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ORIGINAL RESEARCH **Open Access**

ANALYSIS OF A SINGLE SERVER SERVING THREE QUEUES $M^{[X_1]}/G_1/1$, $M^{[X_2]}/G_2/1$, $M^{[X_3]}/G_3/1$ WITH PRIORITY SERVICES, WORKING BREAKDOWN, MODIFIED BERNOULLI VACATION

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Abstract

In this paper considers $M^{[X_1]}/G_1/1$, $M^{[X_2]}/G_2/1$, $M^{[X_3]}/G_3/1$ general queueing system with priority services . Three types of customers from different classes arrive at the system in different independent Poisson process. The server follows the non preemptive priority rule subject to working breakdown, and modified Bernoulli vacation with general (arbitrary) vacation periods. After completing the service, if there is no high priority customers present in the system. The time dependent probability generating functions have been obtained in terms of their Laplace transforms and the corresponding steady state results are obtained. Also the average number of customer in the priority and non priority, preemptive priority queue and the average waiting time are derived.

Key Words: Preemptive priority queueing systems , modified Bernoulli vacation, working breakdown, supplementary variable technique.

AMS Classification: 68M20,60K25

1 Introduction

The study on queueing models has become an indispensable area due to its wide applicability in real-life situations; all the models considered have had the property that units proceed to service on a first come ,first served basis. This is obviously not only the manner of service, and there are many alternatives, such as last come first- served, selection in random order, and selection by priority . In order to offer different qualities of service for different kinds of customers, we often control a queueing system by priority mechanism.

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This phenomenon is common in practice. For example, in telecommunication transfer protocol, for guaranteeing different layers of service for different customers, priority classes control may appear in the header of an IP package or in an ATM cell. Priority control is also widely used in production practice, transportation management, etc.

A few papers appear on bulk arrival priority queueing system. Hawkes (1956) considered the time dependent solution of a priority queue with bulk arrivals. Haghighi and Mishev (2006) have studied a parallel priority queueing system with finite buffers. Vacation queues have been studied by several author including Doshi (1986), Takagi (1990), and Chae et al. (2001). Ayyappan and Muthu Ganapathi Subramanian (2009) have studied single server retrial queueing system with non preemptive priority queueing system with a single server serving two queues with optional server vacation exhaustive service of the priority units. Thangaraj and Vanitha (2010) have studied an M/G/1 queue with two-stage heterogeneous service compulsory server vacation and random breakdowns, and Jau- Chauan and Fu- Min (2009) have studied modified vacation policy for M/G/1 retrial queue with balking and feedback. Here we dervice time dependent probability generating functions for both priority , non priority and preemptive priority units in terms of Laplace transforms. We also derive the average queue size and average waiting time in the queue.

2 Section
Definitions and Notations :

We define the following notations :

1. $Q^1_{pqr}(x,t)$ = Probability that at time t, the server is active providing service and there are p(p $\geq$ 0) priority units,q (q $\geq$ 0) non priority units and r(r$\geq$ 0) preemptive units in the queue excluding the one priority unit in service with elapsed service time for this customer is x. Accordingly ,

   $Q^1_{pqr}(t) = \int_0^\infty Q^1_{pqr}(x,t)dx.$
2. $Q^2_{pqr}(x,t)$ = Probability that at time t, the server is active providing service and there are p(p $\geq$ 0) priority units,q (q $\geq$ 0) non priority units and r(r$\geq$ 0) preemptive units in the queue excluding the one priority unit in service with elapsed service time for this customer is x. Accordingly ,

   $Q^2_{pqr}(t) = \int_0^\infty Q^2_{pqr}(x,t)dx.$.

3. Probability that at time t, the server is active providing service and there are p(p $\geq$ 0) priority units,q (q $\geq$ 0) non priority units and r(r$\geq$ 0) preemptive units in the queue excluding the one priority unit in service with elapsed service time for this customer is x. Accordingly ,

$$Q^3_{pqr}(t) = \int_0^\infty Q^3_{pqr}(x,t)dx.$$

4. $R_{pqr}(x,t)$ = Probability that at time t, the server is on vacation with elapsed vacation time x and there are

p(p $\geq$ 0) priority units,q (q $\geq$ 0) non priority units and r(r$\geq$ 0) preemptive units in the queue.

$$R_{pqr}(t) = \int_0^\infty R_{pqr}(x,t)\,dx.$$

5. $B_{pqr}(x,t)$ = Probability that at time t, the server is on vacation with elapsed vacation time x and there are p(p $\geq$ 0) priority units,q (q $\geq$ 0) non priority units and r(r$\geq$ 0) preemptive units in the queue.

$$B_{pqr}(t) = \sum_0^\infty B_{pqr}(x,t)\,dx.$$

6. S(t) = Probability that at time t, there are no priority and non priority customers in the system and the serve is idle but available in the system.

