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Search of Arrivals of an M/G/1 retrial Queueing system with Delayed repair and Optional re-service using Modified Bernoulli Vacation

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Abstract

Single server queueing system in retrial is investigated, as well as optional re-service, customer search and delayed repair. Furthermore, the server is permitted to take a vacation under Modified Bernoulli vacation. In this model, some of the customer's seek voluntary re-service for the identical service taken, dissatisfied with the first attempt without entering the retrial group (probability r_1) or leaving the service area (probability $1 - r_1$). The failing server's repair begins after an arbitrary length of time termed delay time has passed. The supplementary variable technique is used to calculate orbit size, server status, and performance metrics. We explore several special situations of our suggested model and compare the results to previous research.

Key Words: Orbital search, Retrial Queue, Delayed repair, Optional re-services, Modified Bernoulli vacation.

AMS Classification: 90C05, 90C46, 90C90.

1 Introduction

When a customer notices that a server is free in a retry queueing system, it begins serving that customer. If the server is overburdened, exit the service area and become a member of the retrial group to receive service later. Vacation time and server breakdowns in which the server has other jobs have piqued the curiosity of many academics. Network systems in computer and wireless, switching systems in telephone, all use retrial queues as mathematical models, and they perform well. In some cases, the assistance facility of a queueing system is disrupted, and the service channel is broken during brief periods of time. Artalejo [1, 2] conducted extensive surveys of retrial waits. For a full analysis of the principles of queuing in retrial, see Artalejo and Gomez – Corral [3]. Rehab F. Khalaf, Kailash C. Madan and Cormac A. Lukas [10], PavaiMadheswari, Krishnakumar, and Suganthi [12] all considered a modified Bernoulli vacation schedule. Wang and Li [14] investigated a similar model that included breakdown and repair. Finally, Gomez-Corral [7] took into account the single server retrial queue, as well as the overall guideline on retrials, as well as the general service and seeking times.

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As a result of the disturbance, there is a delay in starting repairs. Repairs and interruption have been examined by authors such as Choudhury and Deka [4]. One of the most significant aspects of queueing theory is reserving, which has a variety of real-world applications. Re-service is necessary in a variety of situations, including hospitals, banks, and post offices, where a customer receives poor treatment on the first attempt and requests re-service. Jeyakumar and Arumuganathan [8] investigated the concept of re-service. The idea behind introducing orbital search of consumers after service completion is to minimize the server's free time. Neuts [11] published check for orbital arrivals, in which authors explored the effects of combining a traditional queue with orbital search on service fulfilment. Search of consumers after service has looked into Krishnamoorthy [9], Deepak [5], Gao and Wang [6] and Rajadurai [13]
The following is the rest of the paper: The mathematical model is described in section 2 under consideration, provides practical explanation for our paradigm. Section 3 investigates our model's stability requirement. Section 4 calculates the steady state joint distribution of the server state and the number of customers in the service area and in retrial group. Section 5 discusses the system's performance. Section 6 looks at a few unique situations. Finally, the last Section brings the paper to a close.

2 The Model

We explore a single server queueing system in retrial with regular and voluntary re-service, orbital search categorized as modified Bernoulli server vacation where the regular, optional re-service is due to delaying repair, and repair in this section
Pattern of Arrival: Customers arrive to the system as mentioned by a Poisson process with rate λ
Rule of Retrial: If an arrival note that the server is not free, He enters the service station right away to begin his servicing. Otherwise the arrival will be added to the orbit group of barred customers and will request service again until the server becomes available. The retrial time's is determined by arbitrary distribution $A(x)$ and LST $A^*(x)$
The method for giving service: The assistance time, according to random distribution, $S_1(x)$ for regular service, $S_2(x)$ for voluntary re-service with corresponding LST $S_1^*(x)$, $S_2^*(x)$. The moments of normal and optional re-service are denoted by s_1, s_2
Searching in Orbit: During the end of vacation, the assistance starts searching for the one in the group of retrial accompanying probability θ_1 or at rest accompanying equivalent probability $1-\theta_1$. The search period is considered to be insignificant
Taking a vacation: The server is on vacation if the queue is free after service is performed. If in the empty queue, service will commence accompanying probability of 1 - a_1, or a period in vacation will begin with probability a_1. If a customer is in line at the end of a vacation period, service will begin. Otherwise, the server awaits the arrival of the initial customer. The vacation time follows random distribution with the probability distribution function $V(x)$, LST $V^*(x)$. The moments are denoted by v_1, v_2
The process of delaying repair: Exogenous Poisson stream with mean breakdown rates as stated by α_1 for regular service and α_2 for optional re-service when the work fails. The delay time take it an arbitrary distribution $Q_1(t)$, $Q_2(t)$ and LST $D_1^*(y)$, $D_2^*(y)$ for regular service and optional re-service respectively. The moments of delaying repair on normal and optional re-service are denoted by d_1, d_2

