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Computing Topological Indices of Certain Windmill Graphs

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Abstract

Let $V(G)$, be the vertex set of graph G, we define $V_d = \{v \in V(G) : S_G(u) = d\}$ in which $S_G(u) = \sum_{v \in N_G(u)} d_G(v)$ and $N_G(u) = \{v \in V(G) : uv \in E(G)\}$. In this paper we express the explicit formula for Sanskruti index $S(G)$, fifth M_1 and M_2 Zagreb indices, fifth M_1 and M_2 multiplicative Zagreb indices for Dutch windmill graph, Kulli cyclic windmill graph, Kulli path windmill graph and French windmill graph.

Key words: Topological index, Dutch windmill graph, Kulli cyclic windmill graph, Kulli path windmill graph, French windmill graph.

AMS classification:05C09, 05C92, 05C99

1. Introduction

Let G be a simple graph having vertex set $V(G)$ and edge set $E(G)$. Let p the order of a graph i.e. $|V(G)| = p$ and the size of a graph be q i.e. $|E(G)| = q$. Generally, the degree of a vertex v is denoted by $d_G(v)$ which is the number of edges incident to the vertex v. Many topological indices are defined based on the degree of the vertex; the topological index is a numerical value associated with the graph. Topological indices have many applications in the various disciplines like chemistry, biology, etc. Some of the standard topological indices are listed below.

The Sanskruti index was introduced by S. Hosamani [4], which is defined as follow

$$S(G) = \sum_{uv \in E(G)} \left(\frac{S_G(u)S_G(v)}{S_G(u) + S_G(v) - 2} \right)^3$$

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The fifth M_1 and M_2 Zagreb indices introduced by Graovac [1] and defined as

$$M_1G_5(G) = \sum_{uv \in E(G)} (S_G(u) + S_G(v))$$

$$M_2G_5(G) = \sum_{uv \in E(G)} (S_G(u) \cdot S_G(v))$$

The fifth M_1 and M_2 multiplicative Zagreb indices proposed by Kulli [6]defined as

$$M_1G_5 \prod(G) = \prod_{uv \in E(G)} (S_G(u) + S_G(v))$$

$$M_2G_5 \prod(G) = \prod_{uv \in E(G)} (S_G(u) \cdot S_G(v))$$

2. Dutch Windmill Graph

For $m \geq 2$,$n \geq 5$, the graph constructed by taking m copies of the cycle C_n with a vertex in common is called Dutch windmill graph, and is generally denoted by D_n^m, which is shown in Figure 1.

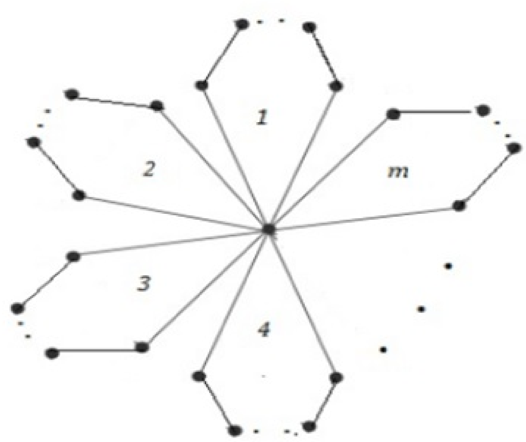

Figure 1: Dutch windmill graph D_n^m

For $m \geq 2$, $n \geq 5$, let $G = D_n^m$ be a Dutch windmill graph then G has $1+m(n-1)$ vertices and mn edges, moreover, G has 3 types of edge partitions.

Table 1.

(S_u, S_v) where $uv \in E(G)$	$(4m, 2(m+1))$	$(2(m+1), 4)$	(4,4)
No. of edges	$2m$	$2m$	$mn - 4m$

Theorem 2.1 For the Dutch windmill graph $G = D_n^m$, Sanskruti index is
$S(G) = \frac{128m}{27(m+2)^3}(m^6 + 9m^5 + 33m^4 + 90m^3 + 143m^2 + 117m + 4n + 19)$.

