stochastic process were explained and its various properties were derived. In particular, for the modelling of the system maintenance process a homogeneous, a non homogeneous and a compound Poisson process were investigated. To apply processes to reliability models, we took up periodic replacements as well as shock and damage models.

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