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**ORIGINAL RESEARCH** **Open Access**

# Dominator Color class Dominating sets on Ladder, Open Ladder and Slanting Ladder Graphs

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**Abstract**

A proper coloring $\mathcal{C}=\{\mathcal{C}_1,\mathcal{C}_2,\ldots\ldots\ldots\mathcal{C}_\chi\}$ of a graph $G=(V,E)$ is a dominator color class dominating set of $G$ if each $v\in V(G)$ is dominated by a color class $\mathcal{C}_i\in$ and each $\mathcal{C}_i\in\mathcal{C}$ dominates a vertex $u\in V(G)$.The minimum number of color classes that satisfy the above condition is known as dominator color class domination number of $G$ and is denoted by $\gamma_\chi^d(G)$. In this paper, we find the dominator color class domination number of ladder, open ladder and slanting ladder graphs.

**Key Words:** dominator color class dominating sets, dominated color class domination number.

**AMS Classification:** 05C69.

## 1 Introduction

In this paper, we only consider ladder,open ladder and slanting ladder graphs. Further terminology can be found in [3].

A proper coloring $\mathcal{C}=\{\mathcal{C}_1,\mathcal{C}_2,\ldots\ldots\ldots\mathcal{C}_\chi\}$ of a graph $G=(V,E)$ is a dominator color class dominating set of $G$ if each $v\in V(G)$ is dominated by a color class $\mathcal{C}_i\in$ and each $\mathcal{C}_i\in\mathcal{C}$ dominates a vertex $u\in V(G)$.The minimum number of color classes that satisfy the above condition is known as dominator color class domination number of $G$ and is denoted by $\gamma_\chi^d(G)$.This notion was introduced by A. Vijayalekshmi and P. Niju in [2].

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The cartesian product of two graphs $G_1 = (V_1, E_1)$and $G_2 = (V_2, E_2)$ , denoted by $G_1 \times G_2$ is a graph with vertex set $V_1 \times V_2$ and edges between two vertices $(u_1, v_1)$ and $(u_2, v_2)$ if and only if either $u_1 = u_2$ and $v_1v_2 \in E(G_2)$ (or)$u_1u_2 \in E(G_1)$ $and$ $v_1 = v_2$. A ladder graph can be defined as $P_2 \times P_n$ where $n \geq 2$ and is denoted by $and\ L_p \left|V(L_p)\right| = 2n\ , n \geq 2$.An open ladder graph$O(L_p)$, is obtained from two paths of length $n-1$ with $V(L_p) = \{u_iv_i / \ 1 \leq i \leq n\}$ and $E(L_p) = \{u_iu_{i+1}, v_iv_{i+1} / \ 1 \leq i \leq n-1\} \cup \{u_iv_i :\ 2 \leq i \leq n-1\}$. A slanting ladder graph $SL_p = SL_{2n}$ $(n \geq 3)$ is agraph obtained from two paths $u_1, u_2 \ldots\ldots u_n$ and $v_1, v_2 \ldots\ldots v_n$by joining each $u_i$with $v_{i+1}(\ 1 \leq i \leq n-1)$. In this paper, we obtain dominator color class domination number of ladder graph, open ladder and slanting ladder graph.

## 2 Main Results

**Theorem 2.1**. Let $L_p$ be a ladder graph of order $2n$. Then

$$\gamma_\chi^d(L_p) = \begin{cases} p & if\ p\ is\ even \\ p+1 & if\ p\ is\ odd \end{cases}.$$

**Proof:** Let $L_p = L_{2n} = p_2 \times p_n$ and let $V(L_p) = \{u_1, u_2, \ldots\ldots u_{2n}\}$withdeg$u_i = 2$ for

$i = 1, n, (n+1), 2n$ and deg$u_j = 3$ for all $j \neq i$.

We take $N(u_i) = \{u_{i-1}, u_{i+1}, u_{i+n}\}$ for $i = 2,3, \ldots\ldots\ldots (n-1)$ and

$N(u_j) = \{u_{j-1}, u_{j+1}, u_{j-n}\}$ for $j = (n+2), (n+3), \ldots\ldots (2n-1)$. We consider two cases

**Case (1):** When $p \equiv 0 (mod 2)$

Decompose $L_p$ in to $\frac{p}{2}$ copies of $L_2$. Assign new colors, say $2i-1, 2i$ $(1 \leq i \leq \frac{p}{2})$to the vertices $\{u_{2i-1}, u_{2i+n}\}$ and $\{u_{2i}, u_{2i+(n-1)}\}$ respectively. We attain a $\gamma_\chi^d$ coloring of $L_p$. So $\gamma_\chi^d(L_p) = p$.