The Kolmogorov forward equations:

$$\frac{d}{dt}Q^1_{pqr}(x,t) + \frac{d}{dt}Q^1_{pqr}(x,t) = -(\lambda_{11}+\lambda_{12}+\lambda_{13}+\mu_{11}(x))Q^1_{pqr}(x,t) + \lambda_{11}\sum_{i=1}^{p} d_i\, Q^1_{p-iqr}(x,t) + \lambda_{12}\, b\sum_{j=1}^{q} d_j\, Q^1_{pq-jr}(x,t) + \lambda_{13}\sum_{k=1}^{r} d_k\, Q^1_{pqr-k}(x,t)\ ;\ p\geq 1,\ q\geq 1,\ r\geq 1.$$

(1)

$$\frac{d}{dt}Q^1_{pq0}(x,t) + \frac{d}{dt}Q^1_{pq0}(x,t) = -(\lambda_{11}+\lambda_{12}+\lambda_{13}+\mu_{11}(x))Q^1_{pq0}(x,t) + \lambda_{11}\sum_{i=1}^{p} d_i\, Q^1_{p-iq0}(x,t) + \lambda_{12}\, b\sum_{j=1}^{q} d_j\, Q^1_{pq-j0}(x,t) + \lambda_{13}\sum_{k=1}^{r} d_k\, Q^1_{pq0}(x,t)\ ;\ p\geq 1,\ q\geq 1,\ r = 0.$$

(2)

$$\frac{d}{dt}Q^1_{p0r}(x,t) + \frac{d}{dt}Q^1_{p0r}(x,t) = -(\lambda_{11}+\lambda_{12}+\lambda_{13}+\mu_{11}(x))Q^1_{p0r}(x,t) + \lambda_{11}\sum_{i=1}^{p} d_i\, Q^1_{p-i0r}(x,t) + \lambda_{12}\, b\sum_{j=1}^{q} d_j\, Q^1_{p0r}(x,t) + \lambda_{13}\sum_{k=1}^{r} d_k\, Q^1_{pqr-k}(x,t)\ ;\ p\geq 1,\ q = 0,\ r\geq 1.$$

(3)

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$\frac{d}{dt} Q^1_{0qr}(x,t) + \frac{d}{dt} Q^1_{0qr}(x,t) = -(\lambda_{11} + \lambda_{12} + \lambda_{13} + \mu_{11}(x))\, Q^1_{0qr}(x,t) + \lambda_{11} \sum_{i=1}^{p} d_i\, Q^1_{0qr}(x,t) + \lambda_{12}$

$b\sum_{j=1}^{q} d_j\, Q^1_{0q\text{-}jr}(x,t) + \lambda_{13} \sum_{k=1}^{r} d_k\, Q^1_{0qr\text{-}k}(x,t)$; $p = 0,\ q \geq 1,\ r \geq 1.$

(4)

$\frac{d}{dt} Q^1_{000}(x,t) + \frac{d}{dt} Q^1_{000}(x,t) = -(\lambda_{11} + \lambda_{12} + \lambda_{13} + \mu_{11}(x))\, Q^1_{000}(x,t) + \lambda_{11} \sum_{i=1}^{p} d_i\, Q^1_{000}(x,t) + \lambda_{12}$

$b\sum_{j=1}^{q} d_j\, Q^1_{000}(x,t) + \lambda_{13} \sum_{k=1}^{r} d_k\, Q^1_{000}(x,t)$; $p = q = r = 0.$

(5)

$\frac{d}{dt} R_{pqr}(x,t) + \frac{d}{dt} R_{pqr}(x,t) = -(\lambda_{11} + \lambda_{12} + \lambda_{13} + \delta(x))\, R_{pqr}(x,t) + \lambda_{11} \sum_{i=1}^{p} d_i\, R_{p\text{-}iqr}(x,t) + \lambda_{12}$

$b\sum_{j=1}^{q} d_j\, R_{pq\text{-}jr}(x,t) + \lambda_{13} \sum_{k=1}^{r} d_k\, R_{pqr\text{-}k}(x,t)$; $p \geq 1,\ q \geq 1,\ r \geq 1.$

(6)

$\frac{d}{dt} R_{pq0}(x,t) + \frac{d}{dt} R_{pq0}(x,t) = -(\lambda_{11} + \lambda_{12} + \lambda_{13} + \delta(x))\, R_{pq0}(x,t) + \lambda_{21} \sum_{i=1}^{p} d_i\, R_{p\text{-}iq0}(x,t) + \lambda_{22}$

$b\sum_{j=1}^{q} d_j\, R_{pq\text{-}j0}(x,t) + \lambda_{23} \sum_{k=1}^{r} d_k\, R_{pq0}(x,t)$; $p \geq 1,\ q \geq 1,\ r = 0.$