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The procedure for repair: Following the repair process, the server is as good as new. The repair period succeed with probability distribution R_1 (t), R_2 (t) for regular service, and optional re-service are taken it to be arbitrarily distributed and LST $G_1^*(y)$, $G_2^*(y)$ respectively. The moments of repair on normal and optional re-service are denoted by g_1, g_2

Explanation of the recommended Model in Detail

The Inventory incorporates the requested model's practical application. Inventory refers to all of the materials or items that are entered in a register and kept in stock for a specific amount of time so that they can be used whenever and wherever they are needed and to meet the customer's service. Raw supplies such as milk, sugar, and flour are obtained from a dealer (server) in a cookie or biscuits producing facility so that work does not bear at the time of the customer's requirement. Raw materials are shared by all units of a shaped machine (comparable to positive customers). If the machine is running when the new arrivals are shaped, they are brought in for quality inspection before final packaging, and a new positive entry is added to a queue that is similar to the retry queue. Otherwise, the section will be given immediately. All of the biscuits or cookies are in line for a quality check and are expected to work in sequence. The machine is shut off by the manufacturer if there is no section (the system is idle) at the service end. Otherwise, the manufacturer will approach the new section on the list, with the exception of an additional external section that arrives before the approach. The approach time is thought to be randomly distributed (comparing to the retrial time). The shaped dough is then placed in the oven to bake.

In the case of a machinery-related issue, the machine may fail unexpectedly while in operation. If we want to prevent machine failures, we may have to wait longer for the food to cook (comparing to the delaying repair). To ensure the highest level of service, many extra pieces of moulded dough may be sent to the oven for cooking as long as the machine is serviced (comparing to repair). Because the raw materials are not correctly moulded, a quality inspection is required before final packing (comparing to optional re-service). After the service is terminated, there are no units in the system, the huge production skill comes to an end, and the skill is available to carry out the next unit to enter (comparing on vacation). Furthermore, to reduce the server's free time, he will check if there are units in the retrial group as soon as the vacation is over, same like (FCFS), and the checking time is assumed to be in the general distribution. The final cookie or biscuit packets that are shipped to retail for sale should be subjected to a quality inspection and be treated as a satisfactory result.

3.Stability Criterion

Let $A^0(t), S_1^0(t), S_2^0(t), D_1^0(t), D_2^0(t), G_1^0(t), G_2^0(t), V^0(t)$ be the passed time in retrial, time in regular service, time in voluntary re – service, delaying repair time on normal service and re – service, repair time on normal service and re – service and vacation period sequentially at time t.

To a greater extend assume that

$A(0)=0,\ A(\infty)=1,\ S_1(0)=0,\ S_1(\infty)=1,\ S_2(0)=0,\ S_2(\infty)=1,\ V(0)=0,\ V(\infty)=1$ at $x = 0$, are continuous and $D_1(0)=0, D_1(\infty)=1, G_1(0)=0, G_1(\infty)=1, D_2(0)=0, D_2(\infty)=1, G_2(0)=0, G_2(\infty)=1$ are continuous at $y = 0$.

Let $\{t_n;\ n\in N\}$ be a sequence in which either completion of service (regular and optional re-service or vacation) times or the time it takes for a repair to be completed. The queueing system's series of random variables is then placed in a Markov chain. The system condition at time t can be related by the Markov process $\{C(t),\ X(t),\ t\geq 0\}$ where the state of the server is denoted by C (t) = 0, 1, 2, 3, 4, 5, 6, 7 mentioning to the server is inactive, active with regular, optional re-service, is delaying repair with regular service, is delaying repair with optional re-service, repair with regular service, repair with optional re-service, on vacation.

The functions are $a(x)dx, \mu_1(x)dx, \mu_2(x)dx, v(x)dx, \eta_1(y)dy, \eta_2(y)dy,\ r_1(y)dy,\ r_2(y)dy$ the conditional termination probability of repetition attempts, regular, re-service, vacation, delaying repair and repair period is $a(x)dx = \frac{dA(x)}{1-A(x)}$, $\mu_1(x)dx = \frac{dS_1(x)}{1-S_1(x)}$, $\mu_2(x)dx = \frac{dS_2(x)}{1-S_2(x)}$, $v(x)dx = \frac{dV(x)}{1-V(x)}$, $\eta_1(y)dy = \frac{dD_1(y)}{1-D_1(y)}$, $\eta_2(y)dy = \frac{dD_2(y)}{1-D_2(y)}$, $r_1(y)dy = \frac{dG_1(y)}{1-G_1(y)}$, $r_2(y)dy = \frac{dG_2(y)}{1-G_2(y)}$