Proof: By the definition of Sanskruti index and the information from Table 1, we have

$$\begin{aligned}
S(G) &= 2m\Big(\frac{4m\cdot 2(m+1)}{4m\cdot 2(m+2)-2}\Big)^3 + 2m\Big(\frac{2(m+1)\cdot 4}{2(m+2)+4-2}\Big)^3 + (mn-4m)\Big(\frac{16}{6}\Big)^3 \\
&= 2m\Big(\frac{8m^2+8m}{6m}\Big)^3 + 2m\Big(\frac{8m+m}{2m+4}\Big)^3 + (mn-4m)\Big(\frac{8}{3}\Big)^3 \\
&= \frac{128m}{27}\Big(m+1\Big)^3 + 128m\Big(\frac{m+1}{m+2}\Big)^3 + (mn-4m)\Big(\frac{8}{3}\Big)^3 \\
&= \frac{128m}{27}\Big((m+1)^3 + 27\Big(\frac{m+1}{m+2}\Big)^3 + 4(n-4)\Big) \\
&= \frac{128m}{27(m+2)^3}\Big((m+1)^3(m+2)^3 + 27(m+1)^3 + (4n-16)(m+2)^3\Big)
\end{aligned}$$

Theorem 2.2 For the Dutch windmill graph $G = D_n^m$, the fifth M_1 and M_2 Zagreb indices are
$M_1G_5(G) = 16m^2 - 16m + 8mn$
$M_2G_5(G) = 16m^3 + 32m^2 + 16mn - 48m$

Proof: By the definition of the fifth M_1 and M_2 Zagreb indices and the inform from Table 1, we have
Case:1

$$\begin{aligned}
M_1G_5(G) &= 2m[4m + 2(m+1)] + 2m[2(m+1)+4] + (mn-4m)[4+4] \\
&= 2m[6m+2] + 2m[2m+6] + 8mn - 32m \\
&= 16m^2 - 16m + 8mn
\end{aligned}$$

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case:2

$$\begin{aligned} M_2G_5(G) &= 2m[4m\cdot 2(m+1)]+2m[2(m+1)\cdot 4]+(mn-4m)[4\cdot 4] \\ &= 2m[8m^2+8m]+2m[8m+8]+16mn-64m \\ &= 16m^3+32m^2+16mm-48m \end{aligned} \tag{1}$$

Theorem 2.3 For the Dutch windmill graph $G=D_n^m$,fifth M_1 and M_2 multiplicative Zagreb indices are
$M_1G_5\prod(G)=(12m^2+40m+12)^{2m}\cdot 8^{(mn-4m)}$.
$M_2G_5\prod(G)=(64m^3+128m^2+64m)^{2m}\cdot 16^{(mn-4m)}$.

Proof: By the definition of the fifth M_1 and M_2 multiplicative Zagreb indices and from Table 1. we have
Case:1

$$\begin{aligned} M_1G_5\prod(G) &= (4m+2(m+1))^{2m}\cdot(2(m+1)+4)^{2m}\cdot(4+4)^{(mn-4m)} \\ &= (6m+2)^2m\cdot(2m+6)^{2m}\cdot 8^{(mn-4m)} \\ &= ((6m+2)(2m+6))^{2m}\cdot 8^{(mn-4m)} \\ &= (12m^2+40m+12)^{2m}\cdot 8^{(mn-4m)} \end{aligned}$$

Case:2

$$\begin{aligned} M_2G_5\prod(G) &= (4m\cdot 2(m+1))^{2m}\cdot(2(m+1)\cdot 4)^{2m}\cdot(4\cdot 4)^{(mn-4m)} \\ &= (8m^2+8m)^{2m}\cdot(8m+8)^{2m}\cdot 16^{(mn-4m)} \\ &= ((8m^2+8m)(8m+8))^{2m}\cdot 16^{(mn-4m)} \\ &= (64m^3+128m^2+64m)^{2m}\cdot 16^{(mn-4m)} \end{aligned}$$

3.Kulli Cyclic Windmill Graph

For $n\geq 3$ with a vertex K_1 in common, the graph constructed by taking m copies of the cycle K_1+C_n is called Dutch windmill graph, and is generally denoted by C_{n+1}^m which is shown in Figure 2 .

