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The Average Degree-Distance Index of Graph Operations

Shailaja Shirakol [1], Manjula Kalyanshetti [2]

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Abstract

Let G be a finite connected graph with vertex set V , the average distance $\nu(G)$ is defined to be the average of all distances in G, $\nu(G) = \frac{1}{n(n-1)} \sum_{u,v \in V(G)} d(u,v)$. In this paper, we obtain the explicit formulae for the average degree-distance index of generalized transformation graphs.

Key words: Transformation graph, Average distance, Zagreb indices
AMS classification: 05C90; 05C35; 05C12.

1 Introduction

Graphs provide a natural representation of the structure of covalently bond molecules, and hence of essentially all organic molecules, as is well known today and established about a century ago. Each atom is represented by a vertex, and each (covalent) chemical bond by an edge between the respective two vertices, in the usual and most direct manner to construct a graph representation of a molecule. The relevant characteristics of molecular structure, referred known as "molecular topology," are reflected in a clear manner by such a "molecular graph." Molecular graphs created in the manner described above have been used in thousands of studies and have innumerable chemical applications. The great majority of currently published studies in mathematical chemistry focus on such molecular graphs.
However, there are alternative, less immediate ways to depict molecular topology with a graph. If G is a molecular graph that can be transformed into another graph G in some fashion so that the correspondence between G and G^* is one-to-one, then the transformation $G \to G^*$ maintains all of G molecular topological information.

There have been a few previous attempts in the chemical literature to shift from regular molecular graphs to their transforms. The usage of graph complements has just been reported [2, 6].
The application of topological indices is a clear advantage of employing transformation graphs instead of conventional molecular graphs. A topological index of a transformation graph must reflect different structural aspects than a topological index of a regular molecular graph. A number of different structural features of the underlying molecules could be described using one and the same class of topological indices.

Although Zagreb indices are among the most well-researched topological indices, their characteristics and chemical applications have traditionally been explored for conventional molecular graphs [9]. We've recently concentrated on the Zagreb indices (and coindices) of some transformation graphs [2, 11].

[1] Department of Mathematics, SDMCET, Dharwad, Karnataka, India
Email: shailajashirkol@gmail.com
[2] Department of Mathematics, Jain College of Engineering, Belagavi, Karnataka, India
Email:mailto:kalyanshettimanjula@gmail.com

2 Terminology and Notations

Let $G=(V,E)$ be a graph. We denote the number of vertices of G by n and the number of edges by m. Thus $|V(G)|=n$ and $|E(G)|=m$. The degree of a vertex v is denoted by $deg_G(v)$. The complement of G, denoted by $\overline{G}$, is a graph which has the same vertex set as G, in which two vertices are adjacent if and only if they are not adjacent in G.
The first and second Zagreb indices[9] are defined as:

$$M_1(G)=\sum_{i=1}^{n} deg_G(v_i)^2$$

$$M_2(G)=\sum_{uv\in E(G)} deg_G(u)deg_G(v)$$

The degree-distance [1] and Gutman indices [10] are defined as:

$$DD(G)=\sum_{\{u,v\}\subseteq V(G)} d(u,v)[deg_G(u)+deg_G(v)]$$

$$GI(G)=\sum_{\{u,v\}\subseteq V(G)} d(u,v)[deg_G(u)deg_G(v)]$$

The Wiener-index[8]is defined as:

$$W(G)=\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d(u,v)$$

In this paper we conceive the average degree-distance index of molecular graph, which is defined as:
The degree-distance and Gutman indices are defined as:

$$DD^a(G)=\frac{1}{n(n-1)}\sum_{\{u,v\}\subseteq V(G)} d(u,v)[deg_G(u)deg_G(v)]$$

3. Graph Operations

Sampathkumar, [4], introduced the concepts of semitotal-point graph and semitotal-line graph which are stated as follows: Let $G=(V,E)$ be a graph. The semitotal-line graph T_1G is a graph with $V(T_1(G))=V(G)\cup E(G)$ and any two vertices $u,v\in T_1G$ are adjacent if and only if (1) u and v are adjacent edges in G and (2) one is a vertex of G and other is an edge of G incident with it.

Note that the definitions of semitotal-line graph and middle graphs given in [4], are identical. These two concepts have been introduced in the same year.

The semitotal-point graph T_2G is a graph with $V(T_2G)=V(G)\cup E(G)$ and any two vertices $u,v\in T_2(G)$ are adjacent if and only if (1) u and v are adjacent vertices in G and (2) one is a vertex of G and other is an edge of G incident with it.