Theorem: The Markov chain $\{Z_n, n \in N\}$ is Ergodic provided
$\rho = A^*(\lambda) + (1 - A^*(\lambda))\theta_1 a_1 + \lambda\{s_1[1+\alpha_1(d_1+g_1)] + r_1 s_2[1+\alpha_2(d_2+g_2)] + a_1 v\} < 1$

4 Steady state Probabilistic Analysis

In this part, the steady state distribution function of the system under study is investigated. The stability criterion is assumed to be met. We define the process's $\{Y(t), t \geq 0\}$ probability $P_0(t) = P\{C(t)=0, Y(t)=0\}$ for the limiting densities for $t \geq 0,\ x \geq 0$ and $n \geq 1$

$P_n(x,t) = P\{C(t)=1, Y(t)=n, x \leq A^0(t) < x+dx\}$, $\pi_{1,n}(x,t) = P\{C(t)=2, Y(t)=n, x \leq S_1^0(t) < x+dx\}$

$\pi_{2,n}(x,t) = P\{C(t)=3, Y(t)=n,\ x \leq S_2^0(t) < x+dx\}$

$Q_{1,n}(x,t) = P\{C(t)=4, Y(t)=n,\ y \leq D_1^0(t) < y+dy / S_1^0(t) = x\}$

$Q_{2,n}(x,t) = P\{C(t)=5, Y(t)=n,\ y \leq D_2^0(t) < y+dy / S_2^0(t) = x\}$

$R_{1,n}(x,t) = P\{C(t)=6, Y(t)=n,\ y \leq G_1^0(t) < y+dy / S_1^0(t) = x\}$

$R_{2,n}(x,t) = P\{C(t)=7, Y(t)=n,\ y \leq G_2^0(t) < y+dy / S_2^0(t) = x\}$

$V_n(x,t) = P\{C(t)=8, Y(t)=n, x \leq V^0(t) < x+dx\}$

We assume that the sequence meets the stability condition, allowing us to set

$P_0 = \lim_{t\to\infty} P_0(t),\ P_n(x) = \lim_{t\to\infty} P_n(x,t), V_n(x) = \lim_{t\to\infty} V_n(x,t),\ Q_{i,n}(x,y) = \lim_{t\to\infty} Q_{i,n}(x,y,t),\ R_{i,n}(x,y) = \lim_{t\to\infty} R_{i,n}(x,y,t)$

i = 1, 2. We attain the following set of equations as a result that governs the dynamic behavior using the supplementary variable technique.

$$\lambda P_\circ = (1-\theta_1)\int_0^\infty V_0(x)v(x)dx + \bar{r}_1(1-a_1)\int_0^\infty \pi_{1,0}(x)\mu_1(x)dx + (1-a_1)\int_0^\infty \pi_{2,0}(x)\mu_2(x)dx \tag{1}$$

$$\frac{dP_n(x)}{dx} + (\lambda + a(x))P_n(x) = 0 \tag{2}$$

$$\frac{d\pi_{1,0}(x)}{dx} = -(\lambda + \alpha_1 + \mu_1(x))\pi_{1,0}(x) + \int_0^\infty R_{1,0}(x,y)r_1(y)dy \tag{3}$$

$$\frac{d\pi_{1,n}(x)}{dx} = -(\lambda + \alpha_1 + \mu_1(x))\pi_{1,n}(x) + \lambda\pi_{1,n-1}(x) + \int_0^\infty R_{1,n}(x,y)r_1(y)dy,\ n \geq 1 \tag{4}$$

$$\frac{d\pi_{2,0}(x)}{dx} = -(\lambda + \alpha_2 + \mu_2(x))\pi_{2,0}(x) + \int_0^\infty R_{2,0}(x,y)r_2(y)dy \tag{5}$$

$$\frac{d\pi_{2,0}(x)}{dx} = -(\lambda + \alpha_2 + \mu_2(x))\pi_{2,0}(x) + \lambda\pi_{2,n-1}(x) + \int_0^\infty R_{2,n}(x,y)r_2(y)dy,\ n \geq 1 \tag{6}$$

$$\frac{dQ_{1,0}(x,y)}{dy} = -(\lambda + \eta_1(y))Q_{1,0}(x,y) \tag{7}$$

$$\frac{dQ_{1,n}(x,y)}{dy} = -(\lambda + \eta_1(y))Q_{1,n}(x,y) + \lambda Q_{1,n-1}(x,y) \tag{8}$$