(7)

$\frac{d}{dt} R_{p0r}(x,t) + \frac{d}{dt} R_{p0r}(x,t) = -(\lambda_{11} + \lambda_{12} + \lambda_{13} + \delta(x))\, R_{p0r}(x,t) + \lambda_{11} \sum_{i=1}^{p} d_i\, R_{p\text{-}i0r}(x,t) + \lambda_{12}$

$b\sum_{j=1}^{q} d_j\, R_{p0r}(x,t) + \lambda_{13} \sum_{k=1}^{r} d_k\, R_{pqr\text{-}k}(x,t)$; $p \geq 1,\ q = 0,\ r \geq 1.$

(8)

$\frac{d}{dt} R_{0qr}(x,t) + \frac{d}{dt} R_{0qr}(x,t) = -(\lambda_{11} + \lambda_{12} + \lambda_{13} + \delta(x))\, R_{0qr}(x,t) + \lambda_{11} \sum_{i=1}^{p} d_i\, R_{0qr}(x,t) + \lambda_{12}$

$b\sum_{j=1}^{q} d_j\, R_{0q\text{-}jr}(x,t) + \lambda_{13} \sum_{k=1}^{r} d_k\, R_{0qr\text{-}k}(x,t)$; $p = 0,\ q \geq 1,\ r \geq 1.$

(9)

$\frac{d}{dt} R_{000}(x,t) + \frac{d}{dt} R_{000}(x,t) = -(\lambda_{11} + \lambda_{12} + \lambda_{13} + \delta(x))\, R_{000}(x,t) + \lambda_{11} \sum_{i=1}^{p} d_i\, R_{000}(x,t) + \lambda_{12}$

$b\sum_{j=1}^{q} d_j\, R_{000}(x,t) + \lambda_{13} \sum_{k=1}^{r} d_k\, R_{000}(x,t)$; $p = q = r = 0.$

(10)

$\frac{d}{dt} Q^2_{pqr}(x,t) + \frac{d}{dt} Q^2_{pqr}(x,t) = -(\lambda_{21} + \lambda_{22} + \lambda_{23} + \mu_{21}(x))\, Q^2_{pqr}(x,t) + \lambda_{21} \sum_{i=1}^{p} d_i\, Q^2_{p-iqr}(x,t) + \lambda_{22}$

$b\sum_{j=1}^{q} d_j\, Q^1_{pq-jr}(x,t) + \lambda_{23} \sum_{k=1}^{r} d_k\, Q^1_{pqr-k}(x,t)$; $p \geq 1,\ q \geq 1,\ r \geq 1.$

(11)

$\frac{d}{dt} Q^2_{pq0}(x,t) + \frac{d}{dt} Q^2_{pq0}(x,t) = -(\lambda_{21} + \lambda_{22} + \lambda_{23} + \mu_{21}(x))\, Q^2_{pq0}(x,t) + \lambda_{21} \sum_{i=1}^{p} d_i\, Q^2_{p-iq0}(x,t) + \lambda_{22}$

$b\sum_{j=1}^{q} d_j\, Q^2_{pq-j0}(x,t) + \lambda_{23} \sum_{k=1}^{r} d_k\, Q^2_{pq0}(x,t)$; $p \geq 1,\ q \geq 1,\ r = 0.$

(12)

$\frac{d}{dt} Q^2_{p0r}(x,t) + \frac{d}{dt} Q^2_{p0r}(x,t) = -(\lambda_{21} + \lambda_{22} + \lambda_{23} + \mu_{21}(x))\, Q^2_{p0r}(x,t) + \lambda_{21} \sum_{i=1}^{p} d_i\, Q^2_{p-i0r}(x,t) + \lambda_{22}$

$b\sum_{j=1}^{q} d_j\, Q^2_{p0r}(x,t) + \lambda_{23} \sum_{k=1}^{r} d_k\, Q^2_{pqr-k}(x,t)$; $p \geq 1,\ q = 0,\ r \geq 1.$

(13)

$\frac{d}{dt} Q^2_{0qr}(x,t) + \frac{d}{dt} Q^2_{0qr}(x,t) = -(\lambda_{21} + \lambda_{22} + \lambda_{23} + \mu_{21}(x))\, Q^2_{0qr}(x,t) + \lambda_{21} \sum_{i=1}^{p} d_i\, Q^2_{0qr}(x,t) + \lambda_{22}$

$b\sum_{j=1}^{q} d_j\, Q^2_{0q-jr}(x,t) + \lambda_{23} \sum_{k=1}^{r} d_k\, Q^2_{0qr-k}(x,t)$; $p = 0,\ q \geq 1,\ r \geq 1.$

(14)

