2456-8686, 6(1),2022:260-295
https://doi.org/10.26524/cm134

ORIGINAL RESEARCH **Open Access**

Non Skolem Mean Labeling Of A Six Star Graph

$$\mathbf{G = K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} = K_{1,2} \cup K_{1,2} \cup K_{1,2} \cup K_{1,2} \cup K_{1,2} \cup K_{1,15} = 5K_{1,2} \cup K_{1,15}\ with |m-n| > 4 + 4\ell = 12.}$$

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Received: 05 February 2022/ Accepted: 08 March 2022 / Published online: 30 March 2022.

Abstract

A graph G = (V, E) with p vertices and q edges is said to be a **Skolem mean graph** if there exists a function f from the vertex set of G to $\{1, 2, 3, \ldots, p\}$ such that the induced map f^* from the edge set of G to $\{2, 3, 4, \ldots, p\}$ defined by $f^*(e = uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$ then the resulting edges get distinct labels from the set $\{2, 3, 4, \ldots, p\}$.

In this paper we prove that the six star graph $G = 5K_{1,2} \cup K_{1,15}$ is not a skolem mean graph if $|m - n| > 4 + 4\ell = 12$.

Key Words: Skolem mean labeling, Non skolem mean graph

AMS Classification: **52A30, 05C78.**

1 Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [4]. The symbols V(G) and E(G) will denote the vertex set and edge set of the graph G. A graph with p vertices and q edges is called a

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(p, q) graph. In this paper, we prove that the six star graph $G = 5K_{1,2} \cup K_{1,15}$ is not a skolem mean graph if $|m - n| > 4 + 4\ell = 12$.

Skolem mean labeling

Definition 1.1: A graph G is a finite non-empty set of objects called **vertices** together with a set of unordered pairs of distinct vertices of G called **edges**. The vertex set and the edge set of G are denoted by V(G) and E (G) respectively. $|V(G)| = p$is called the order of G and $|E(G)| = q$ is called the size of G. A graph of order p and size q is called a **(p, q)-graph**. If e = uv is an edge of G, we say that u and v are **adjacent** and that u and v are incident with e.

Definition 1.2: A **vertex labeling** of a graph G is an assignment of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels f(x) and f(y). Similarly, an **edge labeling** of a graph G is an assignment of labels to the edges of G that induces for each vertex v a label depending on the edge labels incident on it. **Total labeling** involves a function from the vertices and edges to some set of labels.

Definition 1.3: A graph G with p vertices and q edges is called a **mean graph** if it is possible to label the verticesx ∈ V with distinct elements f(x) from 0, 1, 2, …, q in such a way that when each edge e = uv is labeled with$\frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then the resulting edge labels are distinct. The labeling f is called a **mean labeling** of G.

Definition 1.4: A graph G = (V, E) with p vertices and q edges is said to be skolem mean if it is possible to label the verticesx ∈ V with distinct elements f(x) from 1, 2, …q in such a way that when each edge e = uv is labeled with$\frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd, then the resulting edges get distinct labels from 2, 3, . . . , p f is called a skolem mean labeling of G. A graph G = (V, E) with p vertices and q edges is said to be a **skolem mean graph** if there exists a function f from the vertex set of G to {1, 2, . . . ,p} such that the induced map f* from

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the edge set of G to {2, 3, . . . , p} defined by$f^*(e = uv) = \begin{Bmatrix} \frac{f(u)+f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{Bmatrix}$

then the resulting edges get distinct labels from the set {2, 3, . . . , p}.

Definition 1.5 The six star is the disjoint union of $K_{1,a}, K_{1,b}, K_{1,c}, K_{1,d}, K_{1,e}$ and $K_{1,f}$and is denoted by$K_{1,a} \cup K_{1,b} \cup K_{1,c} \cup K_{1,d} \cup K_{1,e} \cup K_{1,f}$.

Theorem 2.1 The six star$G = K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n} = K_{1,2} \cup K_{1,2} \cup K_{1,2} \cup K_{1,2} \cup K_{1,2} \cup K_{1,15} = 5K_{1,2} \cup K_{1,15}$ is not a Skolem mean graph if $|m-n| > 4+4\ell = 12$

Proof: Let $G = 5K_{1,2} \cup K_{1,15}$ where$V(G) = \{v_{I,j}: 1 \le i \le 5; 0 \le j \le 2\} \cup \{v_{6,j}: 0 \le j \le 15\}$ and $E(G) = \{v_{I,0}v_{I,j}: 1 \le i \le 5; 1 \le j \le 2\} \cup \{v_{6,0}v_{6,j}: 1 \le j \le 15\}$.

Suppose G is a Skolem mean graph. Let p = $|V|(= 31)$.

Then there exists a function f from the vertex set of G to {1,2,3,….,p} such that the induced map f^* from the edge set of G to {2,3,…,p} defined by

$$f^*(e = uv) = \begin{Bmatrix} \dfrac{f(u)+f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{Bmatrix}$$

with the resulting edges get distinct labels from the set{2, 3, …,p}.

Let $t_{i,j}$ be the label given to the vertex $v_{i,j}$ for $1 \le i \le 5; 0 \le j \le 2$ and $v_{6,j}$ for $0 \le j \le 15$ and $x_{i,j}$ be the corresponding edge label of the edge $v_{i,0}v_{i,j}$ for $1 \le i \le 5; 1 \le j \le 2$ and $v_{6,0}v_{6,j}$ for $1 \le j \le 15$.

Let us first consider the case that $t_{6,0} = 31$.

If $t_{6,j} = 2n$ and $t_{6,k} = 2n+1$ for some n and for some j and k; then $f^*(v_{6,0}v_{6,j}) = \left\lceil \frac{31+2n}{2} \right\rceil = 16 + n = \left\lceil \frac{31+2n+1}{2} \right\rceil = f^*(v_{6,0}v_{6,k})$.

This is not possible as f^* is a bijection.

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Therefore, the fifteen numbers $t_{6,j}$ for $1 \le j \le 15$ are among the 16 numbers 1,(2 or 3), (4 or 5), (6 or 7), (8 or 9), (10 or 11), (12 or 13), (14 or 15), (16 or 17), (18 or 19), (20 or 21), (22 or 23), (24 or 25), (26 or 27), (28 or 29), 30.

If $30 \notin \{v_{6,j} : 0 \le j \le 15\}$, then $1 \in \{v_{6,j} : 0 \le j \le 15\}$. Then $x_{6,j}$ takes all the values of $\{16,17,18,...,30\}$. If $30 \in \left\{t_{i,j} : 1 \le i \le 5; 0 \le j \le 2\right\}$, then $x_{i,j} > 15$ for which $t_{i,j} = 30$. This is not possible as $x_{6,j}$ takes all the values of {16, 17,18, …, 30}.Therefore $t_{6,j} = 30$ for some j. Without loss of generality let us assume that $t_{6,1} = 30$. Then, we get that $x_{6,1} = 31$.

Next let us consider the case that $t_{6,j} \ne 1$ for any j. In this case, $x_{6,j}$ takes all the values of {17, 18, 19, …, 31} and $x_{i,j}$ for $1 \le i \le 5; 1 \le j \le 2$ take fifteen values out of {1, 2, …, 16}.

Now, $t_{6,2}$ is either 28 or 29.

First we consider the case that $t_{6,2} = 28$.

Case-A: $t_{6,2} = 28$

(Now we have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 28$)

Now, 29 is a label of either $t_{i,0}$ for $1 \le i \le 5$, or $t_{i,j}$ for $1 \le i \le 5; 1 \le j \le 2$. That is, 29 is a label of a pendent vertex or a non-pendent vertex in a $K_{1,2}$ component of G. We consider first that 29 is a label of a non-pendent vertex.Let us assume that $t_{1,0} = 29$.

Case-A1: $t_{1,0} = 29$

(We have now $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 28;\ t_{1,0} = 29$)

If $t_{1,0} = 29$, then $t_{1,1}$ and $t_{1,2}$ take the values 1 or one of 2 and 3. (As $t_{1,1} \ge 4$ would imply that $x_{1,1} \ge 17$ and is not possible). The corresponding edge labels are $x_{1,1} = 15$ and $x_{1,2} = 16$.

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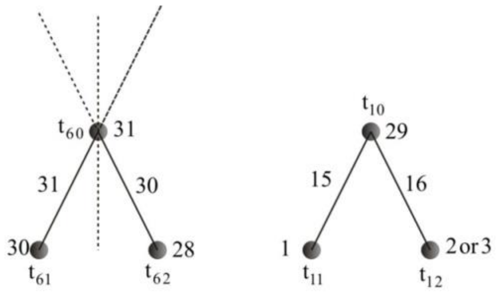


Now $t_{6,3}$ is either 26 or 27.

If $t_{6,3} = 26$, then $x_{6,3} = \left[\frac{31+26}{2}\right] = 29$, $t_{2,0} = 27$ and also $t_{2,j} \geq 4$. This is not possible. (As $t_{2,j} \geq 4$ would imply that $x_{2,j} \geq 16$ and is not possible).

Similarly, if $t_{6,3} = 27$, then $x_{6,3} = \left[\frac{31+27}{2}\right] = 29$, and also $t_{2,j} \geq 4$. This is also not possible. (As $t_{2,j} \geq 4$ would imply that $x_{2,j} \geq 16$ and is not possible).

Hence, it is not possible that $t_{1,0} = 29$.

That is, 29 is not a label of a non-pendent vertex in a $K_{1,2}$ component of G. Next we consider the case that 29 is a label of a pendent vertex in a $K_{1,2}$ component of G. Let us assume that $t_{1,1} = 29$.

Case-A2: $t_{1,1} = 29$.

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 28;\ t_{1,1} = 29$)

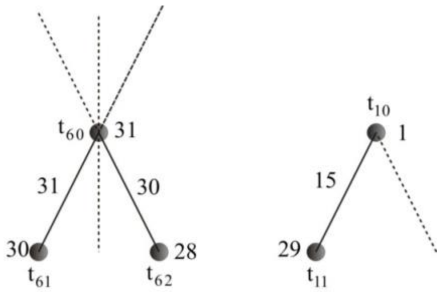
If $t_{1,0} \geq 4$, then $x_{1,1} \geq 17$ and is not possible. Hence the value of $t_{1,0}$ can either be 1 or (2 or 3).

There exist two cases viz. $t_{1,0} = 1$ and $t_{1,0} = 2$ or 3.

Case-A2.1:First let us consider that $t_{1,0} = 1$.

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 28;\ t_{1,0} = 1;\ t_{1,1} = 29$)

Then, $x_{1,1} = 15$.



Now $t_{6,3}$ is either 26 or 27.

Let us now consider the case that $t_{6,3} = 26$. Hence $t_{2,1} = 27$.

If $t_{2,0} \geq 6$, then $x_{2,1} \geq 17$ and is not possible. If $t_{2,0} = 2 \text{ or } 3$, then

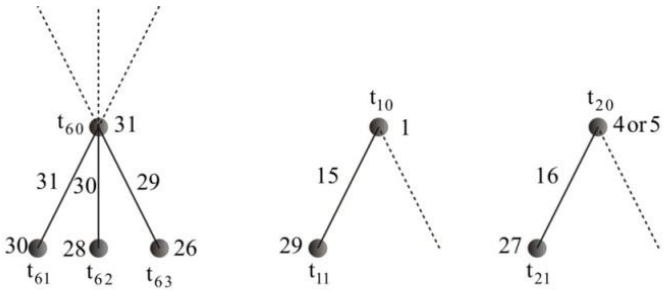
$x_{2,1} = \left\lceil \frac{27 + 2(\text{or}3)}{2} \right\rceil = 15$. This is not possible.

Hence $t_{2,0}$ is either 4 or 5.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 26$; $t_{1,1} = 29$; $t_{1,0} = 1$; $t_{2,1} = 27$;

$t_{2,0} = 4 \text{ or } 5$)

Then, $x_{1,1} = 15$; $x_{2,1} = 16$.



Now $t_{6,4}$ is either 24 or 25.

Now let us consider the case that $t_{6,4} = 24$. Hence $t_{3,1} = 25$.

If $t_{3,0} \geq 6$, then $x_{3,1} \geq 16$ and is not possible. Hence $t_{3,0}$ is either 2 or 3.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 26$; $t_{6,4} = 24$; $t_{1,1} = 29$; $t_{1,0} = 1$;

$t_{2,1} = 27$; $t_{2,0} = 4 \text{ or } 5$; $t_{3,1} = 25$; $t_{3,0} = 2 \text{ or } 3$)

Then $x_{1,1} = 15$; $x_{2,1} = 16$; $x_{3,1} = 14$.