$$\frac{dQ_{2,0}(x,y)}{dy} = -(\lambda + \eta_2(y))Q_{2,0}(x,y) \tag{9}$$

$$\frac{dQ_{2,n}(x,y)}{dy} = -(\lambda+\eta_2(y))Q_{2,n}(x,y)+\lambda Q_{2,n-1}(x,y) \quad (10)$$

$$\frac{dR_{1,0}(x,y)}{dy} = -(\lambda+r_1(y))R_{1,0}(x,y),\ n=0 \quad (11)$$

$$\frac{dR_{1,n}(x,y)}{dy} = -(\lambda+r_1(y))R_{1,n}(x,y)+\lambda R_{1,n-1}(x,y),\ n\geq 1 \quad (12)$$

$$\frac{dR_{2,0}(x,y)}{dy}+(\lambda+r_2(y))R_{2,0}(x,y)=0 \quad (13)$$

$$\frac{dR_{2,n}(x,y)}{dy} = -(\lambda+r_2(y))R_{2,n}(x,y)+\lambda R_{2,n-1}(x,y) \quad (14)$$

$$\frac{dV_0(x)}{dx}+(\lambda+v(x))V_0(x)=0 \quad (15)$$

$$\frac{dV_n(x)}{dx}+(\lambda+v(x))V_n(x)+\lambda V_{n-1}(x)=0 \quad (16)$$

Boundary criteria are as follows,

$$P_n(0)=(1-\theta_1)\int_0^\infty V_n(x)v(x)dx+\bar{r}_1(1-a_1)\int_0^\infty \pi_{1,n}(x)\mu_1(x)dx+(1-a_1)\int_0^\infty \pi_{2,n}(x)\mu_2(x)dx \quad (17)$$

$$\pi_{1,0}(0)=\lambda P_0+\theta_1\int_0^\infty V_1(x)v(x)dx+\int_0^\infty P_1(x)a(x)dx \quad (18)$$

$$\pi_{1,n}(0)=\lambda\int_0^\infty P_n(x)dx+\theta_1\int_0^\infty V_{n+1}(x)v(x)dx+\int_0^\infty P_{n+1}(x)a(x)dx \quad (19)$$

$$\pi_{2,n}(0)=r_1\int_0^\infty \pi_{1,n}(x)\mu_1(x)dx \quad (20)$$

$$Q_{1,n}(x,0)=\alpha_1\pi_{1,n}(x) \quad (21)$$

$$Q_{2,n}(x,0)=\alpha_2\pi_{2,n}(x) \quad (22)$$

$$R_{1,n}(x,0)=\int_0^\infty Q_{1,n}(x,y)\eta_1(y)dy \quad (23)$$

$$R_{2,n}(x,0)=\int_0^\infty Q_{2,n}(x,y)\eta_2(y)dy \quad (24)$$

$$V_0(0)=\bar{r}_1\int_0^\infty \pi_{1,0}(x)\mu_1(x)dx+\int_0^\infty \pi_{2,0}(x)\mu_2(x)dx \quad (25)$$

$$V_n(0)=a_1\bar{r}_1\int_0^\infty \pi_{1,n}(x)\mu_1(x)dx+a_1\int_0^\infty \pi_{2,n}(x)\mu_2(x)dx \quad (26)$$

The condition of normalizing as follows

$$P_0+\sum_{n=1}^\infty\int_0^\infty P_n(x)dx+\sum_{n=0}^\infty\left[\begin{array}{l}\int_0^\infty V_n(x)dx+\int_0^\infty \pi_{1,n}(x)dx+\int_0^\infty \pi_{2,n}(x)dx+\\ \int_0^\infty\int_0^\infty Q_{1,n}(x,y)dxdy+\int_0^\infty\int_0^\infty Q_{2,n}(x,y)dxdy+\int_0^\infty\int_0^\infty R_{1,n}(x,y)dxdy+\int_0^\infty\int_0^\infty R_{2,n}(x,y)dxdy\end{array}\right]=1 \quad (27)$$

The queueing model steady state solution for under consideration was acquired using the technique of probability generating function. To solve the system equations, create the following generating functions for $|z|\leq 1$.

$$P(x,z)=\sum_{n=1}^{\infty}P_n(x)z^n,\ P(0,z)=\sum_{n=1}^{\infty}P_n(0)z^n,\ \pi_1(x,z)=\sum_{n=0}^{\infty}\pi_{1,n}(x)z^n,\ \pi_1(0,z)=\sum_{n=0}^{\infty}\pi_{1,n}(0)z^n,\ \pi_2(x,z)=\sum_{n=0}^{\infty}\pi_{2,n}(x)z^n$$

$$\pi_2(x,z)=\sum_{n=0}^{\infty}\pi_{2,n}(x)z^n,\ V(x,z)=\sum_{n=0}^{\infty}V_n(x)z^n,\ V(0,z)=\sum_{n=0}^{\infty}V_n(0)z^n,\ Q_i(x,y,z)=\sum_{n=0}^{\infty}Q_{i,n}(x,y)z^n,\ Q_i(x,0,z)=\sum_{n=0}^{\infty}Q_{i,n}(x,0)z^n$$

$$R_i(x,y,z)=\sum_{n=0}^{\infty}R_{i,n}(x,y)z^n,\ R_i(x,0,z)=\sum_{n=0}^{\infty}R_{i,n}(x,0)z^n,\ i=1,2$$