$\frac{d}{dt} Q^2_{000}(x,t) + \frac{d}{dt} Q^2_{000}(x,t) = -(\lambda_{21} + \lambda_{22} + \lambda_{23} + \mu_{21}(x))\, Q^2_{000}(x,t) + \lambda_{21} \sum_{i=1}^{p} d_i\, Q^2_{000}(x,t) + \lambda_{22}$

$b\sum_{j=1}^{q} d_j\, Q^2_{000}(x,t) + \lambda_{23} \sum_{k=1}^{r} d_k\, Q^2_{000}(x,t)$; $p = q = r = 0.$

(15)

$\frac{d}{dt} Q^3_{pqr}(x,t) + \frac{d}{dt} Q^3_{pqr}(x,t) = -(\lambda_{31} + \lambda_{32} + \lambda_{33} + \mu_{31}(x))\, Q^3_{pqr}(x,t) + \lambda_{31} \sum_{i=1}^{p} d_i\, Q^3_{p-iqr}(x,t) + \lambda_{32}$

$b\sum_{j=1}^{q} d_j\, Q^3_{pq-jr}(x,t) + \lambda_{33} \sum_{k=1}^{r} d_k\, Q^3_{pqr-k}(x,t)$; $p \geq 1,\ q \geq 1,\ r \geq 1.$

(16)

$\frac{d}{dt} Q^3_{pq0}(x,t) + \frac{d}{dt} Q^3_{pq0}(x,t) = -(\lambda_{31} + \lambda_{32} + \lambda_{33} + \mu_{31}(x))\, Q^3_{pq0}(x,t) + \lambda_{31} \sum_{i=1}^{p} d_i\, Q^3_{p-iq0}(x,t) + \lambda_{32}$

$b\sum_{j=1}^{q} d_j\, Q^3_{pq-j0}(x,t) + \lambda_{33} \sum_{k=1}^{r} d_k\, Q^3_{pq0}(x,t)$; $p \geq 1,\ q \geq 1,\ r = 0.$

(17)

$\frac{d}{dt} Q^3_{p0r}(x,t) + \frac{d}{dt} Q^3_{p0r}(x,t) = -(\lambda_{31} + \lambda_{32} + \lambda_{33} + \mu_{31}(x)) Q^3_{p0r}(x,t) + \lambda_{31} \sum_{i=1}^{p} d_i Q^3_{p-i0r}(x,t) + \lambda_{32}$

$b\sum_{j=1}^{q} d_j Q^3_{p0r}(x,t) + \lambda_{33} \sum_{k=1}^{r} d_k Q^3_{pqr-k}(x,t)$; $p \geq 1, q = 0, r \geq 1.$

(18)

$\frac{d}{dt} Q^3_{0qr}(x,t) + \frac{d}{dt} Q^3_{0qr}(x,t) = -(\lambda_{31} + \lambda_{32} + \lambda_{33} + \mu_{31}(x)) Q^3_{0qr}(x,t) + \lambda_{31} \sum_{i=1}^{p} d_i Q^3_{0qr}(x,t) + \lambda_{32}$

$b\sum_{j=1}^{q} d_j Q^3_{0q-jr}(x,t) + \lambda_{33} \sum_{k=1}^{r} d_k Q^3_{0qr-k}(x,t)$; $p = 0, q \geq 1, r \geq 1.$

(19)

$\frac{d}{dt} Q^3_{000}(x,t) + \frac{d}{dt} Q^3_{000}(x,t) = -(\lambda_{31} + \lambda_{32} + \lambda_{33} + \mu_{31}(x)) Q^3_{000}(x,t) + \lambda_{31} \sum_{i=1}^{p} d_i Q^3_{000}(x,t) + \lambda_{32}$

$b\sum_{j=1}^{q} d_j Q^3_{000}(x,t) + \lambda_{33} \sum_{k=1}^{r} d_k Q^3_{000}(x,t)$; $p = q = r = 0.$

(20)

$\frac{d}{dt} B_{pqr}(x,t) + \frac{d}{dt} B_{pqr}(x,t) = -(\lambda_{11} + \lambda_{12} + \lambda_{13} + \gamma(x)) B_{pqr}(x,t) + \lambda_{11} \sum_{i=1}^{p} d_i B_{p-iqr}(x,t) + \lambda_{12} b\sum_{j=1}^{q} d_j$

$B_{pq-jr}(x,t) + \lambda_{13} \sum_{k=1}^{r} d_k B_{pqr-k}(x,t)$; $p \geq 1, q \geq 1, r \geq 1.$

(21)

$\frac{d}{dt} B_{pq0}(x,t) + \frac{d}{dt} B_{pq0}(x,t) = -(\lambda_{11} + \lambda_{12} + \lambda_{13} + \gamma(x)) B_{pq0}(x,t) + \lambda_{21} \sum_{i=1}^{p} d_i B_{p-iq0}(x,t) + \lambda_{22}$