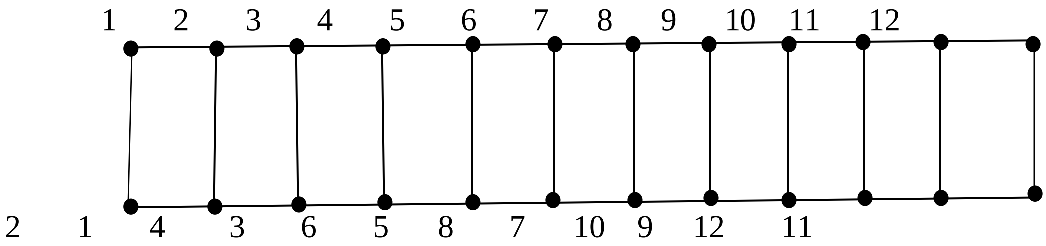


**Figure1** $\gamma_\chi^d(L_{12}) = 12$

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**Case (2):** When$p \equiv 1(mod 2)$

Since $p-1 \equiv 0(mod 2)$, $L_p$ is obtained from, $L_{p-1}$followed by $p_2$. As in case (i) $\gamma_\chi^d(L_{p-1}) = p-1$. Assign two new colors, say, $p \; and \; p+1$ to the vertices $\{u_n\}$ and $\{u_{2n}\}$ respectively. We get a $\gamma_\chi^d$- coloring of $L_p$.Thus

$$\gamma_\chi^d(L_p) = p+1.$$

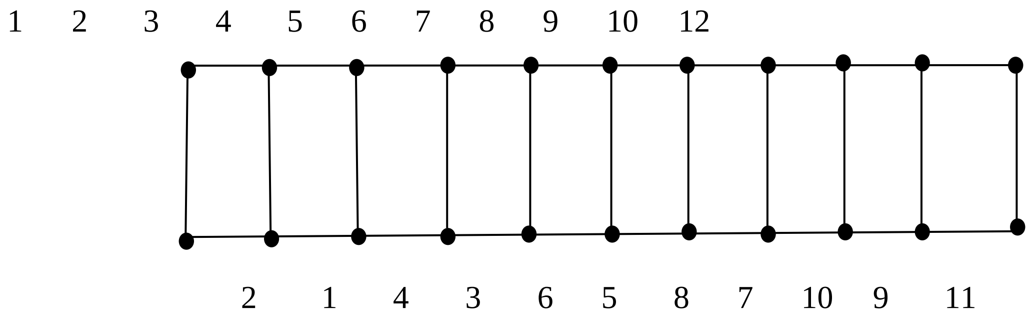


**Figure2** $\gamma_\chi^d(L_{11}) = 12$

**Theorem 2.2**. Let $O(L_p)$ be an open ladder graph. Then

$$\gamma_\chi^d(O(L_p) = \begin{cases} p+1 & if \quad p \equiv 1(mod\ 4) \\ p+2 & otherwise \end{cases}$$

**Proof:** Let $V(O(L_p))$={$u_1, u_2, \ldots \ldots \ldots \ldots \ldots \ldots u_{2n}$}with deg $u_i = 1$ $for \; i = 1, n, (n+1), 2n$ and deg $u_j$=3 $\forall \; j \neq i$ .

We have 2 cases.

**Case (1):** When $p \equiv 1(mod\ 4)$

Assign four distinct colors, say 1,2, $p \; and \; p+1$ to the pendent vertices $\{u_1\}, \{u_n\}, \{u_{n+1}\}$and $\{u_{2n}\}$respectively.

Let $H = \langle u_2, \ldots\ldots u_{n-1}, u_{n+1}, \ldots \ldots u_{2n-1} \rangle$. Then $H \cong L_{p-2}$. Decompose $H$ into $\left\lfloor \frac{P-2}{4} \right\rfloor$ copies of $L_4$ and one copy of $L_3$.

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Assign distinct colors $4i-1, 4i, 4i+1$ $and$ $4i+2$ $(1 \le i \le \left\lfloor\frac{p-2}{4}\right\rfloor$ to the vertices $\{u_{4i-1},\ u_{n+4i-2},\ u_{n+4i}\}, \{u_{4i-2},\ u_{4i},\ u_{n+4i-1}\}, \{u_{4i+1}\}$ $and$ $\{u_{n+4i+1}\}$

respectively. Also assign 2 distinct colors say, $(p-2)$and $(p-1)$ to the vertices

$\{u_{n-3}, u_{n-1}, u_{2n-2}\}$ and $\{u_{n-2}, u_{2n-3}, u_{2n-1}\}$ respectively, we attain a $\gamma_\chi^d$ – coloring of $O(L_p)$. Thus $\gamma_\chi^d\left(O(L_p)\right) = p+1$.