Summing over n, multiply the equations in steady state and boundary conditions (1)–(26) by z^n .The partial differential equations are as follows:

$$\frac{\partial}{\partial x}P(x,z)+(\lambda+a(x))P(x,z)=0 \tag{28}$$

$$\frac{\partial}{\partial x}\pi_1(x,z)+(\lambda(1-z)+\alpha_1+\mu_1(x))\pi_1(x,z)-\int_0^{\infty}R_1(x,y,z)r_1(y)dy=0 \tag{29}$$

$$\frac{\partial}{\partial x}\pi_2(x,z)+(\lambda(1-z)+\alpha_2+\mu_2(x))\pi_2(x,z)-\int_0^{\infty}R_2(x,y,z)r_2(y)dy=0 \tag{30}$$

$$\frac{\partial}{\partial x}Q_1(x,y,z)+(\lambda(1-z)+\eta_1(y))Q_1(x,y,z)=0 \tag{31}$$

$$\frac{\partial}{\partial x}Q_2(x,y,z)+(\lambda(1-z)+\eta_2(y))Q_2(x,y,z)=0 \tag{32}$$

$$\frac{\partial}{\partial x}R_1(x,y,z)+(\lambda(1-z)+\xi_1(y))R_1(x,y,z)=0 \tag{33}$$

$$\frac{\partial}{\partial x}R_2(x,y,z)+(\lambda(1-z)+\xi_2(y))R_2(x,y,z)=0 \tag{34}$$

$$\frac{\partial}{\partial x}V(x,z)+(\lambda(1-z)+v(x))V(x,z)=0 \tag{35}$$

$$P(0,z)=(1-\theta_1)\int_0^{\infty}V(x,z)v(x)dx+\bar{r}_1(1-a_1)\int_0^{\infty}\pi_1(x,z)\mu_1(x)dx+(1-a_1)\int_0^{\infty}\pi_2(x,z)\mu_2(x)dx-\lambda P_0 \tag{36}$$

$$\pi_1(0,z)=\int_0^{\infty}P(x,z)a(x)dx+\lambda P_0+\frac{\theta_1}{z}\int_0^{\infty}V(x,z)v(x)dx+\lambda\int_0^{\infty}P(x,z)dx \tag{37}$$

$$\pi_2(0,z)=r_1\int_0^{\infty}\pi_1(x,z)\mu_1(x)dx \tag{38}$$

$$Q_1(x,0,z)=\alpha_1\pi_1(x,z) \tag{39}$$

$$Q_2(x,0,z)=\alpha_2\pi_2(x,z) \tag{40}$$

$$R_1(x,0,z)=\int_0^{\infty}Q_1(x,y,z)\eta_1(y)dy \tag{41}$$

$$R_2(x,0,z)=\int_0^{\infty}Q_2(x,y,z)\eta_2(y)dy \tag{42}$$

$$V(0,z)=\bar{r}_1a_1\int_0^{\infty}\pi_1(x,z)\mu_1(x)dx+a_1\int_0^{\infty}\pi_2(x,z)\mu_2(x)dx \tag{43}$$

We attain the following equations,

$$P(x,z)=P(0,z)(1-A(x))e^{-\lambda x} \tag{44}$$

$$\pi_1(x,z)=\pi_1(0,z)(1-S_1(x))e^{-C_1(z)x} \tag{45}$$

$$\pi_2(x,z)=\pi_2(0,z)(1-S_2(x))e^{-C_2(z)x} \tag{46}$$

$$Q_1(x,y,z)=Q_1(x,0,z)(1-D_1(y))e^{-C(z)y} \tag{47}$$

$$Q_2(x,y,z)=Q_2(x,0,z)(1-D_2(y))e^{-C(z)y} \tag{48}$$

$$R_1(x,y,z) = R_1(x,0,z)(1-G_1(y))e^{-C(z)y} \tag{49}$$
$$R_2(x,y,z) = R_2(x,0,z)(1-G_2(y))e^{-C(z)y} \tag{50}$$
$$V(x,z) = V(0,z)(1-V(x))e^{-C(z)x} \tag{51}$$