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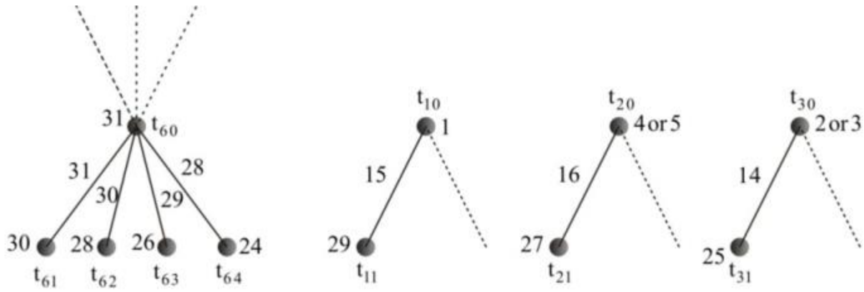


Now $t_{6,5}$ is either 22 or 23.

Now let us consider the casethat $t_{6,5} = 22$. Hence $t_{4,1} = 23$.

(Wehave $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 26$; $t_{6,4} = 24$; $t_{6,5} = 22$; $t_{1,1} = 29$; $t_{1,0} = 1$; $t_{2,1} = 27$; $t_{2,0} = 4 \text{ or } 5$; $t_{3,1} = 25$; $t_{3,0} = 2 \text{ or } 3$; $t_{4,1} = 23$)

Here the value $t_{4,0} \geq 6$, then $x_{3,1} \geq 15$ and is not possible.

Now let us consider the case that $t_{6,5} = 23$. Hence $t_{4,1} = 22$.

(Here we have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 26$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{1,1} = 29$; $t_{1,0} = 1$; $t_{2,1} = 27$; $t_{2,0} = 4 \text{ or } 5$; $t_{3,1} = 25$; $t_{3,0} = 2 \text{ or } 3$; $t_{4,1} = 22$)

Here the value $t_{4,0} \geq 6$, then $x_{3,1} \geq 14$ and is not possible.

Hence $t_{6,4} \neq 24$. Similarly, we can prove that $t_{6,4} \neq 25$. Therefore, $t_{6,3} \neq 26$.

Let us consider the case that $t_{6,3} = 27$. Hence $t_{2,1} = 26$.

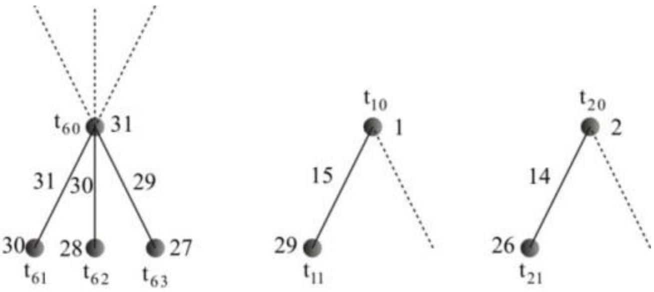
If $t_{2,0} \geq 8$, then $x_{2,1} \geq 17$ and is not possible. Hence the value of $t_{2,0}$ can either be (2 or 3) or (4 or 5) or (6 or 7).

There exist three cases viz. $t_{2,0} = 2$; $t_{2,0} = 5$ and $t_{2,0} = 6$.

Case2.1b (i): First let us consider $t_{2,0} = 2$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{1,1} = 29$; $t_{1,0} = 1$; $t_{2,1} = 26$; $t_{2,0} = 2$)

Then, $x_{1,1} = 15$; $x_{2,1} = 14$.



Now $t_{6,4}$ is either 24 or 25.

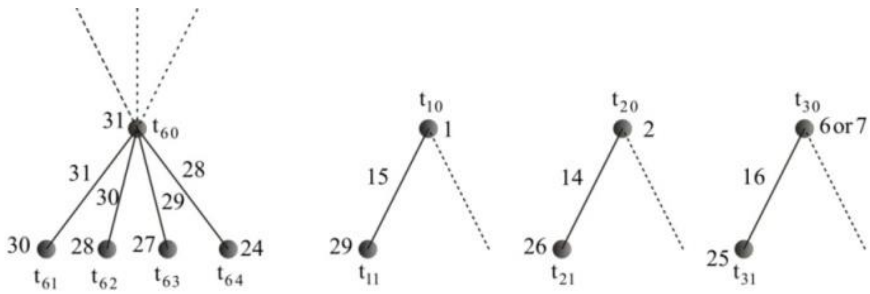
Now let us consider the casethat $t_{6,4} = 24$. Hence $t_{2,1} = 25$.

If $t_{3,0} \geq 8$, then $x_{3,1} \geq 17$ and is not possible. Hence the value of $t_{3,0}$ can be either (4 or 5) or (6 or 7).

If $t_{3,0} = 4 \text{ or } 5$, then $x_{3,1} = \left[\dfrac{25 + 4(\text{or } 5)}{2} \right] = 15$. This is not possible. Hence $t_{3,0}$ is either (6 or 7).

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{1,1} = 29$; $t_{1,0} = 1$; $t_{2,1} = 26$; $t_{2,0} = 2$; $t_{3,1} = 25$; $t_{3,0} = 6 \text{ or } 7$)

Then $x_{1,1} = 15$; $x_{2,1} = 14$; $x_{3,1} = 16$.



Now $t_{6,5}$ is either 22 or 23.

Now let us consider the case that $t_{6,5} = 22$. Hence $t_{4,1} = 23$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 22$; $t_{1,1} = 29$; $t_{1,0} = 1$; $t_{2,1} = 26$; $t_{2,0} = 2$; $t_{3,1} = 25$; $t_{3,0} = 6 \text{ or } 7$; $t_{4,1} = 23$)

Here the value of $t_{4,0} \geq 8$, this is not possible.

Now let us consider the case that $t_{6,5} = 23$. Hence $t_{4,1} = 22$.

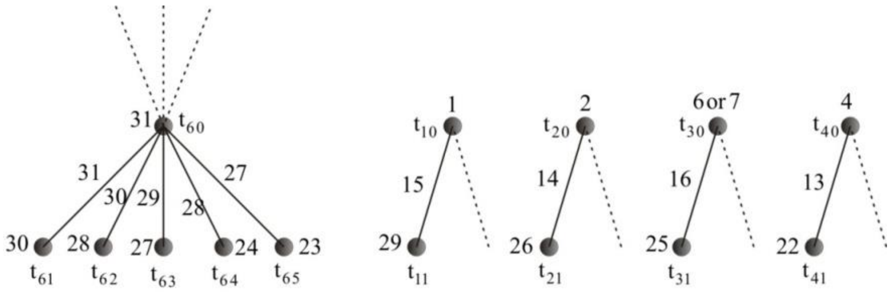
If $t_{4,0} \geq 8$, then $x_{3,1} \geq 15$ and is not possible. Hence $t_{4,0}$ is either 4 or 5.

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 28;\ t_{6,3} = 27;\ t_{6,4} = 24;\ t_{6,5} = 23;\ t_{1,1} = 29;$

$t_{1,0} = 1;\ t_{2,1} = 26;\ t_{2,0} = 2;\ t_{3,1} = 25;\ t_{3,0} = 6 \text{ or } 7;\ t_{4,1} = 22;$

$t_{4,0} = 4$)

Then $x_{1,1} = 15;\ x_{2,1} = 14;\ x_{3,1} = 16;\ x_{4,1} = 13.$



Now $t_{6,6}$ is either 20 or 21.

Now let us consider the case that $t_{6,6} = 20$. Hence $t_{5,1} = 21.$

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 28;\ t_{6,3} = 27;\ t_{6,4} = 24;\ t_{6,5} = 23;\ t_{6,6} = 20;$

$t_{1,1} = 29;\ t_{1,0} = 1;\ t_{2,1} = 26;\ t_{2,0} = 2;\ t_{3,1} = 25;\ t_{3,0} = 6 \text{ or } 7;\ t_{4,0} = 4;\ t_{4,1} = 22;$
$t_{5,1} = 21$)

Here the value of $t_{5,0} \geq 8,$ this is not possible.

Now let us consider the case that $t_{6,6} = 21$. Hence $t_{5,1} = 20.$

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 28;\ t_{6,3} = 27;\ t_{6,4} = 24;\ t_{6,5} = 23;\ t_{6,6} = 21;$

$t_{1,1} = 29;\ t_{1,0} = 1;\ t_{2,1} = 26;\ t_{2,0} = 2;\ t_{3,1} = 25;\ t_{3,0} = 6 \text{ or } 7;\ t_{4,0} = 4;\ t_{4,1} = 22;$
$t_{5,1} = 20$)

Here the value of $t_{5,0} \geq 8,$ this is not possible.

Hence $t_{6,4} \neq 24.$

Similarly, $t_{6,4} \neq 25.$ Hence $t_{2,0} \neq 2.$

Case2.1b (ii): Next let us consider $t_{2,0} = 5.$

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 28;\ t_{6,3} = 27;\ t_{1,1} = 29;\ t_{1,0} = 1;\ t_{2,1} = 26;$

$t_{2,0} = 5$)

Then, $x_{1,1} = 15;\ x_{2,1} = 14.$

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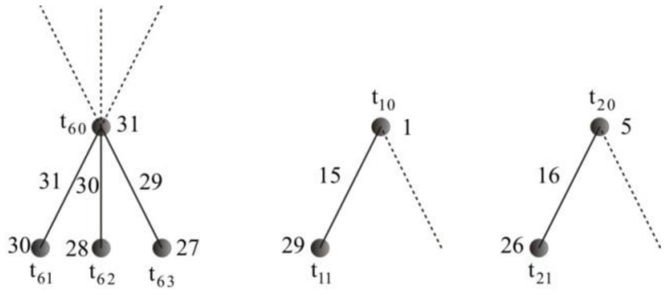


Now $t_{6,4}$ is either 24 or 25.

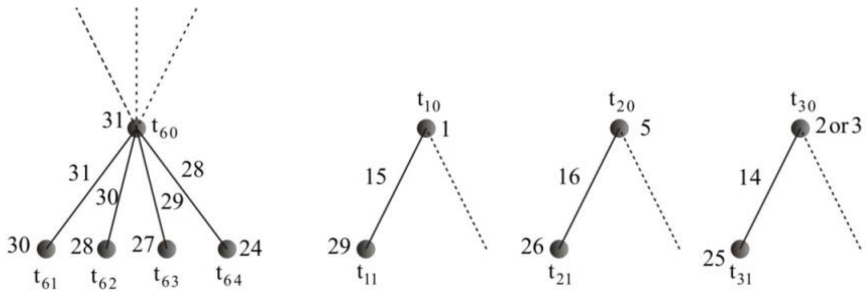
Now let us consider the case that $t_{6,4} = 24$. Hence $t_{3,1} = 25$.

If $t_{3,0} \geq 6$, then $x_{3,1} \geq 16$ and is not possible. Hence $t_{3,0}$ is either 2 or 3.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{1,1} = 29$; $t_{1,0} = 1$;

$t_{2,1} = 26$; $t_{2,0} = 5$; $t_{3,1} = 25$; $t_{3,0} = 2 \text{ or } 3$)

Then $x_{1,1} = 15$; $x_{2,1} = 16$; $x_{3,1} = 14$.



Now $t_{6,5}$ is either 22 or 23.

Now let us consider the case that $t_{6,5} = 22$. Hence $t_{4,1} = 23$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 22$; $t_{1,1} = 29$;

$t_{1,0} = 1$; $t_{2,1} = 26$; $t_{2,0} = 5$; $t_{3,1} = 25$; $t_{3,0} = 2 \text{ or } 3$; $t_{4,1} = 23$)

Here the value of $t_{4,0} \geq 6$, this is not possible.

Now let us consider the case that $t_{6,5} = 23$. Hence $t_{4,1} = 22$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{1,1} = 29$;

$t_{1,0} = 1$; $t_{2,1} = 26$; $t_{2,0} = 5$; $t_{3,1} = 25$; $t_{3,0} = 2 \text{ or } 3$; $t_{4,1} = 22$)

Here the value of $t_{4,0} \geq 6$, this is not possible.

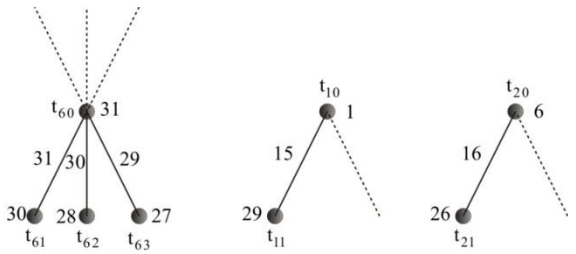
Hence $t_{6,4} \neq 24$.

Similarly, $t_{6,4} \neq 25$. Hence $t_{2,0} \neq 5$.