Basavanagoud et.al [2] have generalized the semitotal point graph and introduced

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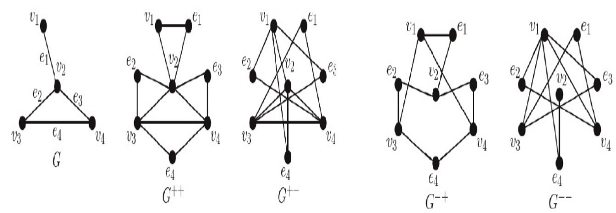
four new types of graph transformations viz., G^{++}, G^{+-}, G^{-+} and G^{--} . In this paper we explore the average degree-distance index of these generalized transformation graphs in the form of explicit formulae. The following figure depicts a graph G and its generalized transformation graphs:



4. Results

We begin with the following three straightforward, previously known, auxiliary results.

Lemma 4.1 Let G be an $(n;m)-$graph. Then the degrees of point and line vertices in G^{ab} are

1. $deg_{G^{++}}(v_i) = 2deg_G(v_i)$ and $deg_{G^{++}}(e_i) = 2$
2. $deg_{G^{+-}}(v_i) = m$ and $deg_{G^{+-}}(e_i) = n-2$
3. $deg_{G^{-+}}(v_i) = n-1$ and $deg_{G^{-+}}(e_i) = 2$
4. $deg_{G^{--}}(v_i) = n+m-1-2(v_i)$ and $deg_{G^{--}}(e_i) = n-2$

Lemma 4.2 Let G be an $(n;m)-$graph. Then the order of $G^{ab}\, n+m$

1. The size of G^{++} is $3m$
2. The size of G^{+-} is $m(n-1)$
3. The size of G^{-+} is $m+\frac{1}{2}n(n-1)$
4. The size of G^{--} is $m(n-3)+\frac{1}{2}n(n-1)$

Now we prove our main results.

Theorem 4.3 Let $G=(n,m)$ graph. Then

$$DD^a(G^{++}) = \frac{4}{m+n(m+n-1)}[GI(G)+W^*(G)]$$

where $W^*(G) = \sum_{\{u,v\}\subseteq V(G)} d(u,v)deg_G(u)$.

proof: Let $G=(n,m)$ graph i.e order and size of G are n and m respectively. By

Lemma 1 and 2, the order and size of G^{++} are $m+n$ and $3m$. By employing the definition of average degree-distance on G^{++} we get,

$$DD^a(G^{++}) = \frac{1}{n(n-1)} \sum_{\{u,v\}\subseteq V(G)} d(u,v)[deg_{G^{++}}(u)deg_{G^{++}}(v)]$$

By Lemma 4.2, we have

$$\begin{aligned}
DD^a(G^{++}) &= \frac{1}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G^{++})} d(u,v)[deg_{G^{++}}(u)deg_{G^{++}}(v)] \\
&= \frac{1}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G^{++})\cap V(G)} d(u,v)[deg_{G^{++}}(u)deg_{G^{++}}(v)] \\
&+ \frac{1}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G^{++})-V(G)} d(u,v)[deg_{G^{++}}(u)deg_{G^{++}}(v)] \\
&= \frac{1}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G^{++})\cap V(G)} d(u,v)[2deg_G(u)2deg_G(v)] \\
&+ \frac{1}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G^{++})-V(G)} d(u,v)[2\cdot 2deg_G(v)] \\
&= \frac{4}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G)\cap V(G)} d(u,v)[deg_G(u)deg_G(v)] \\
&+ \frac{4}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G)} d(u,v)[deg_G(v)] \\
&= \frac{4}{m+n(m+n-1)} GI(G) + \frac{4}{m+n(m+n-1)} W^*(G)
\end{aligned}$$

Theorem 4.4 Let $G=(n,m)$ graph. Then

$$DD^a(G^{+-}) = \mu(G^{+-})[m^2+m(n-2)].$$

proof: Let $G=(n,m)$ graph i.e order and size of G are n and m respectively. By Lemma 4.1 and 4.2, the order and size of G^{+-} are $m+n$ and $m(n-1)$. By employing the definition of average degree-distance on G^{+-}

$$DD^a(G^{+-}) = \frac{1}{n(n-1)} \sum_{\{u,v\subseteq V(G^{+-})\}} d(u,v)[deg_{G^{+-}}(u)deg_{G^{+-}}(v)]$$

By Lemma 4.2, we have

$$\begin{aligned}
DD^a(G^{+-}) &= \frac{1}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G^{+-})} d(u,v)[deg_{G^{+-}}(u)deg_{G^{+-}}(v)] \\
&= \frac{1}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G^{+-})\cap V(G)} d(u,v)[deg_{G^{+-}}(u)deg_{G^{+-}}(v)]
\end{aligned}$$

$$+\frac{1}{m+n(m+n-1)}\sum_{\{u,v\}\subseteq V(G^{+-})-V(G)} d(u,v)[deg_{G^{+-}}(u)deg_{G^{+-}}(v)]$$
$$=\frac{1}{m+n(m+n-1)}\sum_{\{u,v\}\subseteq V(G^{+-})\cap V(G)} d(u,v)[m\cdot m]$$
$$+\frac{1}{m+n(m+n-1)}\sum_{\{u,v\}\subseteq V(G^{+-})-V(G)} d(u,v)[m(n-2)]$$
$$=\frac{1}{m+n(m+n-1)}\sum_{\{u,v\}\subseteq V(G)\cap V(G)} d(u,v)$$
$$+\frac{1}{m+n(m+n-1)}\sum_{\{u,v\}\subseteq V(G)} d(u,v)$$
$$=m^2\mu(G^{+-})+m(n-2)\mu(G^{+-})$$
$$=\mu(G^{+-})[m^2+m(n-2)].$$