Where $C(z) = \lambda(1-z)$, $C_1(z) = C(z)+\alpha_1-\alpha_1 D_1^*(C(z))G_1^*(C(z)), C_2(z) = C(z)+\alpha_2-\alpha_2 D_2^*(C(z))G_2^*(C(z))$

Substituting the equations (44) to (51) in (36) to (43) and we attain

$$P(x,z) = \lambda P_0 \frac{Nr(z)}{Dr(z)}(1-A(x))e^{-\lambda x} \tag{52}$$

Where

$$Nr(z) = z(\bar{r}_1 + r_1 S_2^*(C_2(z))S_1^*(C_1(z))(a_1(1-\theta_1)V^*(C(z))+(1-a_1)) - \{z - a_1\theta_1 V^*(C(z))(\bar{r}_1 + r_1 S_2^*(C_2(z))S_1^*(C_1(z))\}$$
$$Dr(z) = z - \left[\{z+(1-z)A^*(\lambda)\}(a_1(1-\theta_1)V^*(C(z))+(1-a_1)) + a_1\theta_1 V^*(C(z))\right](\bar{r}_1 + r_1 S_2^*(C_2(z)))S_1^*(C_1(z))$$

$$\pi_1(x,z) = \lambda P_0 A^*(\lambda)\frac{(z-1)}{Dr(z)}\left((1-S_1(x))e^{-C_1(z)x}\right) \tag{53}$$

$$\pi_2(x,z) = \lambda P_0 r_1 A^*(\lambda)\frac{(z-1)}{Dr(z)}\left(S_1^*(C_1(z))(1-S_2(x))e^{-C_2(z)x}\right) \tag{54}$$

$$Q_1(x,y,z) = \alpha_1\lambda P_0 A^*(\lambda)\frac{(z-1)}{Dr(z)}\left((1-S_1(x))e^{-C_1(z)x}(1-D_1(y))e^{-C(z)y}\right) \tag{55}$$

$$Q_2(x,y,z) = \alpha_2 r_1\lambda P_0 A^*(\lambda)\frac{(z-1)}{Dr(z)}\left(S_1^*(C_1(z))(1-S_2(x))e^{-C_2(z)x}(1-D_2(y))e^{-C(z)y}\right) \tag{56}$$

$$R_1(x,y,z) = \alpha_1\lambda P_0 A^*(\lambda)\frac{(z-1)}{Dr(z)}\left(D_1^*(C(z))(1-S_1(x))e^{-C_1(z)x}(1-G_1(y))e^{-C(z)y}\right) \tag{57}$$

$$R_2(x,y,z) = \alpha_2 r_1\lambda P_0 A^*(\lambda)\frac{(z-1)}{Dr(z)}\left(S_1^*(C_1(z))D_2^*(C(z))(1-S_2(x))e^{-C_2(z)x}(1-G_2(y))e^{-C(z)y}\right) \tag{58}$$

$$V(x,z) = \lambda P_0 A^*(\lambda)\frac{(z-1)}{Dr(z)}\left((a_1\bar{r}_1 + a_1 r_1 S_2^*(C_2(z)))S_1^*(C_1(z))(1-V(x))e^{-C(z)x}\right) \tag{59}$$

Up on Integrating, we attain

$$P(z) = \int_0^\infty P(x,z)dx = (1-A^*(\lambda))P_0\frac{Nr(z)}{Dr(z)} \tag{60}$$

Where

$$Nr(z) = z(\bar{r}_1 + r_1 S_2^*(C_2(z))S_1^*(C_1(z))(a_1(1-\theta_1)V^*(C(z))+(1-a_1)) - \{z - a_1\theta_1 V^*(C(z))(\bar{r}_1 + r_1 S_2^*(C_2(z))S_1^*(C_1(z))\}$$
$$Dr(z) = z - \left[\{z+(1-z)A^*(\lambda)\}(a_1(1-\theta_1)V^*(C(z))+(1-a_1)) + a_1\theta_1 V^*(C(z))\right](\bar{r}_1 + r_1 S_2^*(C_2(z)))S_1^*(C_1(z))$$

$$\pi_1(z) = \int_0^\infty \pi_1(x,z)dx = \frac{P_0 A^*(\lambda)C(z)}{Dr(z)}\left[\frac{S_1^*(C_1(z))-1}{C_1(z)}\right] \tag{61}$$

$$\pi_2(z) = \int_0^\infty \pi_2(x,z)dx = \frac{P_0 r_1 A^*(\lambda)C(z)}{Dr(z)}\left[\frac{S_2^*(C_2(z))-1}{C_2(z)}\right]S_1^*(C_1(z)) \tag{62}$$