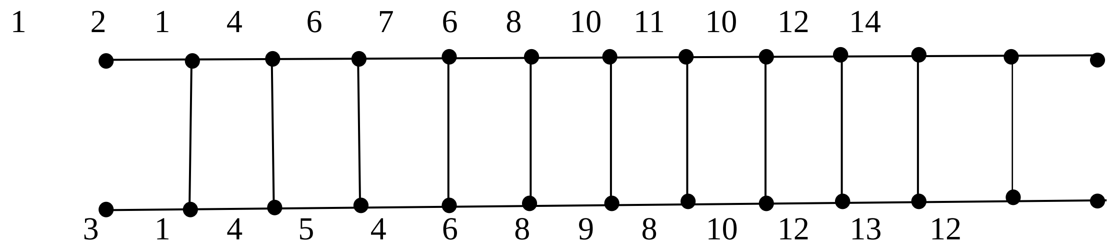


**Figure 3** $\gamma_\chi^d\big(O(L_{13})\big) = 14$

**Case (2):** When $p \equiv 0, 2,3\ (mod 4)$

We have three subcases

**Subcase 2.1:** When $p \equiv 0 (mod 4)$

Assign 4 distinct colors say $1, 2, (p+1)$ $and$ $(p+2)$ to the pendant vertices of $L_p$. As in case (1) $H$ is decomposed into $\left\lfloor\frac{p-2}{4}\right\rfloor$ copies of $L_4$ and one copy of $L_2$.Assign distinct colors, say $4i-1, 4i, 4i+1$ $and$ $4i+2$ $(1 \le i \le \left\lfloor\frac{p-2}{4}\right\rfloor$ to the same vertices as in case (1). Also assign 2 distinct colors say $(p-1)$ and $p$ to the vertices $\{u_{n-2}, u_{2n-1}\}$ and $\{u_{n-1}, u_{2n-2}\}$ respectively. We attain a $\gamma_\chi^d$ – coloring of $L_p$. Thus $\gamma_\chi^d\big(O(L_p)\big) = p+2$.

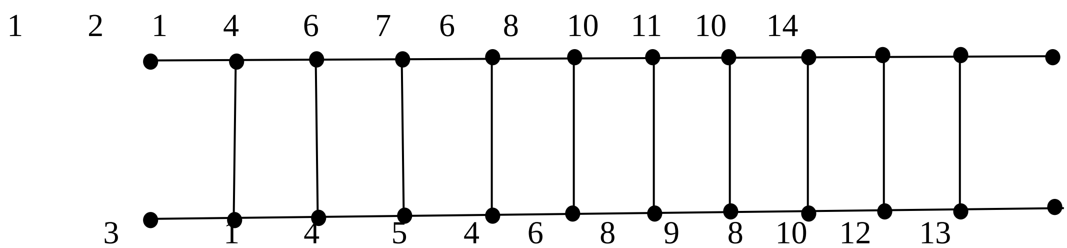


**Figure4** $\gamma_\chi^d(O(L_{12})) = 14$

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**Subcase 2.2:** When $p \equiv 2(mod4)$

Since $p-2 \equiv 0(mod4)$ and as in case (i), $H$ can be decomposed into $\frac{p-2}{4}$ copies of $L_4$. Assign each 4 distinct colors $4i-1, 4i, 4i+1$ $and$ $4i+2$ $(1 \le i \le \frac{p-2}{4})$ to the said vertices above, we get a $\gamma_\chi^d$ – coloring of $L_p$. So $\gamma_\chi^d(L_p) = 4 + \left(\frac{p-2}{4}\right)4 = p+2$.

