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The Numbers of Lakh Place of Mersenne Primes

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Abstract

The numbers $M_n = 2^n - 1, n > 1$ are called Mersenne numbers. A Mersenne number which is also a prime is called Mersenne prime. In this paper, the numbers of lakh place of Mersenne primes $M_p = 2^p - 1$, where $p = 8r+1, 8r+3, 8r+5, 8r+7$ for $r = 1, 2, ...50 \pmod{131069}$ are studied, and the conclusion is presented by Chinese Remainder theorem.

1. Introduction

The numbers $M_n = 2^n - 1, n > 1$ are called Mersenne numbers. A Mersenne number which is also a prime is called Mersenne prime. Mersenne primes have arised naturally from the discussions of perfect number. Infact there is a one-to-one correspondence between the Mersenne primes and even perfect numbers. Evidently $3 = 2^2 - 1$, $7 = 2^3 - 1$, $15 = 2^4 - 1$, $31 = 2^5 - 1$, $63 = 2^6 - 1$, $127 = 2^7 - 1$ are first few Mersenne numbers, out of which $3, 7, 32$ and 127 are primes. Mersenne asserted that for $p = 2, 3, 5, 7, 13, 17, 19, 31, 67, 127$ and $257, M_p$ is prime and composite for all other primes below 257. Since then it has been shown that M_{67} and M_{257} are composite. [4]

Furthermore, M_{61}, M_{89} and M_{107} are primes which were excluded in his list. Till today in all 32 Mersenne primes that are known, the last and the largest such prime is discovered by British scientist in 1992 on a Honey-well computer. Also it is still not decided, if Mersenne primes are finite or infinite. [5]

The obvious problem is to recognize if a Mersenne number is prime and if not, to determine the factors. For this various methods are available. A few results concerning these methods are given.

If $p > 2$, then any prime divisor of M_p must be of the form $2kp+1$ with $k = 1, 2, 3, ...$ the best method presently known for testing the primality of Mersenne numbers is the Lucas-Lehmer Primality test [2]. Specifically, it can be shown that for prime $p > 2$, $M_p = 2^p - 1$ is prime if and only if M divides $S_{(k-2)}$, where $S = 4$ and $S = S(S-1) - 2$ for $k > 0$. The test was originally developed by E. Lucas in 1856,

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and subsequently improved by Lucas in 1878 and D. Lehmer in the 1930. For $p > 2$, every prime divisor of M_p is of the form $8k \pm 1$ [3]. In this paper the numbers of lakh place of Mersenne primes $M_p = 2^p - 1$ where $p = 8k+1,\ 8k+3,\ 8k+5,\ 8k+7$ for $k = 1, 2, 50 \pmod{131069}$ are studied and presented in theorem 2.2. We quote the theorem 2.1 [1] which is already proved by the same authors.

2. Main Results

In order to show the results on Mersenne primes, the following Chinese Remainder theorem is used.
Let $n_1, n_2, ..., n_k$ be pairwise co-prime positive integers then system

$$\begin{aligned} x &\equiv a_1 \pmod{n_1} \\ x &\equiv a_2 \pmod{n_2} \\ &\vdots \\ x &\equiv a_k \pmod{n_k} \end{aligned}$$

has one, and only one, solution in $Z_{n_1,...,n_k}$.

Theorem 2.1 If the power p of M_p fulfils the conditions

$$\begin{cases} p = 8k+1, k = 1, 8, 10, 26, 46 \pmod{8189} \\ p = 8k+3, k = 1, 9, 21, 24, 25, 39, 41 \pmod{8189} \\ p = 8k+5, k = 1, 12, 24, 31 \pmod{8189} \\ p = 8k+7, k = 13, 15, 29, 32 \pmod{8189} \end{cases}$$

then, the digit of ten thousand place of Mersenne primes is 0;
If the power p of M_p fulfils the conditions

$$\begin{cases} p = 8k+1, k = 6, 28, 37, 38, 48 \pmod{8189} \\ p = 8k+3, k = 3, 8, 26, 27, 32, 43 \pmod{8189} \\ p = 8k+5, k = 23, 45, 48 \pmod{8189} \\ p = 8k+7, k = 22, 27, 31, 33, 41 \pmod{8189} \end{cases}$$

then, the digit of ten thousand place of Mersenne primes is 1;
If the power p of M_p fulfils the conditions

$$\begin{cases} p = 8k+1, k = 9, 16, 24, 31, 41 \pmod{8189} \\ p = 8k+3, k = 2, 5, 7, 46 \pmod{8189} \\ p = 8k+5, k = 15, 30, 41, 46 \pmod{8189} \\ p = 8k+7, k = 4, 30, 36, 38, 42 \pmod{8189} \end{cases}$$

then, the digit of ten thousand place of Mersenne primes is 2;
If the power p of M_p fulfils the conditions