For $m\geq 2$,$n\geq 5$, let $G=C_{n+1}^m$ be a Kulli cycle windmill graph with $mn+1$

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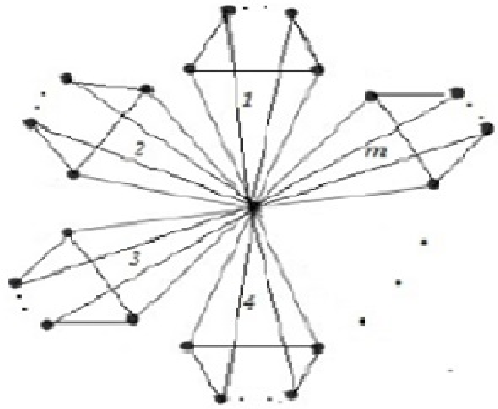

Figure 2: Kulli Cyclic windmill graph $C^m_{(n+1)}$

vertices and $2mn$ edge then G has two type of edge partition as given in Table 2.

Table 2.

(S_u, S_v) where $uv \in E(G)$	$(3mn, mn+6))$	$(nm+6, nm+6)$
No. of edges	nm	nm

Theorem 3.1 For the Kulli cyclic windmill graph $G = C^m_{n+1}$, Sanskruti index is $S(G) = \frac{nm(nm+6)^3}{8}\left(\left(\frac{3nm}{2nm+2}\right)^3 + \left(\frac{nm+6}{nm+5}\right)^3\right)$.

Proof: By the definition of Sanskruti index and the information from Table 2, we have

$$\begin{aligned} S(G) &= nm\Big(\frac{3nm\cdot(nm+6)}{3nm+nm+6-2}\Big)^3 + nm\Big(\frac{(nm+6)\cdot(nm+6)}{nm+6+nm+6-2}\Big)^3 \\ &= nm(nm+6)^3\Big(\Big(\frac{3nm}{4nm+4}\Big)^3 + \Big(\frac{nm+6}{2nm+10}\Big)^3\Big) \\ &= \frac{nm(nm+6)^3}{8}\Big(\Big(\frac{3nm}{2nm+2}\Big)^3 + \Big(\frac{nm+6}{nm+5}\Big)^3\Big) \end{aligned}$$

Theorem 3.2 For the Kulli cyclic windmill graph $G = C^m_{(n+1)}$, fifth M_1 and M_2 Zagreb indices are

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$M_1G_5(G) = 6n^2m^2 + 18nm$
$M_2G_5(G) = 4n^3m^3 + 30n^2m^2 + 36nm$

Proof: By the definition of the fifth M_1 and M_2 Zagreb indices and from Table 2, we have
Case:1

$$\begin{aligned} M_1G_5(G) &= nm(3nm + nm + 6) + nm(nm + 6 + nm + 6) \\ &= nm(4nm + 6) + nm(2nm + 12) \\ &= 4n^2m^2 + 6nm + 2n^2m^2 + 12nm \\ &= 6n^2m^2 + 18nm \end{aligned}$$

case:2

$$\begin{aligned} M_2G_5(G) &= nm[3nm \cdot (nm + 6)] + nm[(nm + 6) \cdot (nm + 6) \\ &= nm(3n^2m^2 + 18nm) + nm(n^2m^2 + 12nm + 36) \\ &= 3n^3m^3 + 18n^2m^2 + n^3m^3 + 12n^2m^2 + 36nm \\ &= 4n^3m^3 + 30n^2m^2 + 36nm \end{aligned}$$

Theorem 3.3 For the Kulli cyclic windmill graph $G = C^m_{(n+1)}$,fifth M_1 and M_2 multiplicative Zagreb indices are
$M_1G_5\prod(G) = (8n^2m^2 + 84nm + 72)^{nm}$
$M_2G_5\prod(G) = (3n^4m^4 + 54n^3m^3 + 324n^2m^2 + 648nm)^{nm}$

Proof: By the definition of the fifth M_1 and M_2 multiplicative Zagreb indices and from Table 2. we have
Case:1

$$\begin{aligned} M_1G_5\prod(G) &= (3nm + nm + 6)^{nm} \cdot (nm + 6 + nm + 6)^{nm} \\ &= (4nm + 6)^{nm} \cdot (2nm + 12)^{nm} \\ &= ((4nm + 6) \cdot (2nm + 12))^{nm} \\ &= [8n^2m^2 + 84nm + 72]^{nm} \end{aligned}$$