$b\sum_{j=1}^{q} d_j B_{pq-j0}(x,t) + \lambda_{23} \sum_{k=1}^{r} d_k B_{pq0}(x,t)$; $p \geq 1, q \geq 1, r = 0.$

(22)

$\frac{d}{dt} B_{p0r}(x,t) + \frac{d}{dt} B_{p0r}(x,t) = -(\lambda_{11} + \lambda_{12} + \lambda_{13} + \gamma(x)) B_{p0r}(x,t) + \lambda_{11} \sum_{i=1}^{p} d_i B_{p-i0r}(x,t) + \lambda_{12} b\sum_{j=1}^{q} d_j$

$B_{p0r}(x,t) + \lambda_{13} \sum_{k=1}^{r} d_k B_{pqr-k}(x,t)$; $p \geq 1, q = 0, r \geq 1.$

(23)

$\frac{d}{dt} B_{0qr}(x,t) + \frac{d}{dt} B_{0qr}(x,t) = -(\lambda_{11} + \lambda_{12} + \lambda_{13} + \gamma(x)) B_{0qr}(x,t) + \lambda_{11} \sum_{i=1}^{p} d_i B_{0qr}(x,t) + \lambda_{12} b\sum_{j=1}^{q} d_j$

$B_{0q-jr}(x,t) + \lambda_{13} \sum_{k=1}^{r} d_k B_{0qr-k}(x,t)$; $p = 0, q \geq 1, r \geq 1.$

(24)

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$\frac{d}{dt} B_{000}(x,t) + \frac{d}{dt} B_{000}(x,t) = -(\lambda_{11} + \lambda_{12} + \lambda_{13} + \gamma(x)) R_{000}(x,t) + \lambda_{11} \sum_{i=1}^{p} d_i B_{000}(x,t) + \lambda_{12} b \sum_{j=1}^{q} d_j$

$B_{000}(x,t) + \lambda_{13} \sum_{k=1}^{r} d_k B_{000}(x,t)$; $p = q = r = 0$.

(25)

$\frac{d}{dt} S(t) = -(\lambda_{11} + \lambda_{22} + \lambda_{33}) S(t) + (1-\theta) \int_0^\infty Q^1_{000}(x,t)\, \mu_{11}(x)dx + \int_0^\infty Q^2_{000}(x,t)\, \mu_{22}(x)dx +$

$\int_0^\infty Q^3_{000}(x,t)\, \mu_{33}(x)dx + \int_0^\infty R_{000}(x,t)\, \delta(x)dx + \int_0^\infty B_{000}(x,t)\, \gamma(x)dx.$

(26)

The above set of equations are to be solved under the following boundary conditions $x = 0$

$Q^1_{pqr}(0,t) = \int_0^\infty \square^1_{P+1qr}(x,t)\, \mu_{11}(x)dx + \int_0^\infty \square^2_{P+1qr}(x,t)\, \mu_{22}(x)dx + \int_0^\infty \square^3_{P+1qr}(x,t)\, \mu_{33}(x)dx +$

$\int_0^\infty R_{P+1qr}(x,t)\, \delta(x)dx + \int_0^\infty B_{p+1qr}(x,t)\, \gamma(x)dx$, ; $p \geq 1$, $q \geq 1$, $r \geq 1$.

(27)

$Q^1_{pq0}(0,t) = \lambda_{11} d_{p+1} s(t) + \int_0^\infty \square^1_{P+1q0}(x,t)\, \mu_{11}(x)dx + \int_0^\infty \square^2_{P+1q0}(x,t)\, \mu_{22}(x)dx + \int_0^\infty \square^3_{P+1q0}(x,t)$

$\mu_{33}(x)dx + \int_0^\infty R_{P+1q0}(x,t)\, \delta(x)dx + \int_0^\infty B_{p+1q0}(x,t)\, \gamma(x)dx$, ; $p \geq 1$, $q \geq 1$, $r = 0$

(28)

$Q^1_{0qr}(0,t) = \int_0^\infty \square^1_{0qr}(x,t)\, \mu_{11}(x)dx + \int_0^\infty \square^2_{0qr}(x,t)\, \mu_{22}(x)dx + \int_0^\infty \square^3_{0qr}(x,t)\, \mu_{33}(x)dx +$

$\int_0^\infty R_{0qr}(x,t)\, \delta(x)dx + \int_0^\infty B_{0qr}(x,t)\, \gamma(x)dx$, ; $p = 0$, $q \geq 1$, $r \geq 1$.

(29)

$Q^1_{p0r}(0,t) = \int_0^\infty \square^1_{P0r}(x,t)\, \mu_{11}(x)dx + \int_0^\infty \square^2_{p0r}(x,t)\, \mu_{22}(x)dx + \int_0^\infty \square^3_{P0r}(x,t)\, \mu_{33}(x)dx +$

$\int_0^\infty R_{P0r}(x,t)\, \delta(x)dx + \int_0^\infty B_{p0r}(x,t)\, \gamma(x)dx$, ; $p \geq 1$, $q=0$, $r \geq 1$.