$$Q_1(z) = \int_0^\infty\int_0^\infty Q_1(x,y,z)dxdy = \frac{\alpha_1 P_0 A^*(\lambda)}{Dr(z)}\left(D_1^*(C(z))-1\right)\left[\frac{1-S_1^*(C_1(z))}{C_1(z)}\right] \tag{63}$$

$$Q_2(z) = \int_0^\infty\int_0^\infty Q_2(x,y,z)dxdy = \frac{\alpha_2 r_1 P_0 A^*(\lambda)}{Dr(z)}S_1^*(C_1(z))\left(D_2^*(C(z))-1\right)\left[\frac{1-S_2^*(C_2(z))}{S_2(z)}\right] \tag{64}$$

$$R_1(z) = \int_0^\infty\int_0^\infty R_1(x,y,z)dxdy = \frac{\alpha_1 P_0 A^*(\lambda)}{Dr(z)}D_1^*(C(z))\left(G_1^*(C(z))-1\right)\left[\frac{1-S_1^*(C_1(z))}{C_1(z)}\right] \tag{65}$$

$$R_2(z) = \int_0^\infty\int_0^\infty R_2(x,y,z)dxdy = \frac{\alpha_2 r_1 P_0 A^*(\lambda)}{Dr(z)}S_1^*(C_1(z))D_2^*(C(z))\left(G_2^*(C(z))-1\right)\left[\frac{1-S_2^*(C_2(z))}{C_2(z)}\right] \tag{66}$$

$$V(z) = \int_0^\infty V(x,z)dx = \frac{P_0 A^*(\lambda)}{Dr(z)}\left(a_1\bar{r}_1 + a_1 r_1 S_2^*(C_2(z))\right)S_1^*(C_1(z))\left[V^*(C(z))-1\right] \tag{67}$$

$$K_s(z) = P_0 + P(z) + V(z) + z\left(\pi_1(z)+\pi_2(z)+Q_1(z)+Q_2(z)+R_1(z)+R_2(z)\right)$$

$$= \frac{P_0 A^*(\lambda)}{Dr(z)}[z-1]\left(r_1 + r_1 S_2^*(C_2(z))\right)S_1^*(C_1(z)) \quad (68)$$

$$K_q(z) = P_0 + P(z) + V(z) + \pi_1(z) + \pi_2(z) + Q_1(z) + Q_2(z) + R_1(z) + R_2(z) = \frac{P_0 A^*(\lambda)}{Dr(z)}[z-1] \quad (69)$$

Using normalizing, we attain

$$P_0 + \lim_{z\to 1}\left[P(z) + \pi_1(z) + \pi_2(z) + Q_1(z) + Q_2(z) + R_1(z) + R_2(z) + V(z)\right] = 1$$

$$P_0 = \frac{A^*(\lambda) + (1 - A^*(\lambda)\theta_1 a - \lambda\{s_1[1+\alpha_1(d_1+g_1)] + r_1 s_2[1+\alpha_2(d_2+g_2)] - a_1 v_1\}}{A^*(\lambda)}$$

5.Performance Characteristics

By the above condition this system, becomes

$$L_s = \lim_{z\to 1} K_s'(z) = \lim_{z\to 1} P_0 A^*(\lambda)\left[\frac{Dr'(1)Nr''(1) - Nr'(1)Dr''(1)}{2[Dr'(1)]^2}\right] \; Nr'(1) = 1; \quad Nr''(1) = 0$$

$$Nr''(1) = (1 - A^*(\lambda))(1 - \theta_1 a_1) - \left[\lambda\left(s_1[1+\alpha_1(d_1+g_1)] + r_1 s_2[1+\alpha_2(d_2+g_2)] + a_1 v_1\right)\right]$$
$$\left\{2\lambda\left(s_1[1+\alpha_1(d_1+g_1)] + r_1 s_2[1+\alpha_2(d_2+g_2)]\right)\right\}$$

$$Dr'(1) = 1 - (1 - A^*(\lambda))(1 - \theta_1 a_1) - \left[\lambda\left(s_1[1+\alpha_1(d_1+g_1)] + r_1 s_2[1+\alpha_2(d_2+g_2)] + a_1 v_1\right)\right]$$