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$$\begin{cases} p = 8k+1, k = 2, 4, 14, 30, 44 \pmod{8189} \\ p = 8k+3, k = 4, 20 \pmod{8189} \\ p = 8k+5, k = 9, 14, 42, 49 \pmod{8189} \\ p = 8k+7, k = 1, 12, 21 \pmod{8189} \end{cases}$$

then, the digit of ten thousand place of Mersenne primes is 3;
If the power p of M_p fulfils the conditions

$$\begin{cases} p = 8k+1, k = 22, 27, 33 \pmod{8189} \\ p = 8k+3, k = 10, 11, 15, 18, 44, 48, 50 \pmod{8189} \\ p = 8k+5, k = 6, 8, 13, 17, 25, 35, 38 \pmod{8189} \\ p = 8k+7, k = 5, 24, 34, 47, 49, 50 \pmod{8189} \end{cases}$$

then, the digit of ten thousand place of Mersenne primes is 4;
If the power p of M_p fulfils the conditions

$$\begin{cases} p = 8k+1, k = 3, 5, 7, 11, 17, 18, 36 \pmod{8189} \\ p = 8k+3, k = 14, 17, 30, 42, 49 \pmod{8189} \\ p = 8k+5, k = 4, 21, 33, 34, 36 \pmod{8189} \\ p = 8k+7, k = 9, 23 \pmod{8189} \end{cases}$$

then, the digit of ten thousand place of Mersenne primes is 5;
If the power p of M_p fulfils the conditions

$$\begin{cases} p = 8k+1, k = 13, 19, 23, 45, 47 \pmod{8189} \\ p = 8k+3, k = 34, 36, 38 \pmod{8189} \\ p = 8k+5, k = 16, 18, 19, 26, 29, 39, 40 \pmod{8189} \\ p = 8k+7, k = 6, 14, 16, 25, 35, 39, 44, 48 \pmod{8189} \end{cases}$$

then, the digit of ten thousand place of Mersenne primes is 6;
If the power p of M_p fulfils the conditions

$$\begin{cases} p = 8k+1, k = 32, 40 \pmod{8189} \\ p = 8k+3, k = 23, 35, 45 \pmod{8189} \\ p = 8k+5, k = 3, 20, 27, 28, 32, 43 \pmod{8189} \\ p = 8k+7, k = 7, 11, 19, 37, 40, 45 \pmod{8189} \end{cases}$$

then, the digit of ten thousand place of Mersenne primes is 7;
If the power p of M_p fulfils the conditions

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$$\begin{cases} p = 8k+1, k = 15, 20, 34, 42, 49, 50 \pmod{8189} \\ p = 8k+3, k = 6, 13, 16, 19, 29, 33 \pmod{8189} \\ p = 8k+5, k = 5, 44, 47, 50 \pmod{8189} \\ p = 8k+7, k = 2, 3, 10, 17, 18, 26, 46 \pmod{8189} \end{cases}$$

then, the digit of ten thousand place of Mersenne primes is 8;
If the power p of M_p fulfils the conditions

$$\begin{cases} p = 8k+1, k = 12, 21, 25, 29, 35, 39, 43 \pmod{8189} \\ p = 8k+3, k = 12, 22, 28, 31, 37, 40, 47 \pmod{8189} \\ p = 8k+5, k = 2, 7, 10, 11, 22, 37 \pmod{8189} \\ p = 8k+7, k = 8, 20, 28 \pmod{8189} \end{cases}$$

then, the digit of ten thousand place of Mersenne primes is 9. [1]