Case2.1b (iii): Now let us consider $t_{2,0} = 6$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{1,1} = 29$; $t_{1,0} = 1$; $t_{2,1} = 26$; $t_{2,0} = 6$)

Then, $x_{1,1} = 15$; $x_{2,1} = 14$.



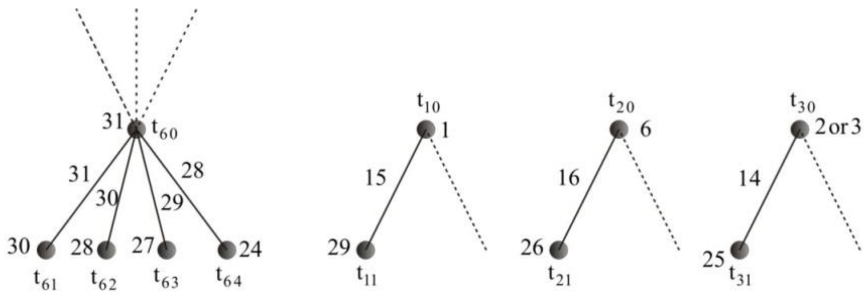
Now $t_{6,4}$ is either 24 or 25.

Now let us consider the case that $t_{6,4} = 24$. Hence $t_{3,1} = 25$.

If $t_{3,0} \geq 8$, then $x_{3,1} \geq 17$ and is not possible. Hence $t_{3,0}$ is either 2 or 3.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{1,1} = 29$; $t_{1,0} = 1$; $t_{2,1} = 26$; $t_{2,0} = 6$; $t_{3,1} = 25$; $t_{3,0} = 2 \text{or} 3$)

Then $x_{1,1} = 15$; $x_{2,1} = 16$; $x_{3,1} = 14$.



Now $t_{6,5}$ is either 22 or 23.

Now let us consider the case that $t_{6,5} = 22$. Hence $t_{4,1} = 23$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 22$; $t_{1,1} = 29$; $t_{1,0} = 1$; $t_{2,1} = 26$; $t_{2,0} = 6$; $t_{3,1} = 25$; $t_{3,0} = 2 \text{or} 3$; $t_{4,1} = 23$)

Here the value of $t_{4,0} = (4 \text{or} 5) \text{or} \geq 8$, this is not possible.

Now let us consider the case that $t_{6,5} = 23$. Hence $t_{4,1} = 22$.

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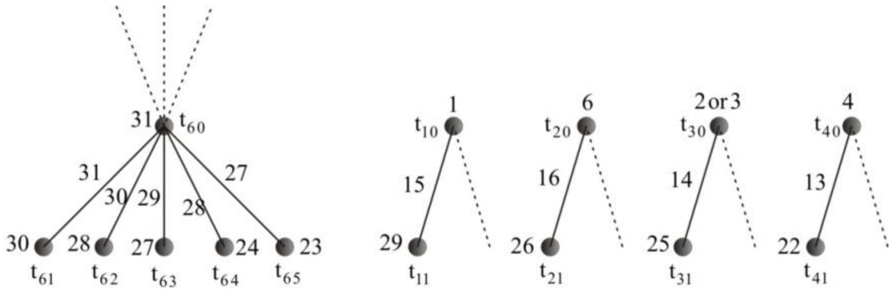
If $t_{4,0} \geq 8$, then $x_{4,1} \geq 15$ and is not possible. Hence $t_{4,0}$ is either 4 or 5.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{1,1} = 29$;

$t_{1,0} = 1$; $t_{2,1} = 26$; $t_{2,0} = 6$; $t_{3,1} = 25$; $t_{3,0} = 2 \text{ or } 3$; $t_{4,1} = 22$)

Then $x_{1,1} = 15$; $x_{2,1} = 16$; $x_{3,1} = 14$ and if $t_{4,0} = 4$, then $x_{4,1} = 13$.



Now $t_{6,6}$ is either 20 or 21.

Now let us consider the case that $t_{6,6} = 20$. Hence $t_{5,1} = 21$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{6,6} = 20$;

$t_{1,1} = 29$; $t_{1,0} = 1$; $t_{2,1} = 26$; $t_{2,0} = 6$; $t_{3,1} = 25$; $t_{3,0} = 2 \text{ or } 3$; $t_{4,0} = 4$; $t_{4,1} = 22$;
$t_{5,1} = 21$)

Here the value of $t_{5,0} \geq 8$, this is not possible.

Now let us consider the case that $t_{6,6} = 21$. Hence $t_{5,1} = 20$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{6,6} = 21$;

$t_{1,1} = 29$; $t_{1,0} = 1$; $t_{2,1} = 26$; $t_{2,0} = 6$; $t_{3,1} = 25$; $t_{3,0} = 2 \text{ or } 3$; $t_{4,0} = 4$; $t_{4,1} = 22$;
$t_{5,1} = 20$)

Here the value of $t_{5,0} \geq 8$, this is not possible.

Hence $t_{6,4} \neq 24$.

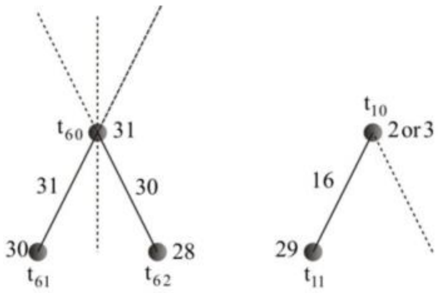
Similarly, $t_{6,4} \neq 25$. Hence $t_{2,0} \neq 6$. Therefore, $t_{6,3} \neq 27$.

Hence $t_{1,0} \neq 1$.

Case-A2.2: Next let us consider $t_{1,0} = 2 \text{ or } 3$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{1,0} = 2 \text{ or } 3$; $t_{1,1} = 29$)

Then, $x_{1,1} = 16$.



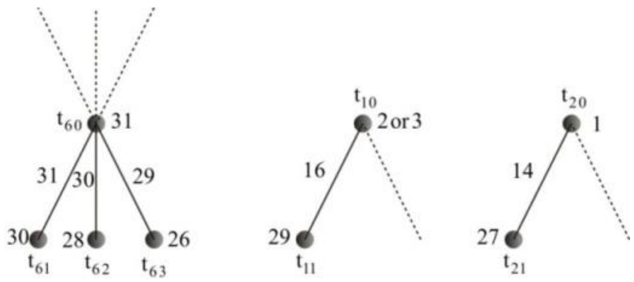
Now $t_{6,3}$ is either 26 or 27.

Let us consider the case that $t_{6,3} = 26.$ Hence $t_{2,1} = 27.$

If $t_{2,0} \geq 4,$ then $x_{2,1} \geq 16$ this is not possible. Hence $t_{2,0}$ is 1.

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 28;\ t_{6,3} = 26;\ t_{1,1} = 29;\ t_{1,0} = 2 \text{ or } 3;$ $t_{2,1} = 27;\ t_{2,0} = 1$)

Then, $x_{1,1} = 16;\ x_{2,1} = 14.$



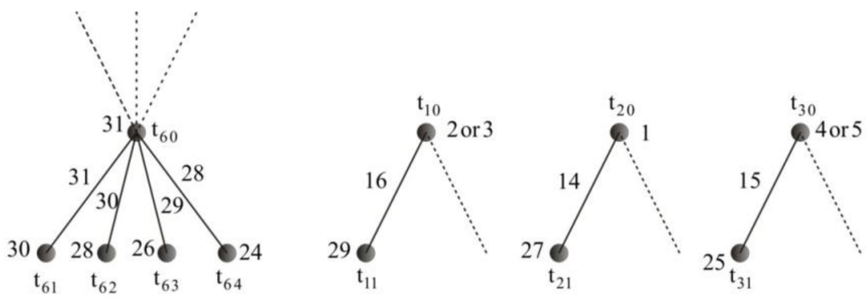
Now $t_{6,4}$ is either 24 or 25.

Now let us consider the case that $t_{6,4} = 24.$ Hence $t_{3,1} = 25.$

If $t_{3,0} \geq 6,$ then $x_{3,1} \geq 16$ and is not possible. Hence $t_{3,0}$ is either 4 or 5.

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 28;\ t_{6,3} = 26;\ t_{6,4} = 24;\ t_{1,1} = 29;$ $t_{1,0} = 2 \text{ or } 3;\ t_{2,1} = 27;\ t_{2,0} = 1;\ t_{3,1} = 25;\ t_{3,0} = 4 \text{ or } 5$)

Then $x_{1,1} = 16;\ x_{2,1} = 14;\ x_{3,1} = 15.$



Now $t_{6,5}$ is either 22 or 23.

Now let us consider the case that $t_{6,5} = 22$. Hence $t_{4,1} = 23$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 26$; $t_{6,4} = 24$; $t_{6,5} = 22$; $t_{1,1} = 29$;

$t_{1,0} = 2 \text{ or } 3$; $t_{2,1} = 27$; $t_{2,0} = 1$; $t_{3,1} = 25$; $t_{3,0} = 4 \text{ or } 5$; $t_{4,1} = 23$)

Here the value of $t_{4,0} \geq 6$, this is not possible.

Now let us consider the case that $t_{6,5} = 23$. Hence $t_{4,1} = 22$.

We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 26$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{1,1} = 29$;

$t_{1,0} = 2 \text{ or } 3$; $t_{2,1} = 27$; $t_{2,0} = 1$; $t_{3,1} = 25$; $t_{3,0} = 4 \text{ or } 5$; $t_{4,1} = 22$)

Here the value of $t_{4,0} \geq 6$, this is not possible.

Hence $t_{6,4} \neq 24$.

Similarly, $t_{6,4} \neq 25$. Hence $t_{6,3} \neq 26$.

Let us consider the case that $t_{6,3} = 27$. Hence $t_{2,1} = 26$.

If $t_{2,0} \geq 6$, then $x_{2,1} \geq 16$ and is not possible. Hence the value of $t_{2,0}$ can either be 1 or (4 or 5).

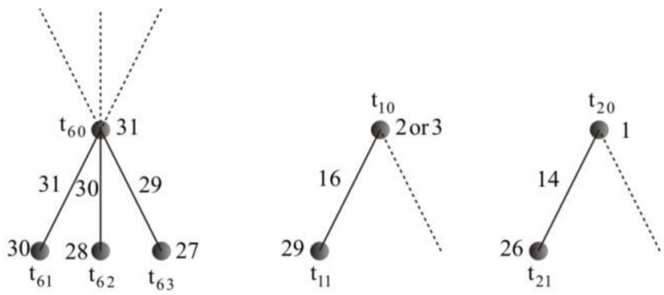
There exist two cases viz. $t_{2,0} = 1$ and $t_{2,0} = 4$.

Case-A2.2b (i): First let us consider $t_{2,0} = 1$

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{1,1} = 29$; $t_{1,0} = 2 \text{ or } 3$;

$t_{2,1} = 26$; $t_{2,0} = 1$)

Then, $x_{1,1} = 16$; $x_{2,1} = 14$.



Now $t_{6,4}$ is either 24 or 25.

Now let us consider the case that $t_{6,4} = 24$. Hence $t_{3,1} = 25$.

If $t_{3,0} \geq 6$, then $x_{3,1} \geq 16$ and is not possible. Hence $t_{3,0}$ is either 4 or 5.

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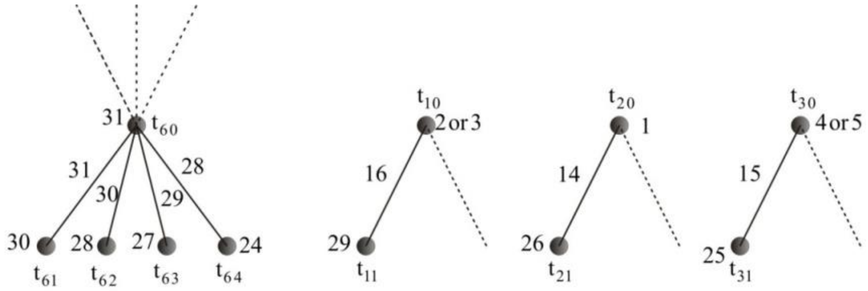
(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{1,1} = 29$; $t_{1,0} = 2 \text{ or } 3$; $t_{2,1} = 26$; $t_{2,0} = 1$; $t_{3,1} = 25$; $t_{3,0} = 4 \text{ or } 5$)

Then, $x_{1,1} = 16$; $x_{2,1} = 14$; $x_{3,1} = 15$.



Now $t_{6,5}$ is either 22 or 23.

Now let us consider the case that $t_{6,5} = 22$. Hence $t_{4,1} = 23$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 22$; $t_{1,1} = 29$; $t_{1,0} = 2 \text{ or } 3$; $t_{2,1} = 26$; $t_{2,0} = 1$; $t_{3,1} = 25$; $t_{3,0} = 4 \text{ or } 5$; $t_{4,1} = 23$)

Here the value of $t_{4,0} \geq 6$, this is not possible.