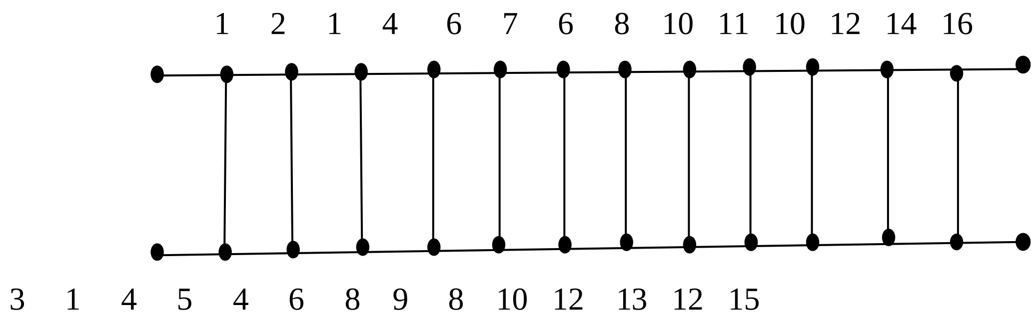


**Figure 5** $\gamma_\chi^d(L_{14}) = 16$

**Subcase 2.3:** When $p \equiv 3(mod4)$

Assign 3 distinct colors say 1,2 and $(p+2)$ to the pendant vertices say$\{u_1\}, \{u_2\}$ $and$ $\{u_{2n}\}$ respectively. Let $H = \langle u_2, \dots . u_{n-2}, u_{n+2}, u_{n+3}, \dots . u_{2n-2}\rangle$and $H$ can be decomposed in$\left\lfloor\frac{p-2}{4}\right\rfloor$ copies of $L_4$. Apply the same coloring to thevertices in $H$. Also apply two distinct colors say, $p$ $and$ $p+1$ to the vertices$\{u_{n-1}\}$ and $\{u_n, u_{2n-1}\}$ respectively, we obtain a $\gamma_\chi^d$- coloring of $L_p$. So$\gamma_\chi^d(L_p) = p+2$.

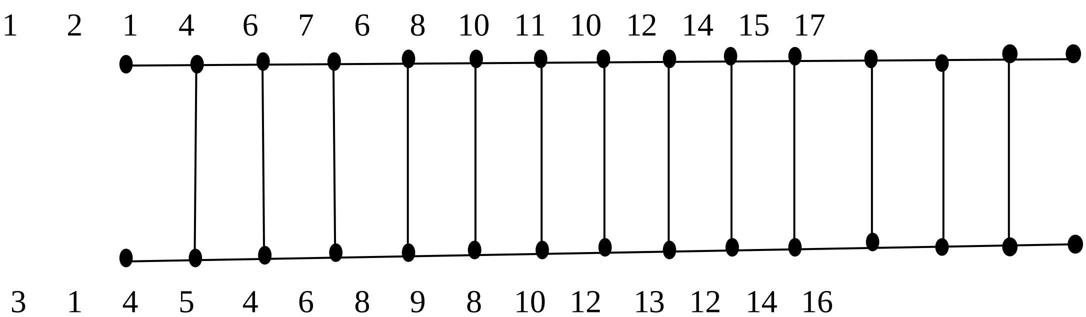


**Figure 6** $\gamma_\chi^d(L_{15}) = 17$

**Theorem 2.3.** Let $SL_n$ be a slanting ladder graph. Then

$\gamma_\chi^d(SL_n) = n+1 \;\forall\, n \geq 3$.

**Proof:** Let $SL_p = SL_{2n} (n \geq 3)$ be a slanting ladder graph with $V(SL_{2n}) = \{u_1, u_2, \dots\dots . . u_n, v_1, v_2, \dots\dots v_n\}$and$E(SL_{2n}) = \{u_i u_{i+1} | i < n\} \cup \{v_i v_{i+1} | i < n\} \cup \{u_i v_{i+1} | 1 \leq i \leq n-1\}$.

Assign 3 distinct colors say $1, n-1$ and $n$ to the vertices say $\{v_1\}$, $\{v_n\}$ and $\{u_n\}$ respectively

Assign distinct colors say $3i+1$ $\left(1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor\right)$ to the vertices $\{v_{3i+1}\}$ $1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor$. Also assign distinct colors say $3j-1$ and $3j$ $\left(1 \leq j \leq \left\lfloor \frac{n}{3} \right\rfloor\right)$ to the vertices $\{u_{3i-2}, u_{3i}, v_{3i}\}$ and $\{u_{3i-1}, v_{3i-1}\}$ respectively, we obtain a $\gamma_\chi^d$ – coloring of $SL_n$. Thus $\gamma_\chi^d(SL_n) = n+1 \;\forall\, n \geq 3$.

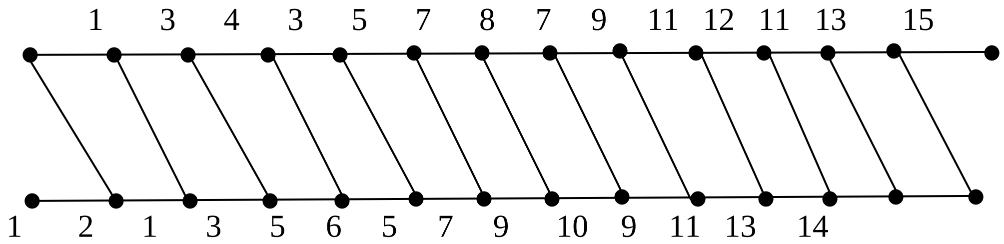


**Figure7** $\gamma_\chi^d(SL_{14}) = 15$

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