Theorem 4.5 Let $G=(n,m)$ graph. Then

$$DD^a(G^{-+})=2(n-1)^2\mu(G^{-+}).$$

proof: Let $G=(n,m)$ graph i.e order and size of G are n and m respectively. By Lemma 4.1 and 4.2, the order and size of G^{-+} are $m+n$ and $m+\frac{1}{2}n(n-1)$. By employing the definition of average degree-distance on G^{-+} we get,

$$DD^a(G^{-+})=\frac{1}{n(n-1)}\sum_{\{u,v\}\subseteq V(G^{-+})} d(u,v)[deg_{G^{-+}}(u)deg_{G^{-+}}(v)]$$

By 4.2, we have

$$DD^a(G^{-+})=\frac{1}{m+n(m+n-1)}\sum_{\{u,v\}\subseteq V(G^{-+})} d(u,v)[deg_{G^{-+}}(u)deg_{G^{-+}}(v)]$$
$$=\frac{1}{m+n(m+n-1)}\sum_{\{u,v\}\subseteq V(G^{-+})\cap V(G)} d(u,v)[deg_{G^{-+}}(u)deg_{G^{-+}}(v)]$$
$$+\frac{1}{m+n(m+n-1)}\sum_{\{u,v\}\subseteq V(G^{-+})-V(G)} d(u,v)[deg_{G^{-+}}(u)deg_{G^{-+}}(v)]$$
$$=\frac{1}{m+n(m+n-1)}\sum_{\{u,v\}\subseteq V(G^{-+})\cap V(G)} d(u,v)[(n-1)\cdot(n-1)]$$
$$+\frac{1}{m+n(m+n-1)}\sum_{\{u,v\}\subseteq V(G^{-+})-V(G)} d(u,v)[(n-1)^2]$$
$$=\frac{(n-1)^2}{m+n(m+n-1)}\sum_{\{u,v\}\subseteq V(G)\cap V(G)} d(u,v)$$
$$+\frac{(n-1)^2}{m+n(m+n-1)}\sum_{\{u,v\}\subseteq V(G)} d(u,v)$$
$$=(n-1)^2\mu(G^{-+})+(n-1)^2\mu(G^{-+})$$

$$= 2(n-1)^2\mu(G^{-+}).$$

Theorem 4.6 Let $G = (n,m)$ graph. Then

$$DD^a(G^{--}) = ((m+n-1-2deg_G(u))^2)W(G) + (n-1)(m+n-1-2deg_G(u))W(S(G)).$$

proof: Let $G = (n,m)$ graph i.e order and size of G are n and m respectively. By Lemma 4.1 and 4.2, the order and size of G^{--} are $m+n$ and $m(n-3)+\frac{1}{2}n(n-1)$. By employing the definition of average degree-distance on G^{--} we get,

$$DD^a(G^{--}) = \frac{1}{n(n-1)} \sum_{\{u,v\}\subseteq V(G^{--})} d(u,v)[deg_{G^{--}}(u)deg_{G^{--}}(v)]$$

By 4.2, we have

$$\begin{aligned}
DD^a(G^{--}) &= \frac{1}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G^{--})} d(u,v)[deg_{G^{--}}(u)deg_{G^{--}}(v)] \\
&= \frac{1}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G^{--})\cap V(G)} d(u,v)[deg_{G^{--}}(u)deg_{G^{--}}(v)] \\
&+ \frac{1}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G^{--})-V(G)} d(u,v)[deg_{G^{--}}(u)deg_{G^{--}}(v)] \\
&= \frac{1}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G^{--})\cap V(G)} d(u,v)[(m+n-1-2deg_G(u))^2] \\
&+ \frac{1}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G^{--})-V(G)} d(u,v)[(n-1)(m+n-1-2deg_G(u))] \\
&= \frac{(m+n-1-2deg_G(u))^2}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G)\cap V(G)} d(u,v) \\
&+ \frac{(n-1)(m+n-1-2deg_G(u))}{m+n(m+n-1)} \sum_{\{u,v\}\subseteq V(G)} d(u,v)
\end{aligned}$$

$$= ((m+n-1-2deg_G(u))^2)W(G) + (n-1)(m+n-1-2deg_G(u))W(S(G))$$

5 Conclusion

In this paper we have obtained the explicit formulae for four kinds of generalized transformation graphs. For further research on this topic, one can work on the mathematical as well as chemical properties of average degree-distance index.

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