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Case:2

$$\begin{aligned} M_2G_5\prod(G) &= (3nm\cdot(nm+6))^{nm}\cdot((nm+6)\cdot(nm+6))^{nm} \\ &= ((3n^2m^2+18nm)\cdot(nm+6)^2)^{nm} \\ &= ((3n^2m^2+18nm)\cdot(n^2m^2+12nm+36))^{nm} \\ &= (3n^4m^4+54n^3m^3+324n^2m^2+648nm)^{nm} \end{aligned}$$

4.Kulli Path Windmill Graph:

Kulli path windmill graph is the graph contains m sets of the graph $K_1 + P_n$ with a common vertex and it is denoted by $P^m_{(n+1)}$ and it is shown in Figure 3. Let

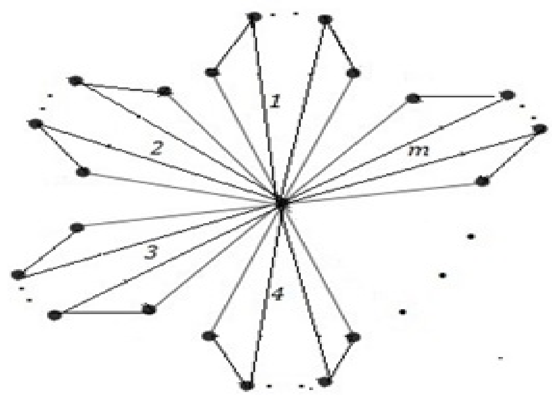

Figure 3: Kulli path windmill graph $P^m_{(n+1)}$

$G = P^m_{(n+1)}$, for $m \geq 2$,$n \geq 6$ be a Kulli path windmill graph then G has $mn+1$ vertices, $2nm-m$ edges and also G has six types of edge partition given in Table 3.
Table 3.

(S_u, S_v) where $uv \in E(G)$	$(3+nm, 3nm-2m)$	$(5+nm, 3+nm)$	$(6+nm, 5+nm)$
No. of edges	$2m$	$2m$	$2m$
(S_u, S_v) where $uv \in E(G)$	$(6+nm, 6+nm)$	$(5+nm, 3nm-2m)$	$(6+nm, 3nm-2m)$
No. of edges	$nm-5m$	$2m$	$nm-4m$

Theorem 4.1 For the Kulli path windmill graph $G = P^m_{(n+1)}$, Sanskruti index is $S(G) = 2m\Big(\frac{3n^2m^2+9nm-2nm^2-6m}{4nm-2m+1}\Big)^3 + 2m\Big(\frac{n^2m^2+8nm+15}{2nm+6}\Big)^3 + 2m\Big(\frac{n^2m^2+11nm+30}{2nm+9}\Big)^3 + (nm - 5m)\Big((\frac{n^2m^2+12nm+36}{nm+10}\Big)^3 + 2m\Big(\frac{3n^2m^2+15nm-2nm^2-10m}{4nm-2m+3}\Big)^3 + (nm - 4m)\Big(\frac{3n^2m^2+18nm-2nm^2-12m}{5nm-2m+4}\Big)^3$.

Proof: By the definition of Sanskruti index and the information from Table 3, we have

$$\begin{aligned}
S(G) &= 2m\Big(\frac{(3+nm)(3nm-2m)}{3+nm+3nm-2m-2}\Big)^3 + 2m\Big(\frac{(5+nm)(3+nm)}{5+nm+3+nm-2}\Big)^3 \\
&\quad +2m\Big(\frac{(6+nm)(5+nm)}{6+nm+5+nm-2}\Big)^3 + (nm-5m)\Big(\frac{(6+nm)(6+nm)}{6+nm+6+nm-2}\Big)^3 \\
&\quad +2m\Big(\frac{(5+nm)(3nm-2m)}{5+nm+3nm-2m-2}\Big)^3 + (nm-4m)\Big(\frac{(6+nm)(3nm-2m)}{6+nm+3nm-2m-2}\Big)^3 \\
&= 2m\Big(\frac{3n^2m^2+9nm-2nm^2-6m}{4nm-2m+1}\Big)^3 + 2m\Big(\frac{n^2m^2+8nm+15}{2nm+6}\Big)^3 \\
&\quad +2m\Big(\frac{n^2m^2+11nm+30}{2nm+9}\Big)^3 + (nm-5m)\Big((\frac{n^2m^2+12nm+36}{nm+10}\Big)^3 \\
&\quad +2m\Big(\frac{3n^2m^2+15nm-2nm^2-10m}{4nm-2m+3}\Big)^3 + (nm-4m)\Big(\frac{3n^2m^2+18nm-2nm^2-12m}{5nm-2m+4}\Big)^3
\end{aligned}$$