Now let us consider the case that $t_{6,5} = 23$. Hence $t_{4,1} = 22$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{1,1} = 29$; $t_{1,0} = 2 \text{ or } 3$; $t_{2,1} = 26$; $t_{2,0} = 1$; $t_{3,1} = 25$; $t_{3,0} = 4 \text{ or } 5$; $t_{4,1} = 22$)

Here the value of $t_{4,0} \geq 6$, this is not possible.

Hence $t_{6,4} \neq 24$.

Similarly, $t_{6,4} \neq 25$. Hence $t_{2,0} \neq 1$.

Case-A2.2b (ii): Next let us consider $t_{2,0} = 4$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{1,1} = 29$; $t_{1,0} = 2 \text{ or } 3$; $t_{2,1} = 26$; $t_{2,0} = 4$)

Then, $x_{1,1} = 16$; $x_{2,1} = 15$.

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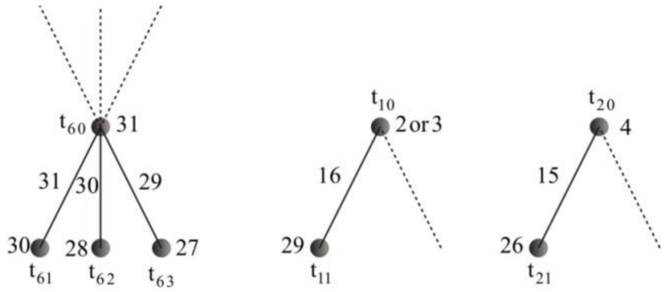


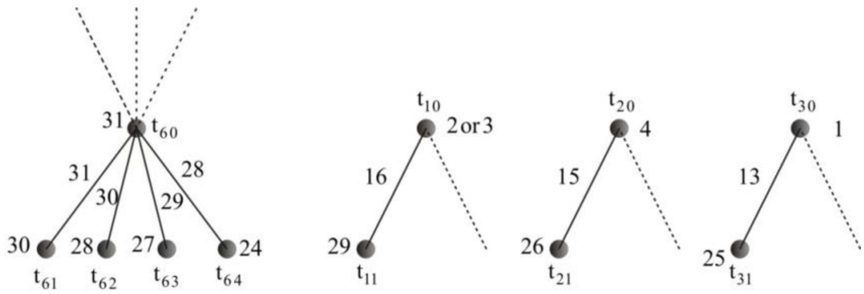
Now $t_{6,4}$ is either 24 or 25.

Now let us consider the case that $t_{6,4}=24$. Hence $t_{3,1}=25$.

If $t_{3,0}\geq 6$, then $x_{3,1}\geq 16$ and is not possible. Hence $t_{3,0}$ is 1.

(We have $t_{6,0}=31$; $t_{6,1}=30$; $t_{6,2}=28$; $t_{6,3}=27$; $t_{6,4}=24$; $t_{1,1}=29$; $t_{1,0}=2\,\text{or}\,3$; $t_{2,1}=26$; $t_{2,0}=4$; $t_{3,1}=25$; $t_{3,0}=1$)

Then, $x_{1,1}=16$; $x_{2,1}=15$; $x_{3,1}=13$.



Now $t_{6,5}$ is either 22 or 23.

Now let us consider the case that $t_{6,5}=22$. Hence $t_{4,1}=23$.

(We have $t_{6,0}=31$; $t_{6,1}=30$; $t_{6,2}=28$; $t_{6,3}=27$; $t_{6,4}=24$; $t_{6,5}=22$; $t_{1,1}=29$; $t_{1,0}=2\,\text{or}\,3$; $t_{2,1}=26$; $t_{2,0}=4$; $t_{3,1}=25$; $t_{3,0}=1$; $t_{4,1}=23$)

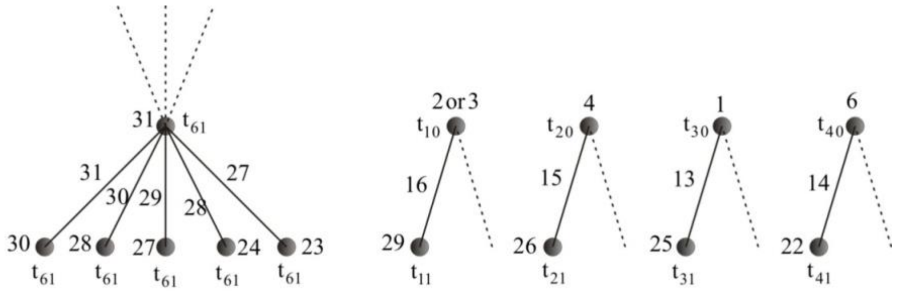
Here the value of $t_{4,0}\geq 6$, this is not possible.

Now let us consider the case that $t_{6,5}=23$. Hence $t_{4,1}=22$.

The value of $t_{4,0}$ can be one of 6 and 7.

(We have $t_{6,0}=31$; $t_{6,1}=30$; $t_{6,2}=28$; $t_{6,3}=27$; $t_{6,4}=24$; $t_{6,5}=23$; $t_{1,1}=29$; $t_{1,0}=2\,\text{or}\,3$; $t_{2,1}=26$; $t_{2,0}=4$; $t_{3,1}=25$; $t_{3,0}=1$; $t_{4,1}=22$)

Then, $x_{1,1}=16$; $x_{2,1}=15$; $x_{3,1}=13$ and if $t_{4,0}=6$, then $x_{4,1}=14$.



Now $t_{6,6}$ is either 20 or 21.

Now let us consider the case that $t_{6,6} = 20$. Hence $t_{5,1} = 21$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{6,6} = 20$;

$t_{1,1} = 29$; $t_{1,0} = 2 \text{ or } 3$; $t_{2,1} = 26$; $t_{2,0} = 4$; $t_{3,1} = 25$; $t_{3,0} = 1$; $t_{4,1} = 22$; $t_{4,0} = 6$;
$t_{5,1} = 21$)

Here the value of $t_{5,0} \geq 8$, this is not possible.

Now let us consider the case that $t_{6,6} = 21$. Hence $t_{5,1} = 20$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 28$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{6,6} = 21$;

$t_{1,1} = 29$; $t_{1,0} = 2 \text{ or } 3$; $t_{2,1} = 26$; $t_{2,0} = 4$; $t_{3,1} = 25$; $t_{3,0} = 1$; $t_{4,1} = 22$; $t_{4,0} = 6$;
$t_{5,1} = 20$)

Here the value of $t_{5,0} \geq 8$, this is not possible.

Hence $t_{6,4} \neq 24$.

Similarly, $t_{6,4} \neq 25$. Hence $t_{2,0} \neq 4$ and hence $t_{6,3} \neq 27$.

Hence $t_{1,0} \neq 2 \text{ or } 3$. Therefore $t_{1,1} \neq 29$ and hence $t_{6,2} \neq 28$.

Next we consider the case that $t_{6,2} = 29$.

Case-B: $t_{6,2} = 29$

Now, 28 is a label of either $t_{i,0}$ $1 \leq i \leq 5$, or $t_{i,j}$ $1 \leq i \leq 5; 1 \leq j \leq 2$. That is, 28 is a label of a pendent vertex or a non pendent vertex in a $K_{1,2}$ component of G. We consider first that 28 is a label of a non-pendent vertex. Let us assume that $t_{1,0} = 28$.

Case-B1.1: $t_{1,0} = 28$

(Here we have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{1,0} = 28$)

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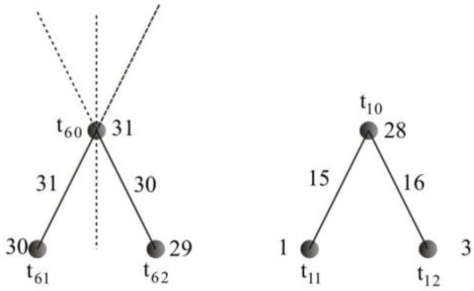
If $t_{1,0} = 28,$ then $t_{1,1}$ and $t_{1,2}$ take the values 1 or (2 or 3) or (4 or 5).

First we consider $t_{1,1} = 1$ **and** $t_{1,2} = 3.$

(As $t_{1,1} \geq 5$ would imply that $x_{1,1} \geq 17$ and is not possible).

The corresponding edge labels are $x_{1,1} = 15$ and $x_{1,2} = 16.$



Now $t_{6,3}$ is either 26 or 27.

If $t_{6,3} = 26,$ then $x_{6,3} = \left\lceil \frac{31+26}{2} \right\rceil = 29,$ $t_{2,0} = 27$ and also $t_{2,j} \geq 4.$ This is not possible. (As $t_{2,j} \geq 4$ would imply that $x_{2,j} \geq 16$ and is not possible).

Similarly, $t_{6,3} = 27$, then $x_{6,3} = \left\lceil \frac{31+27}{2} \right\rceil = 29,$ $t_{2,0} = 26$ and also $t_{2,j} \geq 4.$

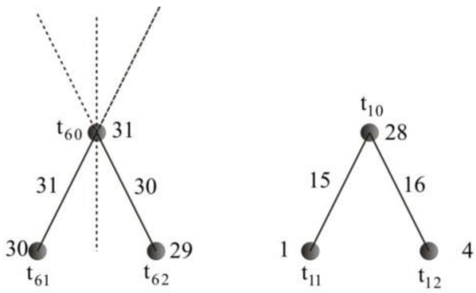
This is not possible. (As $t_{2,j} \geq 4$ would imply that $x_{2,j} \geq 15$ and is not possible).

Case-B1.2: Next we consider $t_{1,1} = 1$ **and** $t_{1,2} = 4.$

(As $t_{1,1} \geq 5$ would imply that $x_{1,1} \geq 17$ and is not possible).

(Here we have $t_{6,0} = 31;$ $t_{6,1} = 30;$ $t_{6,2} = 29;$ $t_{1,0} = 28$)

The corresponding edge labels are $x_{1,1} = 15$ and $x_{1,2} = 16.$



Now $t_{6,3}$ is either 26 or 27.

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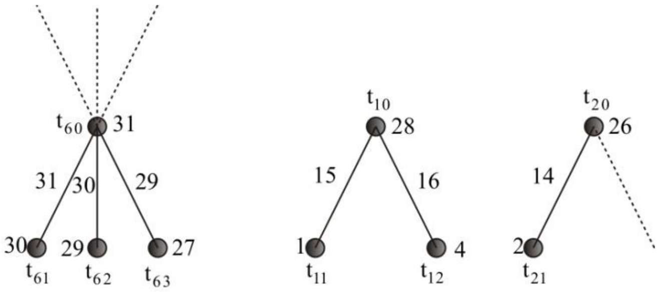
If $t_{6,3}=26$, then $x_{6,3}=\left\lceil\frac{31+26}{2}\right\rceil=29$, $t_{2,0}=27$ and also $t_{2,j}=(2\text{ or }3)$ or ≥ 5. This is not possible. (As $t_{2,j}=(2\text{ or }3)$ or ≥ 5 would imply that $x_{2,j}\geq 15$ and is not possible).

Similarly, if $t_{6,3}=27$, then $x_{6,3}=\left\lceil\frac{31+27}{2}\right\rceil=29$, $t_{2,0}=26$ and also $t_{2,j}=(2\text{ or }3)$ or ≥ 5. (As $t_{2,j}=3$ or ≥ 5 would imply that $x_{2,j}\geq 15$ and is not possible).Hence, $t_{2,j}=2$. Therefore $x_{2,j}=14$.



Now $t_{6,4}$ is either 24 or 25.

If $t_{6,4}=24$, then $x_{6,4}=\left\lceil\frac{31+24}{2}\right\rceil=28$, $t_{3,0}=25$ and also $t_{3,j}\geq 6$. This is not possible. (As $t_{3,j}\geq 6$ would imply that $x_{3,j}\geq 16$ and is not possible).

Similarly, $t_{6,4}=25$, then $x_{6,4}=\left\lceil\frac{31+25}{2}\right\rceil=28$, $t_{3,0}=24$ and also $t_{3,j}\geq 6$. This is not possible. (As $t_{3,j}\geq 6$ would imply that $x_{3,j}\geq 15$ and is not possible).

Hence, it is not possible that $t_{1,0}=28$.

That is, 28 is not a label of a non-pendent vertex in a $K_{1,2}$ component of G. Next we consider the case that 28 is a label of a pendent vertex in a $K_{1,2}$ component of G. Let us assume that $t_{1,1}=28$.

Case-B2: $t_{1,1}=28$.