(30)

$Q^1_{000}(0,t) = \lambda_{11} d_1 s(t) + \int_0^\infty \square^1_{100}(x,t)\, \mu_{11}(x)dx + \int_0^\infty \square^2_{010}(x,t)\, \mu_{22}(x)dx + \int_0^\infty \square^3_{001}(x,t)\, \mu_{33}(x)dx +$

$\int_0^\infty R_{100}(x,t)\, \delta(x)dx + \int_0^\infty B_{100}(x,t)\, \gamma(x)dx$, ; $p=q=r=0$

(31)

$R_{0qr}(0,t) = \Box \int_0^\infty \Box^1{}_{0qr}(x,t)\,\mu_{11}(x)dx \;, q\geq 0, r\geq 0$

(32)

$B_{0qr}(0,t) = \int_0^\infty \Box^1{}_{0qr}(x,t)\,\mu_{11}(x)dx \;, q\geq 0, r\geq 0$

(33)

$Q^2{}_{000}(0,t) = \lambda_{22}d_1s(t) + \int_0^\infty \Box^1{}_{p00}(x,t)\,\mu_{11}(x)dx + \int_0^\infty \Box^2{}_{0q0}(x,t)\,\mu_{22}(x)dx + \int_0^\infty \Box^3{}_{00r}(x,t)\,\mu_{33}(x)dx +$

$\int_0^\infty R_{010}(x,t)\,\delta(x)dx + \int_0^\infty B_{010}(x,t)\,\gamma(x)dx, ;\; p=q=r=0$

(34)

$Q^2{}_{0qr}(0,t) = \int_0^\infty \Box^1{}_{0qr}(x,t)\,\mu_{11}(x)dx + \int_0^\infty \Box^2{}_{0qr}(x,t)\,\mu_{22}(x)dx + \int_0^\infty \Box^3{}_{0qr}(x,t)\,\mu_{33}(x)dx +$

$\int_0^\infty R_{0qr}(x,t)\,\delta(x)dx + \int_0^\infty B_{0qr}(x,t)\,\gamma(x)dx, ;\; p= 0, q\geq 1, r\geq 1.$

(35)

$Q^2{}_{p0r}(0,t) = \int_0^\infty Q^1{}_{P0r}(x,t)\,\mu_{11}(x)dx + \int_0^\infty Q^2{}_{p0r}(x,t)\,\mu_{22}(x)dx + \int_0^\infty Q^3{}_{P0r}(x,t)\,\mu_{33}(x)dx +$

$\int_0^\infty R_{P0r}(x,t)\,\delta(x)dx + \int_0^\infty B_{p0r}(x,t)\,\gamma(x)dx, ;\; p\geq 1, q=0, r\geq 1.$

(36)

$Q^2{}_{pq0}(0,t) = \lambda_{11}d_{p+1}s(t) + \int_0^\infty Q^1{}_{P+1q0}(x,t)\,\mu_{11}(x)dx + \int_0^\infty Q^2{}_{P+1q0}(x,t)\,\mu_{22}(x)dx + \int_0^\infty Q^3{}_{P+1q0}(x,t)$

$\mu_{33}(x)dx + \int_0^\infty R_{P+1q0}(x,t)\,\delta(x)dx + \int_0^\infty B_{p+1q0}(x,t)\,\gamma(x)dx, ;\; p\geq 1, q\geq 1, r = 0$

(37)

$R_{p0r}(0,t) = \Box \int_0^\infty \Box^2{}_{p0r}(x,t)\,\mu_{22}(x)dx \;, p\geq 0, r\geq 0$

(38)

$B_{p0r}(0,t) = \int_0^\infty Q^2{}_{p0r}(x,t)\,\mu_{22}(x)dx \;, p\geq 0, r\geq 0$

(39)

$Q^3{}_{000}(0,t) = \lambda_{33}d_1s(t) + \int_0^\infty \Box^1{}_{p00}(x,t)\,\mu_{11}(x)dx + \int_0^\infty \Box^2{}_{0q0}(x,t)\,\mu_{22}(x)dx + \int_0^\infty \Box^3{}_{00r}(x,t)\,\mu_{33}(x)dx +$

$\int_0^\infty \Box_{010}(x,t)\,\Box(\Box)dx + \int_0^\infty \Box_{010}(x,t)\,\Box(\Box)dx, ;\; p=q=r=0$