Theorem 4.2 For the Kulli path windmill graph $G = P^m_{(n+1)}$ the fifth M_1 and M_2 Zagreb indices are
$M_1G_5(G) = 6n^2m^2 + 18nm - 4nm^2 - 30m$
$M_2G_5(G) = 4n^3m^3 - 3n^2m^3 - 58nm^2 + 30n^2m^2 + 16m^2 + 36nm - 90m$

Proof: By the definition of the fifth M_1 and M_2 Zagreb indices and from Table 3, we have

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Case:1

$$\begin{aligned}
M_1G_5(G) &= 2m(3+nm+3nm-2m)+2m(5+nm+3+nm)+2m(6+nm+5+nm) \\
&\quad +(nm-5m)(6+nm+6+nm)+2m(5+nm+3nm-2m) \\
&\quad +(nm-4m)(6+nm+3nm-2)] \\
&= 2m(4nm-2m+3)+2m(2nm+8)+2m(2nm+11)+(nm-5m)(2nm+12) \\
&\quad +2m(4nm-2m+5)+(nm-4m)(4nm-2m+6) \\
&= 8nm^2-4m^2+6m+4nm^2+16m+4nm^2+22m+2n^2m^2+12nm-10nm^2 \\
&\quad -60m+8nm^2-4m^2+10m+4n^2m^2-2nm^2+6nm-16nm^2+8m^2-24m \\
&= 6n^2m^2+18nm-4nm^2-30m
\end{aligned}$$

case:2

$$\begin{aligned}
M_2G_5(G) &= 2m((3+nm)\cdot(3nm-2m))+2m((5+nm)\cdot(3+nm))+2m((6+nm)\cdot(5+nm) \\
&\quad +(nm-5m)((6+nm)\cdot(6+nm))+2m((5+nm)(3nm-2m)) \\
&\quad +(nm-4m)((6+nm)\cdot(3nm-2m)) \\
&= 2m(3n^2m^2-2nm^2+9nm-6m)+2m(n^2m^2+8nm+15)+2m(n^2m^2+11nm+3)) \\
&\quad +(nm-5m)(n^2m^2+12nm+36)+2m(3n^2m^2-2nm^2+15nm-10m) \\
&\quad +(nm-4m)(3n^2m^2-2nm^2+18nm-12m) \\
&= 6n^2m^3-4nm^3+18nm^2-12m^2+2n^2m^3+16nm^2+30m+2n^2m^3+22nm^2 \\
&\quad +60m+n^3m^3+12n^2m^2+36nm-5n^2m^3-60nm^2-180m+6n^2m^3-4nm^3 \\
&\quad +30nm^2-20m^2+3n^3m^3-2n^2m^3+18n^2m^2-12nm^2-12n^2m^3 \\
&\quad +8nm^3-72nm^2+48m^2 \\
&= 4n^3m^3-3n^2m^3-58nm^2+30n^2m^2+16m^2+36nm-90m
\end{aligned}$$

Theorem 4.3 For the Kulli path windmill graph $G = P^m_{(n+1)}$,fifth M_1 and M_2 multiplicative Zagreb indices are
$M_1G_5\prod(G) = (4nm-2m+3)^{2m}\cdot(2nm+8)^{2m}\cdot(2nm+11)^{2m}\cdot(2nm+12)^{(nm-5m)}\cdot(4nm-2m+5)^{2m}\cdot(2nm-2m+6)^{(nm-4m)}$
$M_2G_5\prod(G) = (3n^2m^2-2nm^2+9nm-6m)^{2m}\cdot(n^2m^2+8nm+15)^{2m}\cdot(n^2m^2+11nm+30)^{2m}\cdot(n^2m^2+12nm+36)^{((nm-5m))}\cdot(3n^2m^2-2nm^2+15nm-10m)^{2m}\cdot(3n^2m^2-$