Theorem 2.2 If the power p of M_p fulfiles the conditions

$$\begin{cases} p = 8r+1, r = 1, 8, 20, 21, 24, 36, 49 \pmod{131069} \\ p = 8r+3, r = 1, 5, 17, 21, 23 \pmod{131069} \\ p = 8r+5, r = 1, 2, 14, 15, 21, 34, 36 \pmod{131069} \\ p = 8r+7, r = 1, 16 \pmod{131069} \end{cases}$$

then, the number of lakh digit of Mersenne prime is 0;
If the power p of M_p fulfills the conditions

$$\begin{cases} p = 8r+1, r = 2, 47 \pmod{131069} \\ p = 8r+3, r = 24, 32, 41 \pmod{131069} \\ p = 8r+5, r = 13 \pmod{131069} \\ p = 8r+7, r = 14, 15, 27, 37, 43, 47, 50 \pmod{131069} \end{cases}$$

then, the number of lakh digit of Mersenne prime is 1;
If the power p of M_p fulfills the conditions

$$\begin{cases} p = 8r+1, r = 5, 7, 23, 30, 32 \pmod{131069} \\ p = 8r+3, r = 3, 11, 13, 26, 36, 39 \pmod{131069} \\ p = 8r+5, r = 5, 17, 22, 27, 31, 42, 43 \pmod{131069} \\ p = 8r+7, r = 19, 21, 23, 34, 36, 46, 48 \pmod{131069} \end{cases}$$

then, the number of lakh digit of Mersenne prime is 2;
If the power p of M_p fulfills the conditions

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$$\begin{cases} p = 8r+1, r = 6, 26, 31, 39 \pmod{131069} \\ p = 8r+3, r = 20, 29, 31, 49 \pmod{131069} \\ p = 8r+5, r = 18, 20, 23, 33, 46 \pmod{131069} \\ p = 8r+7, r = 2, 5, 9, 10, 29, 30 \pmod{131069} \end{cases}$$

then, the number of lakh digit of Mersenne prime is 3;
If the power p of M_p fulfills the conditions

$$\begin{cases} p = 8r+1, r = 9, 10, 14, 16, 28, 44 \pmod{131069} \\ p = 8r+3, r = 7, 8, 25, 40, 45, 48 \pmod{131069} \\ p = 8r+5, r = 12, 24, 32, 38, 41, 49 \pmod{131069} \\ p = 8r+7, r = 3, 18, 26, 33, 39 \pmod{131069} \end{cases}$$

then, the number of lakh digit of Mersenne prime is 4;
If the power p of M_p fulfills the conditions

$$\begin{cases} p = 8r+1, r = 3, 11, 13, 29, 41 \pmod{131069} \\ p = 8r+3, r = 2, 33, 42 \pmod{131069} \\ p = 8r+5, r = 10, 19, 29 \pmod{131069} \\ p = 8r+7, r = 20, 25 \pmod{131069} \end{cases}$$

then, the number of lakh digit of Mersenne prime is 5;
If the power p of M_p fulfills the conditions

$$\begin{cases} p = 8r+1, r = 15, 25, 33, 34, 48 \pmod{131069} \\ p = 8r+3, r = 6, 10, 12, 16, 37, 47 \pmod{131069} \\ p = 8r+5, r = 7, 8, 25 \pmod{131069} \\ p = 8r+7, r = 8, 12, 13, 24, 45 \pmod{131069} \end{cases}$$

then, the number of lakh digit of Mersenne prime is 6;
If the power p of M_p fulfills the conditions

$$\begin{cases} p = 8r+1, r = 38, 43, 46 \pmod{131069} \\ p = 8r+3, r = 4, 9, 14, 15, 34, 44 \pmod{131069} \\ p = 8r+5, r = 6, 16, 37, 47, 50 \pmod{131069} \\ p = 8r+7, r = 7, 35, 41, 49 \pmod{131069} \end{cases}$$

then, the number of lakh digit of Mersenne prime is 7;
If the power p of M_p fulfills the conditions

$$\begin{cases} p = 8r+1, r = 4, 7, 40, 42, 45 \pmod{131069} \\ p = 8r+3, r = 18, 19, 22, 27, 38, 43, 46 \pmod{131069} \\ p = 8r+5, r = 3, 9, 26, 30, 39, 48 \pmod{131069} \\ p = 8r+7, r = 4, 22, 28, 31, 38, 40 \pmod{131069} \end{cases}$$

then, the number of lakh digit of Mersenne prime is 8;
If the power p of M_p fulfills the conditions

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$$\begin{cases} p = 8r+1, r = 12, 18, 19, 22, 27, 35, 37, 50 \pmod{131069} \\ p = 8r+3, r = 28, 30, 35, 50 \pmod{131069} \\ p = 8r+5, r = 4, 11, 28, 35, 40, 44, 45 \pmod{131069} \\ p = 8r+7, r = 6, 11, 17, 32, 42, 44 \pmod{131069} \end{cases}$$

then, the number of lakh digit of Mersenne prime is 9.