(40)

$$Q^3_{0qr}(0,t) = \int_0^\infty \square^1_{0qr}(x,t)\, \mu_{11}(x)dx + \int_0^\infty \square^2_{0qr}(x,t)\, \mu_{22}(x)dx + \int_0^\infty \square^3_{0qr}(x,t)\, \mu_{33}(x)dx + \int_0^\infty \square_{0qr}(x,t)\, \square(\square)dx + \int_0^\infty \square_{0qr}(x,t)\, \square(\square)dx, ;\ p= 0,\ q\geq 1,\ r\geq 1.$$

(41)

$$Q^3_{p0r}(0,t) = \int_0^\infty \square^1_{P0r}(x,t)\, \mu_{11}(x)dx + \int_0^\infty \square^2_{p0r}(x,t)\, \mu_{22}(x)dx + \int_0^\infty \square^3_{P0r}(x,t)\, \mu_{33}(x)dx + \int_0^\infty R_{P0r}(x,t)\, \delta(x)dx + \int_0^\infty B_{p0r}(x,t)\, \gamma(x)dx, ;\ p\geq 1,\ q=0,\ r\geq 1.$$

(42)

$$Q^3_{pq0}(0,t) = \lambda_{11}d_{p+1}s(t) + \int_0^\infty \square^1_{P+1q0}(x,t)\, \mu_{11}(x)dx + \int_0^\infty \square^2_{P+1q0}(x,t)\, \mu_{22}(x)dx + \int_0^\infty \square^3_{P+1q0}(x,t)\, \mu_{33}(x)dx + \int_0^\infty R_{P+1q0}(x,t)\, \delta(x)dx + \int_0^\infty B_{pq0}(x,t)\, \gamma(x)dx, ;\ p\geq 1,\ q\geq 1,\ r = 0$$

(43)

$$R_{pq0}(0,t) = \square \int_0^\infty \square^3_{pq0}(x,t)\, \mu_{33}(x)dx\ ,\ p\geq 0,\ q\geq 0$$

(44)

$$B_{0qr}(0,t) = \int_0^\infty \square^3_{pq0}(x,t)\, \mu_{33}(x)dx\ ,\ p\geq 0,\ q\geq 0$$

(45)

We assume that initially there are no customers in the system and the server is idle. so the initial conditions are

$$Q^1_{pqr}(0) = Q^1_{pq0}(0) = Q^1_{0qr}(0) = Q^1_{p0r}(0) = Q^1_{000}(0) =0$$

$$Q^2_{pqr}(0) = Q^2_{pq0}(0) = Q^2_{0qr}(0) = Q^2_{p0r}(0) = Q^2_{000}(0) =0$$

$$Q^3_{pqr}(0) = Q^3_{pq0}(0) = Q^3_{0qr}(0) = Q^3_{p0r}(0) = Q^3_{000}(0) =0$$

$$R_{pqr}(0) = R_{pq0}(0) = R_{0qr}(0) = R_{p0r}(0) = R_{000}(0) =0$$

$$B_{pqr}(0) = B_{pq0}(0) = B_{0qr}(0) =B_{p0r}(0) = B_{000}(0) = 0 \text{ and } S(0) = 1$$

(46)

The following probability generating functions:

$$\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty} Z^p_{11}Z^q_{22}Z^r_{33}Q^1_{pqr}(x,t) = Q^1(x,Z_{11},Z_{22},Z_{33})$$

$\sum_{p=0}^{\infty} Z^{p}_{11}Q^{1}_{p}(x,t) = Q^{1}(x,Z_{11}), \sum_{q=0}^{\infty} Z^{q}_{22}Q^{1}_{q}(x,t) = Q^{1}(x,Z_{22}), \sum_{r=0}^{\infty} Z^{p}_{33}Q^{1}_{r}(x,t) = Q^{1}(x,Z_{33})$

(47)

$\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty} Z^{p}_{11}Z^{q}_{22}Z^{r}_{33}Q^{2}_{pqr}(x,t) = Q^{2}(x,Z_{11},Z_{22},Z_{33})$

$\sum_{p=0}^{\infty} Z^{p}_{11}Q^{2}_{p}(x,t) = Q^{2}(x,Z_{11}), \sum_{q=0}^{\infty} Z^{q}_{22}Q^{2}_{q}(x,t) = Q^{2}(x,Z_{22}), \sum_{r=0}^{\infty} Z^{p}_{33}Q^{1}_{r}(x,t) = Q^{2}(x,Z_{33})$

(48)

$\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty} Z^{p}_{11}Z^{q}_{22}Z^{r}_{33}Q^{3}_{pqr}(x,t) = Q^{3}(x,Z_{11},Z_{22},Z_{33})$

$\sum_{p=0}^{\infty} Z^{p}_{11}Q^{3}_{p}(x,t) = Q^{3}(x,Z_{11}), \sum_{q=0}^{\infty} Z^{q}_{22}Q^{3}_{q}(x,t) = Q^{3}(x,Z_{22}), \sum_{r=0}^{\infty} Z^{p}_{33}Q^{3}_{r}(x,t) = Q^{3}(x,Z_{33})$