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$2nm^2 + 18nm - 12m)^{((nm-4m))}$

Proof: By the definition of the fifth M_1 and M_2 multiplicative Zagreb indices and from Table 3. we have
Case:1

$$\begin{aligned} M_1G_5\prod(G) &= (3+nm+3nm-2m)^{2m}\cdot(5+nm+3+nm)^{2m}\cdot(6+nm+5+nm)^{2m} \\ &\quad \cdot(6+nm+6+nm)^{(nm-5m)}\cdot(5+nm+3nm-2m)^{2m} \\ &\quad \cdot(6+nm+3nm-2m)^{(nm-4m)} \\ &= (4nm-2m+3)^{2m}\cdot(2nm+8)^{2m}\cdot(2nm+11)^{2m} \\ &\quad \cdot(2nm+12)^{(nm-5m)}\cdot(4nm-2m+5)^{2m}\cdot(2nm-2m+6)^{(nm-4m)} \end{aligned}$$

Case:2

$$\begin{aligned} M_2G_5\prod(G) &= ((3+nm)\cdot(3nm-2m))^{2m}\cdot((5+nm)\cdot(3+nm))^{2m}\cdot((6+nm)\cdot(5+nm))^{2m} \\ &\quad \cdot((6+nm)\cdot(6+nm))^{(}(nm-5m))\cdot(5+nm)\cdot(3nm-2m))^{2m} \\ &\quad \cdot((6+nm)\cdot(3nm-2m))^{((nm-4m))} \\ &= (3n^2m^2-2nm^2+9nm-6m)^{2m}\cdot(n^2m^2+8nm+15)^{2m}\cdot(n^2m^2+11nm+30)^{2m} \\ &\quad \cdot(n^2m^2+12nm+36)^{((nm-5m))}\cdot(3n^2m^2-2nm^2+15nm-10m)^{2m} \\ &\quad \cdot(3n^2m^2-2nm^2+18nm-12m)^{((nm-4m))} \end{aligned}$$

5.French Windmill Graph:

For $m \geq 2$,$n \geq 5$ with a common vertex, the graph constructed by taking m copies of complete graph K_n is called French Windmill Graph, and is generally denoted by F_n^m, which is shown in Figure 4. Let $G = F_n^m$ for $m \geq 2$,$n \geq 2$ be a French Windmill Graph then G has $1+m(n-1)$ vertices, $mn/2(n-1)$ edges and also G has two types of edge partition given in Table 4.

Table 4.

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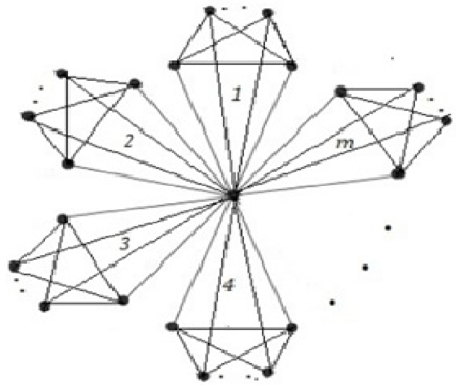

Figure 4: French Windmill Graph F_n^m

(S_u, S_v) where $uv \in E(G)$	$(m(n-1)^2, (n-1)(m+n-2))$	$((n-1)(m+n-2),$ $(n-1)(m+n-2))$
No. of edges	$m(n-1)$	$\frac{1}{2}m(n-1)(n-2)$

Theorem 5.1 For the French Windmill graph $G = F_n^m$ the Sanskruti index is $S(G) = m(n-1)^7(m+n-2)^3\Big(\Big(\frac{m^3(n-1)^3}{(m(n-1)^2+(n-1)(m+n-2)-2)^3}\Big) + \Big(\frac{(n-2)(m+n-2)^3}{16((n-1)(m+n-2)-1)^3}\Big)\Big)$