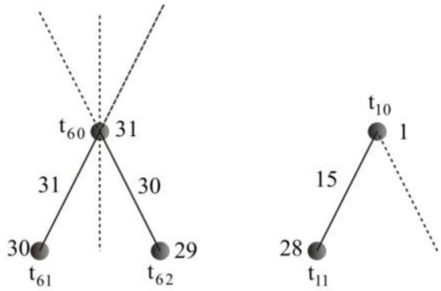
Here the value of $t_{1,0}$ can either be 1 or (2 or 3) or (4 or 5).

There exist four cases viz. $t_{1,0}=1$; $t_{1,0}=2$; $t_{1,0}=3$ and $t_{1,0}=4$.

Case-B2.1: $t_{1,1} = 28$ **and** $t_{1,0} = 1$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{1,0} = 1$)

Then $x_{1,1} = 15$.



Now $t_{6,3}$ is either 26 or 27.

Let us consider the case that $t_{6,3} = 26$. Hence $t_{2,1} = 27$.

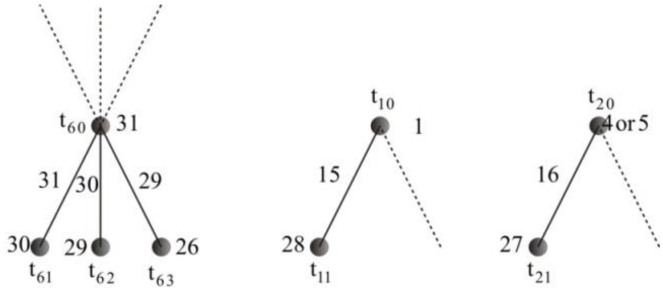
Here the value of $t_{2,0}$ can either be (2 or 3) or (4 or 5).

If $t_{2,0} = 2 \text{ or } 3$, then $x_{2,1} = \left[\dfrac{27 + 2(\text{or } 3)}{2}\right] = 15$. This is not possible.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 26$; $t_{1,1} = 28$; $t_{1,0} = 1$; $t_{2,1} = 27$;

$t_{2,0} = 4 \text{ or } 5$)

Then, $x_{1,1} = 15$ and $x_{2,1} = 16$.



Now $t_{6,4}$ is either 24 or 25.

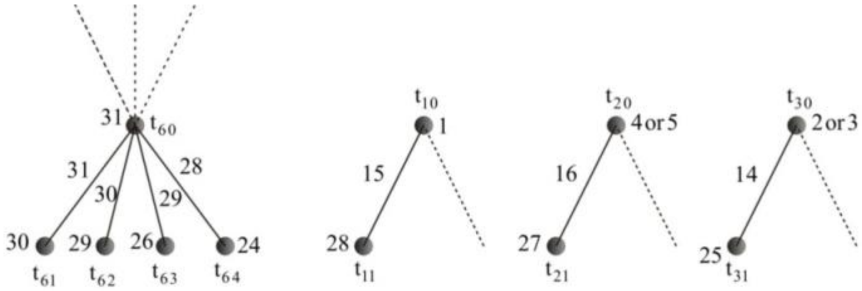
Now let us consider the case that $t_{6,4} = 24$. Hence $t_{3,1} = 25$.

If $t_{3,0} \geq 6$, then $x_{3,1} \geq 16$ and is not possible. Hence $t_{3,0}$ is either 2 or 3.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 26$; $t_{6,4} = 24$; $t_{1,1} = 28$; $t_{1,0} = 1$;

$t_{2,1} = 27$ $t_{2,0} = 4 \text{ or } 5$; $t_{3,1} = 25$; $t_{3,0} = 2 \text{ or } 3$)

Then $x_{1,1} = 15$; $x_{2,1} = 16$; $x_{3,1} = 14$.



Now $t_{6,5}$ is either 22 or 23.

Now let us consider the case that $t_{6,5} = 22.$ Hence $t_{4,1} = 23.$

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 29;\ t_{6,3} = 26;\ t_{6,4} = 24;\ t_{6,5} = 22;\ t_{1,1} = 28;$

$t_{1,0} = 1;\ t_{2,1} = 27\ \ t_{2,0} = 4 \text{ or } 5;\ t_{3,1} = 25;\ t_{3,0} = 2 \text{ or } 3;\ t_{4,1} = 23)$

Here the value of $t_{4,0} \geq 6,$ this is not possible.

Now let us consider the case that $t_{6,5} = 23.$ Hence $t_{4,1} = 22.$

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 29;\ t_{6,3} = 26;\ t_{6,4} = 24;\ t_{6,5} = 23;\ t_{1,1} = 28;$

$t_{1,0} = 1;\ t_{2,1} = 27;\ t_{2,0} = 4 \text{ or } 5;\ t_{3,1} = 25;\ t_{3,0} = 2 \text{ or } 3;\ t_{4,1} = 22)$

Here the value of $t_{4,0} \geq 6,$ this is also not possible.

Hence $t_{6,4} \neq 24.$

Similarly, $t_{6,4} \neq 25.$ Hence $t_{6,3} \neq 26.$

Now let us consider the case that $t_{6,3} = 27.$ Hence $t_{2,1} = 26.$

Here the value of $t_{2,0}$ can either be (2 or 3) or (4 or 5).

There exist two cases viz. $t_{2,0} = 2$ and $t_{2,0} = 5.$

Case-B2.1a (i): First consider $t_{2,0} = 2.$

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 29;\ t_{6,3} = 27;\ t_{1,1} = 28;\ t_{1,0} = 1;\ t_{2,1} = 26;$

$t_{2,0} = 2)$

Then, $x_{1,1} = 15$ and $x_{2,1} = 14.$

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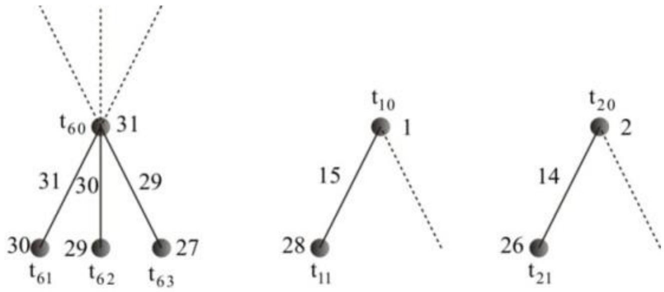


Now $t_{6,4}$ is either 24 or 25.

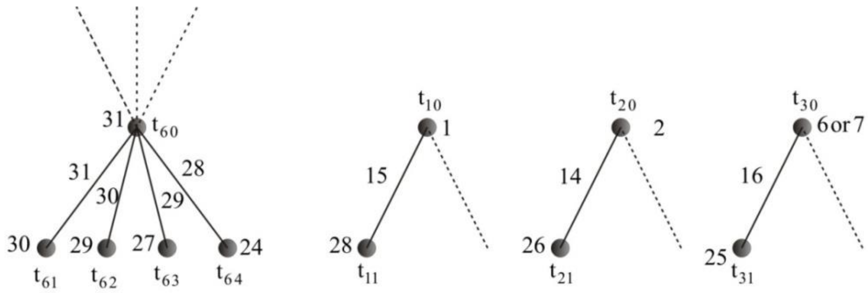
Now let us consider the case that $t_{6,4} = 24$. Hence $t_{3,1} = 25$.

If $t_{3,0} \geq 8$, then $x_{3,1} = 17$ and is not possible. Hence the value of $t_{3,0}$ can either be (2 or 3) or (6 or 7).

If $t_{3,0} = 2 \text{ or } 3$, then $x_{3,1} = \left[\dfrac{25 + 2(\text{or}3)}{2}\right] = 14$. This is not possible.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{1,1} = 28$; $t_{1,0} = 1$; $t_{2,1} = 26$; $t_{2,0} = 2$; $t_{3,1} = 25$; $t_{3,0} = 6 \text{ or } 7$)

Then $x_{1,1} = 15$; $x_{2,1} = 14$ and $x_{3,1} = 16$.



Now $t_{6,5}$ is either 22 or 23.

Now let us consider the case that $t_{6,5} = 22$. Hence $t_{4,1} = 23$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 22$; $t_{1,1} = 28$; $t_{1,0} = 1$; $t_{2,1} = 26$; $t_{2,0} = 2$; $t_{3,1} = 25$; $t_{3,0} = 6 \text{ or } 7$; $t_{4,1} = 23$)

Here the value of $t_{4,0} = (4 \text{ or } 5)$ or ≥ 8, this is not possible.

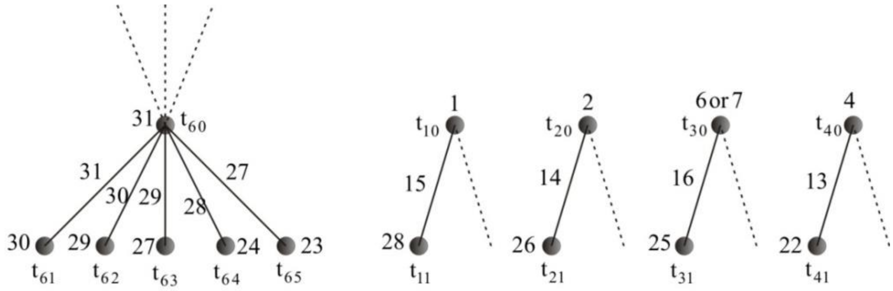
Now let us consider the case that $t_{6,5} = 23$. Hence $t_{4,1} = 22$.

The value of $t_{4,0}$ can be from (4 or 5).

(We have $t_{6,0}=31$; $t_{6,1}=30$; $t_{6,2}=29$; $t_{6,3}=27$; $t_{6,4}=24$; $t_{6,5}=23$; $t_{1,1}=28$;

$t_{1,0}=1$; $t_{2,1}=26$; $t_{2,0}=2$; $t_{3,1}=25$; $t_{3,0}=6$ or 7; $t_{4,1}=22$)

Then $x_{1,1}=15$; $x_{2,1}=14$; $x_{3,1}=16$ and if $t_{4,0}=4$, then $x_{4,1}=13$.



Now $t_{6,6}$ is either 20 or 21.

Now let us consider the case that $t_{6,6}=20$. Hence $t_{5,1}=21$.

(We have $t_{6,0}=31$; $t_{6,1}=30$; $t_{6,2}=29$; $t_{6,3}=27$; $t_{6,4}=24$; $t_{6,5}=23$; $t_{6,6}=20$;

$t_{1,1}=28$; $t_{1,0}=1$; $t_{2,1}=26$; $t_{2,0}=2$; $t_{3,1}=25$; $t_{3,0}=6$ or 7; $t_{4,1}=22$;
$t_{4,0}=4$; $t_{5,1}=21$)

Here the value of $t_{5,0}\geq 8$, this is not possible.

Now let us consider the case that $t_{6,6}=21$. Hence $t_{5,1}=20$.

(We have $t_{6,0}=31$; $t_{6,1}=30$; $t_{6,2}=29$; $t_{6,3}=27$; $t_{6,4}=24$; $t_{6,5}=23$; $t_{6,6}=21$;

$t_{1,1}=28$; $t_{1,0}=1$; $t_{2,1}=26$; $t_{2,0}=2$; $t_{3,1}=25$; $t_{3,0}=6$ or 7; $t_{4,1}=22$;
$t_{4,0}=4$; $t_{5,1}=20$)

Here the value of $t_{5,0}\geq 8$, this is not possible.

Hence $t_{6,4}\neq 24$.

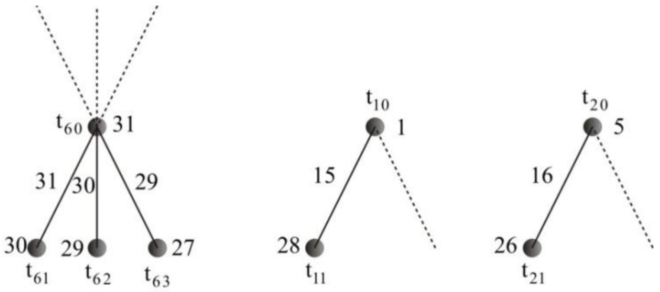
Similarly, $t_{6,4}\neq 25$. Hence $t_{2,0}\neq 2$.

Case-B2.1a (ii): Next we consider $t_{2,0}=5$.

(We have $t_{6,0}=31$; $t_{6,1}=30$; $t_{6,2}=29$; $t_{6,3}=27$; $t_{1,1}=28$; $t_{1,0}=1$; $t_{2,1}=26$;

$t_{2,0}=5$)

Then, $x_{1,1}=15$; $x_{2,1}=16$.



Now $t_{6,4}$ is either 24 or 25.