Proof: Since the power p of M_P is a prime, then $p = 4r+1$ or $p = 4r+3$. when $p = 2, 3, 5, 2^p - 1 < 100$. So, $p = 8r+1$ or $p = 8r+3$ or $p = 8r+5$ or $p = 8r+7$. We have the congruences equations as following, when M_p modulo 64, 15625 separately

$$\left.\begin{array}{l} M_p = 2^{8r+1} - 1 = 256^k \times 2 - 1 \equiv -1 \pmod{64} \\ M_p = 2^{8r+3} - 1 = 256^k \times 2^3 - 1 \equiv -1 \pmod{64} \\ M_p = 2^{8r+5} - 1 = 256^k \times 2^5 - 1 \equiv -1 \pmod{64} \\ M_p = 2^{8r+7} - 1 = 256^k \times 2^7 - 1 \equiv -1 \pmod{64} \end{array}\right\} \quad (1)$$

$$M_p = 2^{8r+1} - 1 \equiv 256^r \times 2 - 1 \pmod{15625} \quad (2)$$
$$M_p = 2^{8r+3} - 1 \equiv 256^r \times 2^3 - 1 \pmod{15625} \quad (3)$$
$$M_p = 2^{8r+5} - 1 \equiv 256^r \times 2^5 - 1 \pmod{15625} \quad (4)$$
$$M_p = 2^{8r+7} - 1 \equiv 256^r \times 2^7 - 1 \pmod{15625} \quad (5)$$

We have to solve congruence equations (1) and (2) to (5) as following, when $r \equiv r_i \pmod{131069}$ and $r_i = 0, 1, ...50$.

$$\begin{aligned} 256^r =& 1, 256, 3036, 11591, 14171, 2776, 7531, 6061, 4741, 10571, 3051, 15431, \\ & 12836, 4766, 1346, 826, 8331, 7736, 11666, 2121, 11726, 1856, 6386, \\ & 9816, 12896, 4501, 11631, 8786, 14841, 2421, 10401, 6406, 14936, \\ & 11116, 1946, 13801, 1806, 9211, 14266, 11471, 14701, 13456, 7236, \\ & 8666, 15371, 13101, 10106, 9011, 9941, 13646, 9001 \pmod{15625}. \end{aligned} \quad (6)$$

Combined congruence Equations(6) and (2), (3), (4), (5) separately, we have

$$\begin{aligned} M_p = 2^{8r+1} - 1 =& 1, 511, 6071, 7556, 12716, 5551, 15061, 12121, 9481, 5516, \\ & 6101, 15236, 10046, 9531, 2691, 1651, 1036, 15471, 7706, 4241, \\ & 7826, 3711, 12771, 4006, 10166, 9001, 7636, 1946, 14056, 4841, \\ & 5176, 12811, 14246, 6606, 3891, 11976, 3611, 2796, 12906, \\ & 731613776, 11286, 14471, 1706, 15116, 10576, 4586, 2396, 4256, \\ & 11666, 2376 \pmod{15625}. \end{aligned} \quad (7)$$

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$$\begin{aligned}M_p =& 2^{8r+3} - 1 = 7, 2047, 8662, 14602, 3992, 6582, 13372, 1612, 6677, 6442,\\ & 8782, 14072, 8937, 6877, 10767, 6607, 4147, 15012, 15202, 1342, 47, 14847,\\ & 4212, 402, 9417, 4757, 14922, 7787, 9352, 3742, 5082, 4372, 10112, 10802,\\ & 15567, 1032, 14447, 11187, 4752, 13642, 8232, 13897, 11012, 6827, 13592,\\ & 11057, 2722, 9587, 1402, 15417, 9507, \pmod{15625}.\end{aligned} \tag{8}$$