Proof: By definition of Sanskruti index and from Table 4, we get

$$\begin{aligned} S(G) &= m(n-1)\Big(\frac{m(n-1)^2\cdot(n-1)(m+n-2)}{m(n-1)^2(n-1)(m+n-2)-2}\Big)^3 \\ &\quad +\frac{1}{2}m(n-1)(n-2)\Big(\frac{(n-1)(m+n-2)\cdot(n-1)(m+n-2)}{(n-1)(m+n-2)+(n-1)(m+n-2)-2}\Big)^3 \\ &= m(n-1)^7(m+n-2)^3\Big(\Big(\frac{m^3(n-1)^3}{(m(n-1)^2+(n-1)(m+n-2)-2)^3}\Big) \\ &\quad +\Big(\frac{(n-2)(m+n-2)^3}{16((n-1)(m+n-2)-1)^3}\Big)\Big). \end{aligned}$$

Theorem 5.2 For the French Windmill graph $G = F_n^m$ the fifth M_1 and M_2 Zagreb indices are
$M_1G_5(G) = m(n-1)^2(n^2+2nm-2m-3n+2)$
$M_2G_5(G) = \frac{1}{2}m(n-1)^3(m+n-2)(n^2+3nm-4m-4n+4)$

Proof: By the definition of the fifth M_1 and M_2 Zagreb indices and from Table 4, we have
Case:1

$$\begin{aligned} M_1G_5(G) &= m(n-1)(m(n-1)^2+(n-1)(m+n-2)) \\ &\quad +\frac{1}{2}m(n-1)(n-2)((n-1)(m+n-2)+(n-1)(m+n-2)) \\ &= m(n-1)^2(m(n-1)+(m+n-2)+(n-2)(m+n-2)) \\ &= m(n-1)^2(n^2+2nm-2m-3n+2) \end{aligned}$$

case:2

$$\begin{aligned} M_2G_5(G) &= m(n-1)(m(n-1)^2\cdot(n-1)(m+n-2)) \\ &\quad +1/2m(n-1)(n-2)((n-1)(m+n-2)\cdot(n-1)(m+n-2)) \\ &= m^2(n-1)^4(m+n-2)+\frac{1}{2}m(n-1)^3(n-2)(m+n-2)^2 \\ &= \frac{1}{2}m(n-1)^3(m+n-2)(2m(n-1)+(n-2)(m+n-2)) \\ &= \frac{1}{2}m(n-1)^3(m+n-2)(n^2+3nm-4m-4n+4) \end{aligned}$$

Theorem 5.3 For the French windmill graph $G=F_n^m$,fifth M_1 and M_2 multiplicative Zagreb indices are
$M_1G_5\prod(G)=(m(n-1)^2+(n-1)(m+n-2))^{m(n-1)}\cdot(2(n-1)(m+n-2))^{(1/2m(n-1)(n-2))}$
$M_2G_5\prod(G)=(m(n-1)^3(m+n-2))^{m(n-1)}$

Proof: By the definition of the fifth M_1 and M_2 multiplicative Zagreb indices and from Table 4, we have
Case:1

$$\begin{aligned} M_1G_5\prod(G) &= (m(n-1)^2+(n-1)(m+n-2))^{m(n-1)}\cdot((n-1)(m+n-2) \\ &\quad +(n-1)(m+n-2))^{(\frac{1}{2}m(n-1)(n-2))} \\ &= (m(n-1)^2+(n-1)(m+n-2))^{m(n-1)}\cdot(2(n-1)(m+n-2))^{(\frac{1}{2}m(n-1)(n-2))} \end{aligned}$$

Case:2

$$
\begin{aligned}
M_2G_5\prod(G) &= (m(n-1)^2(n-1)(m+n-2))^m(n-1)\cdot((n-1)(m+n-2)\\
&\quad (n-1)(m+n-2))^{(\frac{1}{2}m(n-1)(n-2))}\\
&= (m(n-1)^3(m+n-2))^{m(n-1)}((n-1)(m+n-2))^{m(n-1)(n-2)}
\end{aligned}
$$

6 Conclusion

For the given graph G with number of vertex or edges, above results gives explicit formula for Sanskruti index, fifth M_1 and M_2 Zagreb indices, fifth M_1 and M_2 multiplicative Zagreb indices of Dutch windmill graph, Kulli cyclic windmill graph, Path windmill graph and French windmill graph. These developments achieve remarkable advances in chemical sciences in which compounds undergo a similar structure of the considered graph.

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