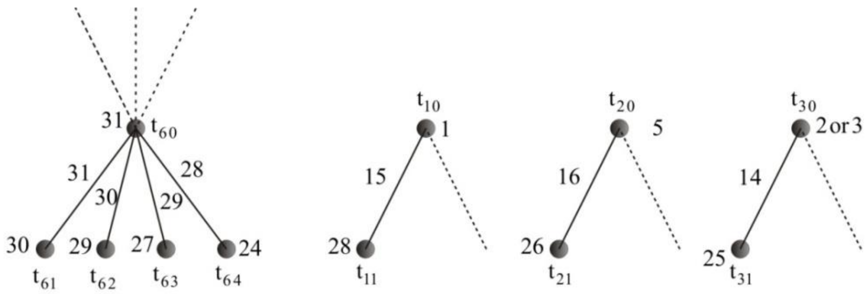
Now let us consider the case that $t_{6,4} = 24.$ Hence $t_{3,1} = 25.$

If $t_{3,0} \geq 6,$ then $x_{3,1} \geq 16$ and is not possible. Hence $t_{3,0}$ is either 2 or 3.

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 29;\ t_{6,3} = 27;\ t_{6,4} = 24;\ t_{1,1} = 28;\ t_{1,0} = 1;$

$t_{2,1} = 26;\ t_{2,0} = 5;\ t_{3,1} = 25$)

Then, $x_{1,1} = 15;\ x_{2,1} = 16$ and if $t_{3,0} = 2 \text{ or } 3,$ then $x_{3,1} = 14.$



Now $t_{6,5}$ is either 22 or 23.

Now let us consider the case that $t_{6,5} = 22.$ Hence $t_{4,1} = 23.$

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 29;\ t_{6,3} = 27;\ t_{6,4} = 24;\ t_{6,5} = 22;\ t_{1,1} = 28;$

$t_{1,0} = 1;\ t_{2,1} = 26;\ t_{2,0} = 5;\ t_{3,1} = 25;\ t_{3,0} = 2 \text{ or } 3;\ t_{4,1} = 23)$

Here the value of $t_{4,0} \geq 6,$ this is not possible.

Now let us consider the case that $t_{6,5} = 23.$ Hence $t_{4,1} = 22.$

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 29;\ t_{6,3} = 27;\ t_{6,4} = 24;\ t_{6,5} = 23;\ t_{1,1} = 28;$

$t_{1,0} = 1;\ t_{2,1} = 26;\ t_{2,0} = 5;\ t_{3,1} = 25;\ t_{3,0} = 2 \text{ or } 3;\ t_{4,1} = 22)$

Here the value of $t_{4,0} \geq 6,$ this is not possible.

Hence $t_{6,4} \neq 24.$

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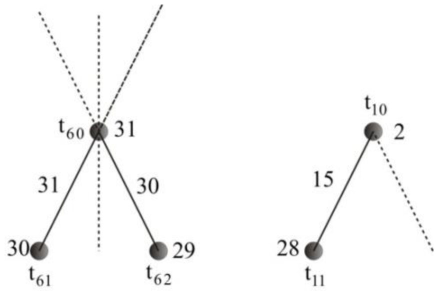
Similarly, $t_{6,4} \neq 25$. Hence $t_{2,0} \neq 5$. Therefore $t_{6,3} \neq 27$.

Hence $t_{1,0} \neq 1$.

Case-B2.2: $t_{1,1} = 28$ **and** $t_{1,0} = 2$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{1,1} = 28$; $t_{1,0} = 2$)

Then $x_{1,1} = 15$.



Now $t_{6,3}$ is either 26 or 27.

Let us consider the case that $t_{6,3} = 26$. Hence $t_{2,1} = 27$.

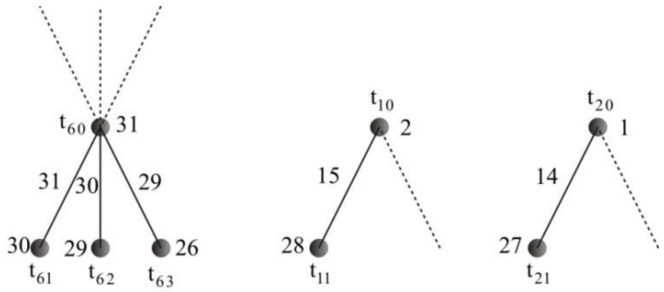
Here the value of $t_{2,0}$ can either be 1 or (4 or 5).

There exist two cases viz. $t_{2,0} = 1$ and $t_{2,0} = 4$ or 5.

Case-B2.2a (i):First consider the case that $t_{2,0} = 1$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 26$; $t_{1,1} = 28$; $t_{1,0} = 2$; $t_{2,1} = 27$; $t_{2,0} = 1$)

Then $x_{1,1} = 15$ and $x_{2,1} = 14$.



Now $t_{6,4}$ is either 24 or 25.

Now let us consider the case that $t_{6,4} = 24$. Hence $t_{3,1} = 25$.

If $t_{3,0} \geq 8$, then $x_{3,1} \geq 17$ and is not possible. Hence the value of $t_{3,0}$ can either be (4 or 5) or (6 or 7).

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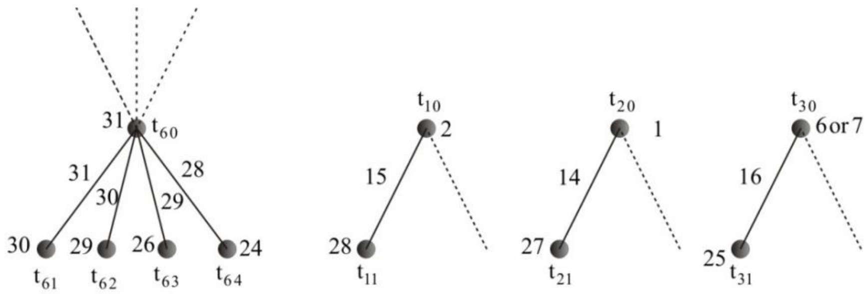
If $t_{3,0}=4\text{or}5,$ then $x_{3,1}=\left[\frac{25+4(\text{or}5)}{2}\right]=15.$ This is not possible.

(We have $t_{6,0}=31;\ t_{6,1}=30;\ t_{6,2}=29;\ t_{6,3}=26;\ t_{6,4}=24;\ t_{1,1}=28;\ t_{1,0}=2;$

$t_{2,1}=27;\ t_{2,0}=1;\ t_{3,1}=25$)

Then $x_{1,1}=15;\ x_{2,1}=14$ and if $t_{3,0}=6\text{or}7,$ then $x_{3,1}=16.$



Now $t_{6,5}$ is either 22 or 23.

Now let us consider the case that $t_{6,5}=22.$ Hence $t_{4,1}=23.$

(We have $t_{6,0}=31;\ t_{6,1}=30;\ t_{6,2}=29;\ t_{6,3}=26;\ t_{6,4}=24;\ t_{6,5}=22;\ t_{1,1}=28;$

$t_{1,0}=2;\ t_{2,1}=27;\ t_{2,0}=1;\ t_{3,1}=25;\ t_{3,0}=6\text{or}7;\ t_{4,1}=23$)

Here the value of $t_{4,0}=(4\text{or}5)\text{or}\geq 8,$ this is not possible.

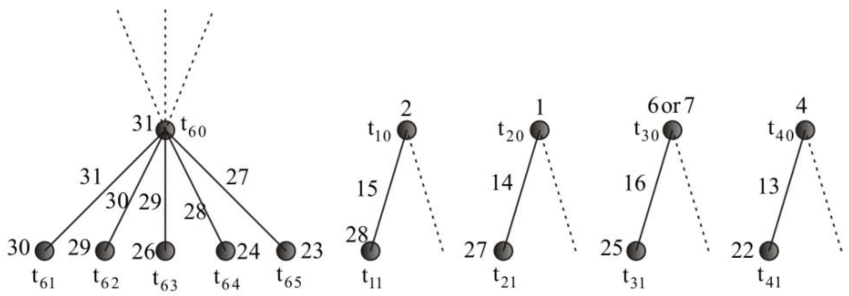
Now let us consider the case that $t_{6,5}=23.$ Hence $t_{4,1}=22.$

The value of $t_{4,0}$ can be from (4 or 5).

(We have $t_{6,0}=31;\ t_{6,1}=30;\ t_{6,2}=29;\ t_{6,3}=26;\ t_{6,4}=24;\ t_{6,5}=23;\ t_{1,1}=28;$

$t_{1,0}=2;\ t_{2,1}=27;\ t_{2,0}=1;\ t_{3,1}=25;\ t_{3,0}=6\text{or}7;\ t_{4,1}=22$)

Then $x_{1,1}=15;\ x_{2,1}=14;\ x_{3,1}=16$ and if $t_{4,0}=4,$ then $x_{4,1}=13.$



Now $t_{6,6}$ is either 20 or 21.

Now let us consider the case that $t_{6,6}=20.$ Hence $t_{5,1}=21.$

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(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 26$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{6,6} = 20$;

$t_{1,1} = 28$; $t_{1,0} = 2$; $t_{2,1} = 27$; $t_{2,0} = 1$; $t_{3,1} = 25$; $t_{3,0} = 6 \text{ or } 7$; $t_{4,1} = 22$; $t_{4,0} = 4$; $t_{5,1} = 21$)

Here the value of $t_{5,0} \geq 8$, this is not possible.

Now let us consider the case that $t_{6,6} = 21$. Hence $t_{5,1} = 20$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 26$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{6,6} = 21$;

$t_{1,1} = 28$; $t_{1,0} = 2$; $t_{2,1} = 27$; $t_{2,0} = 1$; $t_{3,1} = 25$; $t_{3,0} = 6 \text{ or } 7$; $t_{4,1} = 22$; $t_{4,0} = 4$; $t_{5,1} = 20$)

Here the value of $t_{5,0} \geq 8$, this is also not possible.

Hence $t_{6,4} \neq 24$.

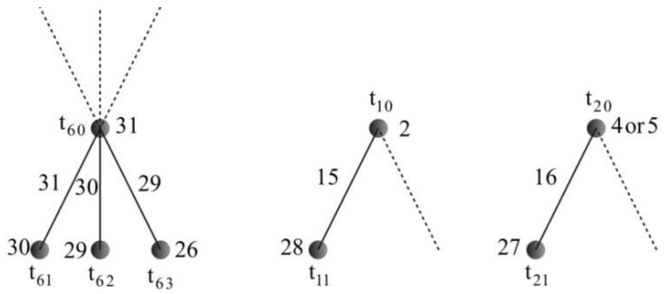
Similarly, $t_{6,4} \neq 25$. Hence $t_{2,0} \neq 1$.

Case-B2.2a (ii): Next we consider the case that $t_{2,0} = 4 \text{ or } 5$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 26$; $t_{1,1} = 28$; $t_{1,0} = 2$; $t_{2,1} = 27$;

$t_{2,0} = 4 \text{ or } 5$)

Then $x_{1,1} = 15$ and $x_{2,1} = 16$.



Now $t_{6,4}$ is either 24 or 25.

Now let us consider the case that $t_{6,4} = 24$. Hence $t_{3,1} = 25$.

If $t_{3,0} \geq 6$, then $x_{3,1} \geq 16$ and is not possible. Hence $t_{3,0}$ is 1.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 26$; $t_{6,4} = 24$; $t_{1,1} = 28$; $t_{1,0} = 2$;

$t_{2,1} = 27$; $t_{2,0} = 4 \text{ or } 5$; $t_{3,1} = 25$)

Then $x_{1,1} = 15$; $x_{2,1} = 16$ and $x_{3,1} = 13$.

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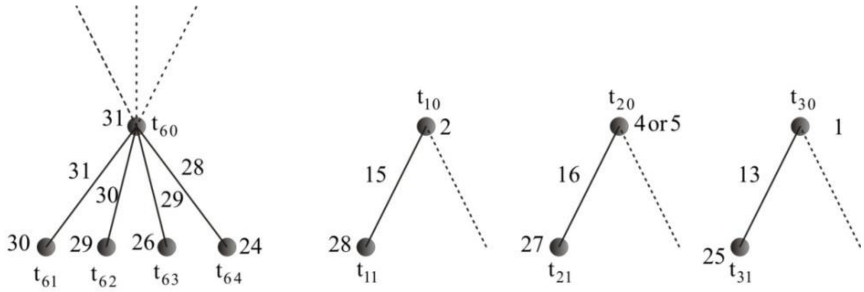


Now $t_{6,5}$ is either 22 or 23.