$$\begin{aligned}M_p = 2^{8r+5} - 1 =& 31, 8191, 3401, 11536, 346, 10706, 6616, 6451, 11807, 10146, 3881,\\ & 9416, 4501, 11886, 11821, 10806, 966, 13176, 13396, 5371, 231, 12516,\\ & 1226, 1611, 6421, 3406, 12816, 15526, 6161, 14971, 4706, 1866, 9201,\\ & 11961, 15396, 10916, 13501, 3386, 7696, 325, 1681, 8716, 12801, 11686,\\ & 7496, 12984, 10891, 7101, 5611, 14796, 6781 \pmod{15625}.\end{aligned} \tag{9}$$

$$\begin{aligned}M_p = 2^{8r+7} - 1 =& 127, 1517, 13607, 14897, 1387, 11577, 10842, 10182, 13097, 9337,\\ & 15527, 6417, 2382, 672, 412, 11977, 3867, 5832, 8872, 5862, 927,\\ & 3192, 4907, 6447, 10062, 13627, 4392, 15232, 9022, 13012, 3202,\\ & 7467, 5557, 972, 14712, 902, 12417, 7132, 13547, 15162, 6727,\\ & 3617, 4332, 15497, 14362, 5052, 12317, 12782, 6822,\\ & 12312, 11502 \pmod{15625}.\end{aligned} \tag{10}$$

Combined congruence Equations (1) and (7), (8), (9), (10) separately , by using the Chinese Remainder theorem, conclusions as follows.

$$\begin{aligned}M_p = 2^{8r+1} - 1 =& 511, 131071, 554431, 934591, 255551, 421311, 855871, 103231,\\ & 427391, 412351, 562111, 900671, 572031, 440191, 689151, 422911,\\ & 265471, 960831, 972991, 859510, 3711, 950271, 269631, 25791,\\ & 602751, 304511, 955071, 498431, 598591, 239551, 325311, 279871,\\ & 647231, 691391, 996351, 66311, 924671, 716031, 304191, 873151,\\ & 526911, 889471, 704831, 436991, 869951, 707711, 174271, 613631,\\ & 89791, 986751 \pmod{1000000}.\end{aligned} \tag{11}$$

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$$\begin{aligned} M_p = 2^{8r+3} - 1 =& 2047, 524287, 217727, 738367, 22207, 685247, 423487, 412927,\\ & 709567, 649407, 248447, 602687, 288127, 760767, 756607,\\ & 691647, 61887, 843327, 891967, 343807, 14847, 801087, 78527,\\ & 103167, 411007, 218047, 820287, 993727, 394367, 958207,\\ & 301247, 119487, 588927, 765567, 985407, 264447, 698687,\\ & 864127, 216767, 492607, 107647, 557887, 819327, 747967, 479807,\\ & 830847, 697087, 454527, 359167, 947007 (\text{mod } 1000000). \end{aligned} \tag{12}$$

$$\begin{aligned} M_p = 2^{8r+5} - 1 =& 8191, 97152, 870911, 953471, 88831, 740991, 693951, 651711,\\ & 838271, 597631, 993791, 410751, 152511, 43071, 26431, 766591,\\ & 247551, 373311, 567871, 375231, 59391, 204351, 314111, 412671,\\ & 644031, 872191, 281151, 974911, 577471, 832831, 204991, 477951,\\ & 355711, 622271, 941631, 57791, 794751, 456511, 867071, 970431,\\ & 430591, 231551, 277311, 991871, 919231, 323391, 788351, 818111,\\ & 436671, 788031 (\text{mod } 1000000). \end{aligned} \tag{13}$$

$$\begin{aligned} M_p = 2^{8r+7} - 1 =& 32767, 388607, 483647, 813887, 355327, 963967, 775807, 606847,\\ & 353087, 390527, 975167, 643007, 610047, 172287, 105727, 66367,\\ & 990207, 493247, 271487, 500927, 237567, 817407, 256447, 650687,\\ & 576127, 488767, 124607, 899647, 309887, 331327, 819967, 911807,\\ & 422847, 249087, 766527, 231167, 179007, 826047, 468287, 881727,\\ & 722367, 926207, 109247, 967487, 676927, 293567, 153407, 272447,\\ & 746687, 152127 (\text{mod } 1000000). \end{aligned} \tag{14}$$

this complete the proof.

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