$$Dr''(1) = -\left\{\lambda^2\left[s_1^2[1+\alpha_1(d_1+g_1)]^2 + r_1 s_2^2[1+\alpha_2(d_2+g_2)]^2 + a_1 {v_1}^2\right]\right\}$$
$$+ \lambda^2\left[\alpha_1 s_1\left\{g_1^2 + d_1^2 + 2d_1 g_1\right\} + r_1\alpha_2 s_2\left\{g_2^2 + d_2^2 + 2d_2 g_2\right\}\right]$$
$$+ \lambda\left[s_1^2[1+\alpha_1(d_1+g_1)]^2 + r_1 s_2^2[1+\alpha_2(d_2+g_2)]^2 + a_1 v_1\right]$$
$$+ 2\lambda^2 r_1\left[s_1[1+\alpha_1(d_1+g_1)]s_2[1+\alpha_2(d_2+g_2)]\right] - (1 - A^*(\lambda))(1 - \theta_1 a_1)$$
$$+ 2a_1\lambda^2 v\left[s_1[1+\alpha_1(d_1+g_1)] + s_2[1+\alpha_2(d_2+g_2)]\right]$$
$$+ 2(1 - A^*(\lambda))\left\{\lambda\left[s_1[1+\alpha_1(d_1+g_1)] + s_2[1+\alpha_2(d_2+g_2)]\right]\right\}(1 - \theta_1 a_1)$$

$$L_q = \lim_{z\to 1} K_q'(z) = \lim_{z\to 1} P_0 A^*(\lambda)\left[\frac{Dr'(1)Nr''(1) - Nr'(1)Dr''(1)}{2[Dr'(1)]^2}\right] \; Nr'(1) = 1, \quad Nr''(1) = 0$$

We gain various performance measures of the system as follows
The probability that the server will be idle during the retry period in the steady state is

$$P = \lim_{z\to 1} P(z) = \frac{(1 - A^*(\lambda))\left[\lambda\left(s_1[1+\alpha_1(d_1+g_1)] + r_1 s_2[1+\alpha_2(d_2+g_2)] + a_1 v_1\right) - \theta_1 a_1\right]}{A^*(\lambda)}$$

The probability that the server will be inactive during standard service hours in a stable state is $\pi_1 = \lim_{z\to 1} \pi_1(z) = \lambda s_1$

The probability that the server is not free on optional re-service in the steady state is

$$\pi_2 = \lim_{z\to 1} \pi_2(z) = \lambda r_1 s_2$$

The probability that the server is postponing normal service repair in the long run is

$$Q_1 = \lim_{z\to 1} Q_1(z) = \alpha_1 \lambda s_1 d_1$$

The likelihood that the server is postponing repair on optional re-service in the steady state is

$$Q_2 = \lim_{z\to 1} Q_2(z) = \alpha_2 \lambda r_1 s_2 d_2$$

The steady-state probability that the server is out of commission during typical operations is

$$R_1 = \lim_{z\to 1} R_1(z) = \alpha_1 \lambda s_1 g_1$$

The likelihood that the server is under repair on voluntary re-service in the steady state is

$$R_2 = \lim_{z\to 1} R_2(z) = \alpha_2 \lambda r_1 s_2 g_2$$

The likelihood that the server is taking vacation in the steady state $V = \lim_{z\to 1} V(z) = \lambda a v_1$

Let *B* be the utilization of the server (server is busy), which is the chance that the server is serving an arrival at any given time is $B = \pi_1(1) + \pi_2(1) = \lambda s_1 + \lambda r_1 s_2$

Let R be the likelihood that the server is idle, down, or on vacation in the steady state. $R = P_0 + P(1) + Q_1(1) + Q_2(1) + R_1(1) + R_2(1) + V(1) = 1 - \{\lambda s_1 + \lambda r_1 s_2\}$. It is verified that $R = 1 - B$

The user's accessibility is $A = P_0 + P + \pi_1 + \pi_2 = 1 - \lambda\{\alpha_1 s_1[d_1 + g_1] + \alpha_2 s_2[d_2 + g_2] + a_1 v_1\}$

The non-performance behavior is $F = \alpha_1\pi_1 + \alpha_2\pi_2 = \lambda[\alpha_1 s_1 + \alpha_2 s_2]$

6. Special Cases

Case (i): If $\alpha_1 = \alpha_2 = 0$ then the system reduces to retrial queues without breakdown

Case (ii): If $\theta = 0$, we obtain that the M/G/1 retrial queues with delayed repair and optional re-service under modified Bernoulli server vacation

Case (iii): If $r = 0$, then the suggested model transforms into a single-server in retrial with Bernoulli server vacation, delaying repair and orbital search

Case (iv): If $a_1 = 0$, we attain the search of arrivals of an M/G/1 retrial queues with delayed repair and optional re-service

7. Conclusion

The system size and orbit size distributions in steady state are investigated using the supplementary variable technique, as well as their averages, for a server in retrials including arrival search, delayed repair, voluntary re-service under Bernoulli server vacation. Other system performances measurements and orbit properties are also computed. By including the batch arrival concepts, service in bulk, impatient consumers, and breakdowns in the workplace, this effort can be expanded in numerous directions. This analysis will be extremely useful to system managers in making informed judgments about the system's size and other issues and a real-world application in a computer processing system that uses a processor to process messages.

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