Now let us consider the case that $t_{6,5} = 22$. Hence $t_{4,1} = 23$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 26$; $t_{6,4} = 24$; $t_{6,5} = 22$; $t_{1,1} = 28$; $t_{1,0} = 2$; $t_{2,1} = 27$; $t_{2,0} = 4 \text{ or } 5$; $t_{3,1} = 25$; $t_{3,0} = 1$; $t_{4,1} = 23$)

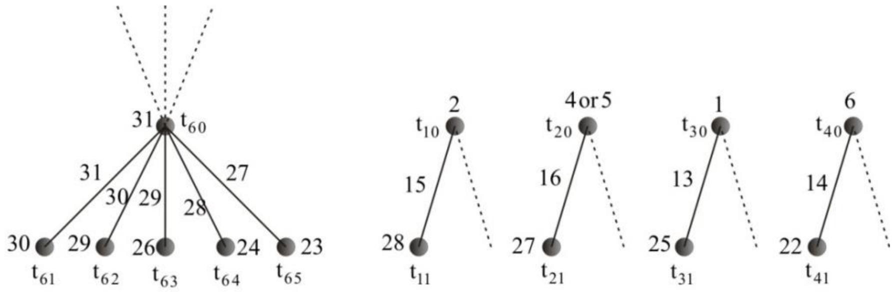
Here the value of $t_{4,0} \geq 6$, this is not possible.

Now let us consider the case that $t_{6,5} = 23$. Hence $t_{4,1} = 22$.

The value of $t_{4,0}$ can be from (6 or 7).

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 26$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{1,1} = 28$; $t_{1,0} = 2$; $t_{2,1} = 27$; $t_{2,0} = 4 \text{ or } 5$; $t_{3,1} = 25$; $t_{3,0} = 1$; $t_{4,1} = 22$)

Then $x_{1,1} = 15$; $x_{2,1} = 16$; $x_{3,1} = 13$ and if $t_{4,0} = 6$, then $x_{4,1} = 14$.



Now $t_{6,6}$ is either 20 or 21.

Now let us consider the casethat $t_{6,6} = 20$. Hence $t_{5,1} = 21$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 26$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{6,6} = 20$; $t_{1,1} = 28$; $t_{1,0} = 2$; $t_{2,1} = 27$; $t_{2,0} = 4 \text{ or } 5$; $t_{3,1} = 25$; $t_{3,0} = 1$; $t_{4,1} = 22$; $t_{4,0} = 6$; $t_{5,1} = 21$)

Here the value of $t_{5,0} \geq 8$, this is not possible.

Now let us consider the case that $t_{6,6} = 21$. Hence $t_{5,1} = 20$.

(We have $t_{6,0}=31;\ t_{6,1}=30;\ t_{6,2}=29;\ t_{6,3}=26;\ t_{6,4}=24;\ t_{6,5}=23;\ t_{6,6}=21;$

$t_{1,1}=28;\ t_{1,0}=2;\ t_{2,1}=27;\ t_{2,0}=4\text{or}5;\ t_{3,1}=25;\ t_{3,0}=1;\ t_{4,1}=22;\ t_{4,0}=6;$ $t_{5,1}=20$)

Here the value of $t_{5,0}\geq 8,$ this is not possible.

Hence $t_{6,4}\neq 24.$

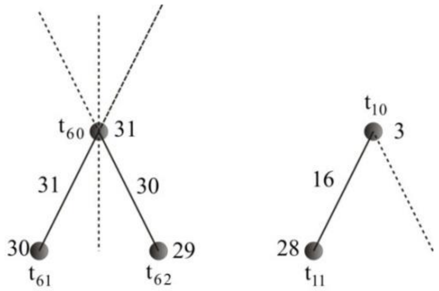
Similarly, $t_{6,4}\neq 25.$ Hence $t_{2,0}\neq 4\text{or}5.$ Therefore $t_{6,3}\neq 26.$

Similarly, $t_{6,3}\neq 27.$ Hence $t_{1,0}\neq 2.$

Case-B2.3: $t_{1,1}=28$ and $t_{1,0}=3.$

(We have $t_{6,0}=31;\ t_{6,1}=30;\ t_{6,2}=29;\ t_{1,1}=28;\ t_{1,0}=3$)

Then $x_{1,1}=16.$



Now $t_{6,3}$ is either 26 or 27.

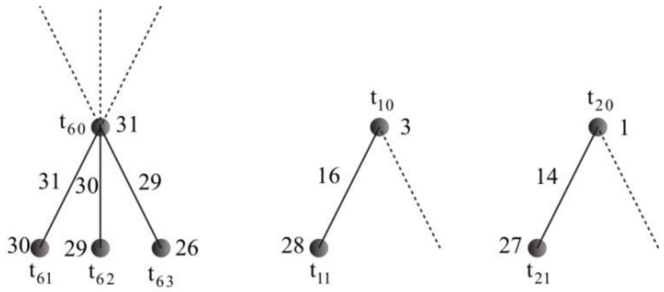
Let us consider the case that $t_{6,3}=26.$ Hence $t_{2,1}=27.$

If $t_{2,0}\geq 4,$ then $x_{3,1}\geq 16$ this is not possible. Hence $t_{2,0}$ is 1

(We have $t_{6,0}=31;\ t_{6,1}=30;\ t_{6,2}=29;\ t_{6,3}=26;\ t_{1,1}=28;\ t_{1,0}=3;\ t_{2,1}=27;$

$t_{2,0}=1$)

Then $x_{1,1}=16$ and $x_{2,1}=14.$



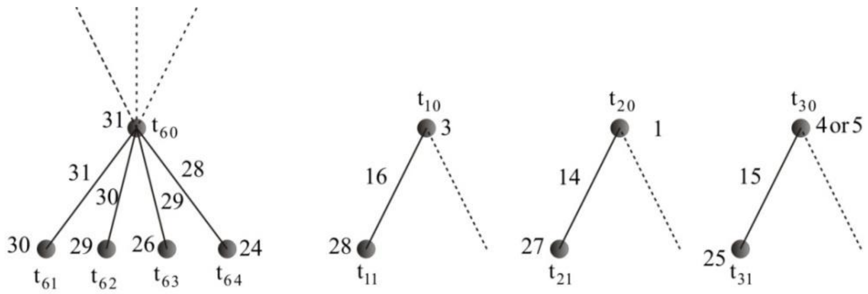
Now $t_{6,4}$ is either 24 or 25.

Now let us consider the case that $t_{6,4} = 24.$ Hence $t_{3,1} = 25.$

If $t_{3,0} \geq 6,$ then $x_{3,1} \geq 16$ and is not possible. Hence $t_{3,0}$ is (4 or 5).

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 29;\ t_{6,3} = 26;\ t_{6,4} = 24;\ t_{1,1} = 28;\ t_{1,0} = 3;$
$t_{2,1} = 27;\ t_{2,0} = 1;\ t_{3,1} = 25$)

Then $x_{1,1} = 16;\ x_{2,1} = 14$ and if we take $t_{3,0} = 4 \text{ or } 5,$ then $x_{3,1} = 15.$



Now $t_{6,5}$ is either 22 or 23.

Now let us consider the case that $t_{6,5} = 22.$ Hence $t_{4,1} = 23.$

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 29;\ t_{6,3} = 26;\ t_{6,4} = 24;\ t_{6,5} = 22;\ t_{1,1} = 28;$
$t_{1,0} = 3;\ t_{2,1} = 27;\ t_{2,0} = 1;\ t_{3,1} = 25;\ t_{3,0} = 4 \text{ or } 5;\ t_{4,1} = 23$)

Here the value of $t_{4,0} \geq 6,$ this is not possible.

Now let us consider the case that $t_{6,5} = 23.$

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 29;\ t_{6,3} = 26;\ t_{6,4} = 24;\ t_{6,5} = 23;\ t_{1,1} = 28;$
$t_{1,0} = 3;\ t_{2,1} = 27;\ t_{2,0} = 1;\ t_{3,1} = 25;\ t_{3,0} = 4 \text{ or } 5;\ t_{4,1} = 22$)

Here the value of $t_{4,0} \geq 6,$ this is not possible.

Hence $t_{6,4} \neq 24.$

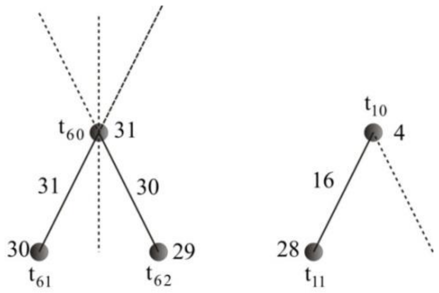
Similarly, $t_{6,4} \neq 25.$ Hence $t_{6,3} \neq 26.$

Similarly, $t_{6,3} \neq 27.$ Hence $t_{1,0} \neq 3.$

Case-B2.4: $t_{1,1} = 28$ and $t_{1,0} = 4.$

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 29;\ t_{1,1} = 28;\ t_{1,0} = 4$)

Then $x_{1,1} = 16.$



Now $t_{6,3}$ is either 26 or 27.

Let us consider the case that $t_{6,3} = 26.$ Hence $t_{2,1} = 27.$

Here the value of $t_{2,0}$ can either be 1or (2 or 3).

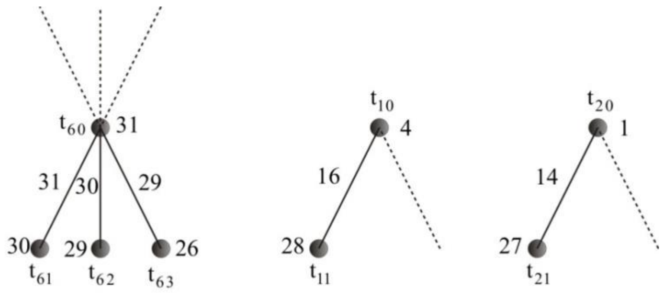
There exist two cases viz. $t_{2,0} = 1$ and $t_{2,0} = 2 \text{ or } 3.$

Case-B2.4a (i): First we consider the case that $t_{2,0} = 1.$

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 29;\ t_{6,3} = 26;\ t_{1,1} = 28;\ t_{1,0} = 4;\ t_{2,1} = 27;$

$t_{2,0} = 1$)

Then $x_{1,1} = 16$ and $x_{2,1} = 14.$



Now $t_{6,4}$ is either 24 or 25.

Now let us consider the case that $t_{6,4} = 24.$ Hence $t_{3,1} = 25.$

Here the value of $t_{3,0} = (2 \text{ or } 3) \text{ or } \geq 6,$ this is not possible.

Hence $t_{6,4} \neq 24.$

Similarly, $t_{6,4} \neq 25.$ Hence $t_{2,0} \neq 1.$

Case-B2.4a (ii): Next we consider the case that $t_{2,0} = 2 \text{ or } 3.$

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 29;\ t_{6,3} = 26;\ t_{1,1} = 28;\ t_{1,0} = 4;\ t_{2,1} = 27;$

$t_{2,0} = 2 \text{ or } 3$)

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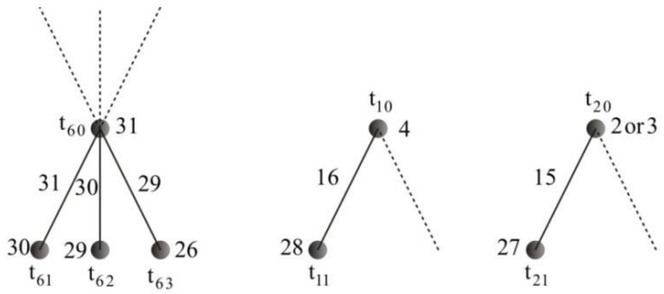
https://doi.org/10.26524/cm134

Then $x_{1,1}=16$ and $x_{2,1}=15.$



Now $t_{6,4}$ is either 24 or 25.

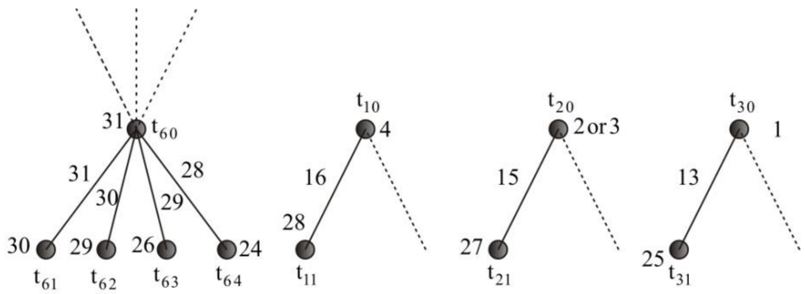
Now let us consider the case that $t_{6,4}=24.$ Hence $t_{3,1}=25.$

If $t_{3,0}\geq 6,$ then $x_{3,1}\geq 16$ and is not possible. Hence $t_{3,0}$ is 1.

(We have $t_{6,0}=31;\ t_{6,1}=30;\ t_{6,2}=29;\ t_{6,3}=26;\ t_{6,4}=24;\ t_{1,1}=28;\ t_{1,0}=4;$

$t_{2,1}=27;\ t_{2,0}=2\,\text{or}\,3;\ t_{3,1}=25)$

Then $x_{1,1}=16;\ x_{2,1}=15$ and if we take $t_{3,0}=1,$ then $x_{3,1}=13.$



Now $t_{6,5}$ is either 22 or 23.