(49)

$\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty} Z^{p}_{11}Z^{q}_{22}Z^{r}_{33}R_{pqr}(x,t) = R(x,Z_{11},Z_{22},Z_{33}),$

$\sum_{p=0}^{\infty} Z^{p}_{11}R_{p}(x,t) = R(x, Z_{11}) , \sum_{q=0}^{\infty} Z^{q}_{22}R_{q}(x,t) = R(x, Z_{22}) , \sum_{r=0}^{\infty} Z^{r}_{33}R_{r}(x,t) = R(x, Z_{33}).$

(50)

$\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\sum_{r=0}^{\infty} Z^{p}_{11}Z^{q}_{22}Z^{r}_{33}B_{pqr}(x,t) = B(x,Z_{11},Z_{22},Z_{33}),$

$\sum_{p=0}^{\infty} Z^{p}_{11}B_{p}(x,t) = B(x, Z_{11}) , \sum_{q=0}^{\infty} Z^{q}_{22}B_{q}(x,t) = B(x, Z_{22}) , \sum_{r=0}^{\infty} Z^{r}_{33}B_{r}(x,t) = B(x, Z_{33}).$

(51)

Which are convergent inside the circle given by $|Z11| \leq 1$, $|Z22| \leq 1$, $|Z33| \leq 1$.

Subsection

STEADY STATE ANALYSIS : LIMITING BEHAVIOR

The steady state probability for an priority queueing system with a single server three queues $M^{[X_1]},M^{[X_2]},M^{[X_3]}$ / G_1,G_2,G_3 /1 and server vacation, breakdown based on exhaustive service of the priority units are given by

$Q^{1}(Z_{11},Z_{22},Z_{33}) = Q^{1}(0,Z_{11},Z_{22},Z_{33}) \frac{1-\overline{B}1\ (\lambda 11[1 - C(Z11)]+ \lambda 22b[\ 1 - C(Z22)]+\lambda 33[\ 1 - C(Z33)])}{(\lambda 11[1 - C(Z11)]+ \lambda 22b[\ 1 - C(Z22)]+\lambda 33b[\ 1 - C(Z33)]}$,

(52)

$Q^2(Z_{11},Z_{22},Z_{33}) = Q^2(0,Z_{22}) \frac{1-\overline{B1}\,(\lambda 11[1-C(Z11)]+\lambda 22b[1-C(Z22)]+\lambda 33[1-C(Z33)])}{(\lambda 11[1-C(Z11)]+\lambda 22b[1-C(Z22)]+\lambda 33b[1-C(Z33)]}$,

(53)

$Q^3(Z_{11},Z_{22},Z_{33}) = Q^3(0,Z_{33}) \frac{1-\overline{B1}\,(\lambda 11[1-C(Z11)]+\lambda 22b[1-C(Z22)]+\lambda 33[1-C(Z33)])}{(\lambda 11[1-C(Z11)]+\lambda 22b[1-C(Z22)]+\lambda 33b[1-C(Z33)]}$,

(54)

$R(Z_{11},Z_{22},Z_{33}) = \theta Q(0,Z_{11},Z_{22},Z_{33})\overline{B}_1(\lambda_{11} + \lambda_{22}b[1 - c(Z_{33})]$

$* \frac{1-\bar{R}\,((\lambda 11[1-C(Z11)]+\lambda 22b[1-C(Z22)]+\lambda 33[1-C(Z33)]}{(\lambda 11[1-C(Z11)]+\lambda 22b[1-C(Z22)]+\lambda 33[1-C(Z33)]}$ (55) $B(Z_{11},Z_{22},Z_{33}) =$

$\theta Q(0,Z_{11},Z_{22},Z_{33})\overline{B}_1(\lambda_{11} + \lambda_{22}b[1 - c(Z_{33})]$

$* \frac{1-\bar{R}\,((\lambda 11[1-C(Z11)]+\lambda 22b[1-C(Z22)]+\lambda 33[1-C(Z33)]}{(\lambda 11[1-C(Z11)]+\lambda 22b[1-C(Z22)]+\lambda 33[1-C(Z33)]}$ (56) In the order to determine Q, we use the normalizing condition

$Q^1(1,1,1) + Q^2(1,1,1) + Q^3(1,1,1) + R(1,1,1) + B(1,1,1) + S = 1.$

CONCLUSION:

In this paper we studied a priority queueing system with optional server vacation and modified Bernoulli breakdown based on exhaustive service of the priority units. The server provides three types of service, namely priority, non priority and preemptive priority rule.We derived the probability generating functions of the number of customers in the priority units are found by using the supplementary variable technique.

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