Now let us consider the case that $t_{6,5}=22.$ Hence $t_{4,1}=23.$

(We have $t_{6,0}=31;\ t_{6,1}=30;\ t_{6,2}=29;\ t_{6,3}=26;\ t_{6,4}=24;\ t_{6,5}=22;\ t_{1,1}=28;$

$t_{1,0}=4;\ t_{2,1}=27;\ t_{2,0}=2\,\text{or}\,3;\ t_{3,1}=25;\ t_{3,0}=1;\ t_{4,1}=23)$

Here the value of $t_{4,0}\geq 6,$ this is not possible.

Now let us consider the case that $t_{6,5}=23.$ Hence $t_{4,1}=22.$

If $t_{4,0}\geq 7,$ then $x_{3,1}\geq 15$ and is not possible. Hence $t_{4,0}$ is (6 or 7).

(We have $t_{6,0}=31;\ t_{6,1}=30;\ t_{6,2}=29;\ t_{6,3}=26;\ t_{6,4}=24;\ t_{6,5}=23;\ t_{1,1}=28;$

$t_{1,0}=4;\ t_{2,1}=27;\ t_{2,0}=2\,\text{or}\,3;\ t_{3,1}=25;\ t_{3,0}=1;\ t_{4,1}=22)$

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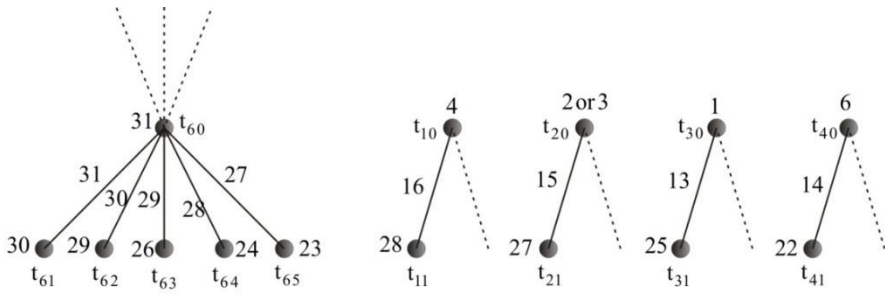
Then $x_{1,1}=16$; $x_{2,1}=15$; $x_{3,1}=13$; and if we take $t_{4,0}=6$, then $x_{4,1}=14$.



Now $t_{6,6}$ is either 20 or 21.

Now let us consider the case that $t_{6,6}=20$. Hence $t_{5,1}=21$.

(We have $t_{6,0}=31$; $t_{6,1}=30$; $t_{6,2}=29$; $t_{6,3}=26$; $t_{6,4}=24$; $t_{6,5}=23$; $t_{6,6}=20$;

$t_{1,1}=28$; $t_{1,0}=4$; $t_{2,1}=27$; $t_{2,0}=2\,\mathrm{or}\,3$; $t_{3,1}=25$; $t_{3,0}=1$; $t_{4,1}=22$;
$t_{4,0}=6$; $t_{5,1}=21$)

Here the value of $t_{5,0}\geq 8$, this is not possible.

Now let us consider the case that $t_{6,6}=21$. Hence $t_{5,1}=20$.

(We have $t_{6,0}=31$; $t_{6,1}=30$; $t_{6,2}=29$; $t_{6,3}=26$; $t_{6,4}=24$; $t_{6,5}=23$; $t_{6,6}=21$;

$t_{1,1}=28$; $t_{1,0}=4$; $t_{2,1}=27$; $t_{2,0}=2\,\mathrm{or}\,3$; $t_{3,1}=25$; $t_{3,0}=1$; $t_{4,1}=22$;
$t_{4,0}=6$; $t_{5,1}=20$)

Here the value of $t_{5,0}\geq 8$, this is not possible.

Hence $t_{6,4}\neq 24$.

Similarly, $t_{6,4}\neq 25$. Hence $t_{2,0}\neq 2\,\mathrm{or}\,3$.

Let us consider the case that $t_{6,3}=27$. Hence $t_{2,1}=26$.

Here the value of $t_{2,0}$ can either be 1or (2 or 3).

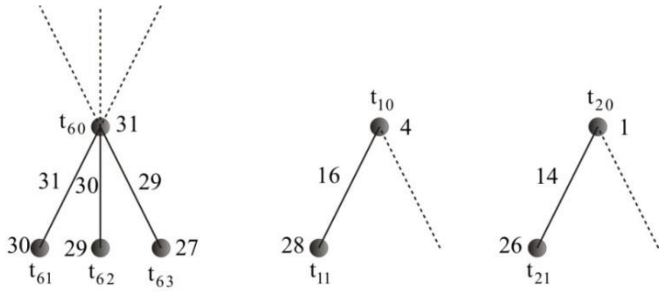
There exist two cases viz. $t_{2,0}=1$ and $t_{2,0}=3$.

Case B 2.4b (i): First we consider the case that $t_{2,0}=1$.

(We have $t_{6,0}=31$; $t_{6,1}=30$; $t_{6,2}=29$; $t_{6,3}=27$; $t_{1,1}=28$; $t_{1,0}=4$; $t_{2,1}=26$;

$t_{2,0}=1$)

Then $x_{1,1}=16$ and $x_{2,1}=14$.



Now $t_{6,4}$ is either 24 or 25.

Now let us consider the case that $t_{6,4} = 24.$ Hence $t_{3,1} = 25.$

Here the value of $t_{3,0} = (2 \text{ or } 3) \text{ or} \geq 6,$ this is not possible.

Hence $t_{6,4} \neq 24.$

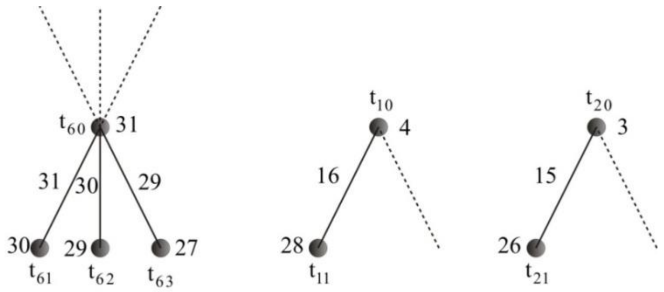
Similarly, $t_{6,4} \neq 25.$ Hence $t_{2,0} \neq 1.$

Case B 2.4b (ii): Next we consider the case that $t_{2,0} = 3.$

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 29;\ t_{6,3} = 27;\ t_{1,1} = 28;\ t_{1,0} = 4;\ t_{2,1} = 26;$

$t_{2,0} = 3$)

Then $x_{1,1} = 16$ and $x_{2,1} = 15.$



Now $t_{6,4}$ is either 24 or 25.

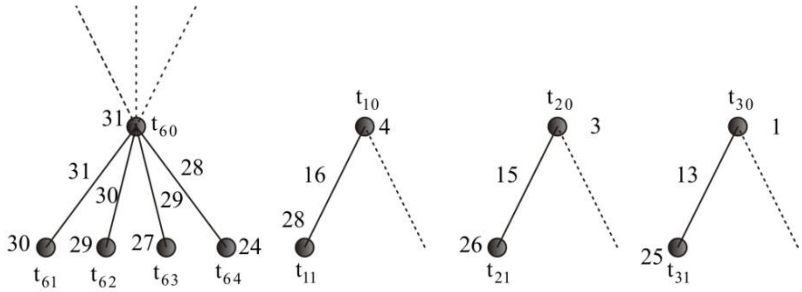
Now let us consider the case that $t_{6,4} = 24.$ Hence $t_{3,1} = 25.$

If $t_{3,0} \geq 6,$ then $x_{3,1} \geq 16$ and is not possible. Hence $t_{3,0}$ is 1.

(We have $t_{6,0} = 31;\ t_{6,1} = 30;\ t_{6,2} = 29;\ t_{6,3} = 27;\ t_{6,4} = 24;\ t_{1,1} = 28;\ t_{1,0} = 4;$

$t_{2,1} = 26;\ t_{2,0} = 3;\ t_{3,1} = 25;\ t_{3,0} = 1$)

Then $x_{1,1} = 16;\ x_{2,1} = 15$ and $x_{3,1} = 13.$



Now $t_{6,5}$ is either 22 or 23.

Now let us consider the case that $t_{6,5} = 22$. Hence $t_{4,1} = 23$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 22$; $t_{1,1} = 28$;

$t_{1,0} = 4$; $t_{2,1} = 26$; $t_{2,0} = 3$; $t_{3,1} = 25$; $t_{3,0} = 1$; $t_{4,1} = 23$)

Here the value of $t_{4,0} \geq 6$, this is not possible.

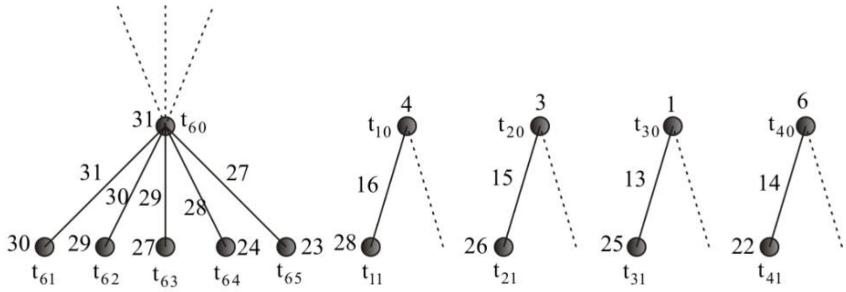
Now let us consider the case that $t_{6,5} = 23$. Hence $t_{4,1} = 22$.

If $t_{4,0} \geq 7$, then $x_{3,1} \geq 15$ and is not possible. Hence $t_{4,0}$ is (6 or 7).

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{1,1} = 28$;

$t_{1,0} = 4$; $t_{2,1} = 26$; $t_{2,0} = 3$; $t_{3,1} = 25$; $t_{3,0} = 1$; $t_{4,1} = 22$; $t_{4,0} = 6$)

Then $x_{1,1} = 16$; $x_{2,1} = 15$; $x_{3,1} = 13$ and $x_{4,1} = 14$.



Now $t_{6,6}$ is either 20 or 21.

Now let us consider the case that $t_{6,6} = 20$. Hence $t_{5,1} = 21$.

(We have $t_{6,0} = 31$; $t_{6,1} = 30$; $t_{6,2} = 29$; $t_{6,3} = 27$; $t_{6,4} = 24$; $t_{6,5} = 23$; $t_{6,6} = 20$;

$t_{1,1} = 28$; $t_{1,0} = 4$; $t_{2,1} = 26$; $t_{2,0} = 3$; $t_{3,1} = 25$; $t_{3,0} = 1$; $t_{4,1} = 22$; $t_{4,0} = 6$;
$t_{5,1} = 21$)

Here the value of $t_{5,0} \geq 8$, this is not possible.

Now let us consider the case that $t_{6,6} = 21$. Hence $t_{5,1} = 20$.

(We have $t_{6,0}=31;\ t_{6,1}=30;\ t_{6,2}=29;\ t_{6,3}=27;\ t_{6,4}=24;\ t_{6,5}=23;\ t_{6,6}=21;$

$t_{1,1}=28;\ t_{1,0}=4;\ t_{2,1}=26;\ t_{2,0}=3;\ t_{3,1}=25;\ t_{3,0}=1;\ t_{4,1}=22;\ t_{4,0}=6;$
$t_{5,1}=20$)

Here the value of $t_{5,0}\geq 8$, this is not possible.

Hence $t_{6,4}\neq 24.$

Similarly, $t_{6,4}\neq 25.$ Hence $t_{2,0}\neq 3.$ Therefore $t_{6,3}\neq 27.$

Hence $t_{1,0}\neq 4.$ Therefore $t_{1,1}\neq 28$ and hence $t_{6,2}\neq 29.$

Therefore $t_{6,1}\neq 30$ and hence $t_{6,0}\neq 31.$

We have proved that if $t_{6,0}=31$, the six star $G=5K_{1,2}\cup K_{1,15}$ does not admit a skolem mean labeling. Similarly, we can prove the result for other values of $t_{6,0}$.

Hence the six star $G=K_{1,\ell}\cup K_{1,\ell}\cup K_{1,\ell}\cup K_{1,\ell}\cup K_{1,m}\cup K_{1,n}=K_{1,2}\cup K_{1,2}\cup K_{1,2}\cup K_{1,2}\cup K_{1,2}\cup K_{1,15}=5K_{1,2}\cup K_{1,15}$ is not a skolem mean graph if $|m-n|>4+4\ell=12.$

In a similar way, we can prove that $G=4K_{1,3}\cup K_{1,4}\cup K_{1,21}$ is not a skolem mean graph if $|m-n|>16.$

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