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Multi Series Solution of q- Gamma Difference Equation

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Abstract

In this paper we defined multi series solution of q-Gamma difference equation deals with q-Gamma difference operator and generalized higher order q-Gamma difference equation and obtain the sum of finite q-Gamma multi summation formula. Suitable numerical examples and theorem verified the main result.

Key words: q-difference equation, generalized Gamma difference equation, Closed from solution, generalized higher order.

AMS classification: 39A13, 39A70, 65J10, 35k41

1 Introduction

In this paper we introduced generalized q-Gamma difference equation

$$\underset{(q_1)\gamma_1}{\Delta}\left(\underset{(q_2)\gamma_2}{\Delta}\left(\cdots \underset{(q_t)\gamma_t}{\Delta} v(e^k)\cdots\right)\right) = u(e^k), e^k \in (-\infty, \infty) \tag{1}$$

where $\underset{(q_i)\gamma_i}{\Delta} v(e^k) = v(q_i e^k) - \gamma_i v(e^k)$ we derive sum of t^{th} order q-Gamma multi series of polynomials, polynomial factorials and function by equation sum of closed form solutions. Also we extended certain results of q-Gamma multi series of q-Gamma multi infinite series and obtain the value of sum of Gamma multi series of polynomials, polynomial factorials and numerical examples

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2 q-Gamma Difference Operator

We present some basic definitions and results on q-Gamma difference operator and its inverse.

Definition 2.1 Let $u(e^k)$ ba a real valued function $(-\infty,\infty)$ and $q \neq 0$. Then the q-Gamma difference operator, denoted by $\underset{(q)\gamma}{\Delta}$ on $u(e^k)$ is defined as

$$\underset{(q)\gamma}{\Delta} v(e^k) = v(qe^K) - \gamma v(e^k) \tag{2}$$

and the inverse of the q-Gamma difference operator $\underset{(q)\gamma}{\Delta^{-1}}$ is defined as

$$if \underset{(q)\gamma}{\Delta} v(e^k) = u(e^k) \quad then \quad v(e^k) = \underset{(q)\gamma}{\Delta^{-1}} u(e^k) \tag{3}$$

Result 2.2. Let $e^k \in (0,\infty)$ and $\gamma \neq 1$. Then we have,

$$\underset{(q)\gamma}{\Delta^{-1}}(1) = \frac{1}{1-\gamma} \tag{4}$$

q-Gamma Summation Formula

Theorem 3.1 Let $q \neq 0$, $m \in N(1)$ and $u(e^k)$ be a function on $(-\infty,\infty)$, Then we have.

$$\underset{(q)\gamma}{\Delta^{-1}} u(qe^k) - \gamma^{m+1} \underset{(q)\gamma}{\Delta^{-1}} u\left(\frac{e^k}{q^m}\right) = \sum_{r=0}^{m} \gamma^r u\left(\frac{e^k}{q^r}\right) \tag{5}$$

is a solution of the $q-\gamma$difference equation $\underset{(q)\gamma}{\Delta} v(e^k) = u(e^k)$ and hence

$$\gamma^p \underset{(q)\gamma}{\Delta^{-1}} u\left(\frac{qe^k}{q^p}\right) - \gamma^{m+1} \underset{(q)\gamma}{\Delta^{-1}} u\left(\frac{e^k}{q^m}\right) = \sum_{r=p}^{m} \gamma^r u\left(\frac{e^k}{q^r}\right) forp < m \tag{6}$$

proof:

By Definition (1.1) and $\underset{(q)\gamma}{\Delta} v(e^k) = u(e^k)$ we have,

$$v(qe^k) = u(e^k) + \gamma v(e^k) \tag{7}$$

Replacing e^k by $\frac{e^k}{q}$ in (7) we get,

$$v(e^k) = u\left(\frac{e^k}{q}\right) + \gamma v\left(\frac{e^k}{q}\right) \tag{8}$$

Again replacing e^k by $\frac{e^k}{q}$, $\frac{e^k}{q^2}$, $\frac{e^k}{q^3} \cdots \frac{e^k}{q^{m-1}}$ in (8) repeatedly and substituting the resultant expressions in (7) we arrive.

$$u(e^k) + \gamma u\left(\frac{e^k}{q}\right) + \gamma^2 u\left(\frac{e^k}{q^2}\right) + \cdots + \gamma^m u\left(\frac{e^k}{q^m}\right) = v(qe^k) - \gamma^{m+1} v\left(\frac{e^k}{q^m}\right)$$

which yields (5)
Now replacing m by $p-1$ in 5
where $p < m$ we get.

$$\underset{(q)\gamma}{\Delta^{-1}} u(qe^k) - \gamma^p \underset{(q)\gamma}{\Delta^{-1}} u\left(\frac{e^k}{q^{p-1}}\right) = \sum_{r=0}^{p-1} \gamma^r u\left(\frac{e^k}{q^r}\right) \tag{9}$$

Hence (6), follows by subtracting (9) from (5)
The following theorem gives q-Gamma multi finite series formula using inverse q-Gamma difference operator.

Theorem 3.2 For $q_i \neq 0 \quad m \in N(1)$ and $e^k \in (-\infty, \infty)$ we have.

$$\sum_{i=1}^{t-1} \gamma_{i+1}^{m+1} \sum_{(r)_{1\to i}}^{m} \prod_{p=1}^{i} \gamma_p^{r_p} \underset{(q,\gamma)_{i+1\to t}}{\overset{-1}{\Delta}} u\left(\frac{\prod_{p=i+2}^{t} q_p e^k}{\prod_{p=1}^{i} q_p^{r_p} q_{i+1}^m}\right) + \sum_{(r)_{1\to i}}^{m} \prod_{p=1}^{t} \gamma_p^{r_p} u\left(\frac{e^k}{\prod_{i=1}^{t} q_i^{r_i}}\right)$$

$$= \underset{(q\gamma)_{1\to t}}{\Delta^{-1}} u\left(\prod_{i=1}^{t} q_i e^k\right) - \gamma_1^{m+1} \underset{(q,\gamma)_{1\to t}}{\overset{-1}{\Delta}} u \prod_{p=2}^{t}\left(\frac{q_p e^k}{q_1^m}\right) \tag{10}$$

proof:
Replacing q by q_1 and γ by γ_1 in (5) we get

$$u(e^k) + \gamma_1 u\left(\frac{e^k}{q_1}\right) + \cdots + \gamma_1^m u\left(\frac{e^k}{q_1^m}\right) = \underset{(q_1)\gamma_1}{\Delta^{-1}} u(q_1 e^k) - \gamma_1^{m+1} \underset{(q_1)\gamma_1}{\Delta^{-1}} u\left(\frac{e^k}{{q_1}^m}\right) \tag{11}$$

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Replacing $u(e^k)$ by $\underset{(q_2)\gamma_2}{\Delta^{-1}} u(q_2e^k)$, $u\left(\frac{e^k}{q_1}\right)$ by $\underset{(q_2)\gamma_2}{\Delta^{-1}} u\left(\frac{q_2e^k}{q_1}\right), u\left(\frac{e^k}{q_1^2}\right)$ by $\underset{(q_2)\gamma_2}{\Delta^{-1}} u\left(\frac{q_2e^k}{q_1^2}\right)\cdots\cdots u\left(\frac{e^k}{q_1^m}\right)$ by $\underset{(q_2)\gamma_2}{\Delta^{-1}} u\left(\frac{q_2e^k}{q_1^m}\right)$ in (11)
We obtain

$$\underset{(q_2)\gamma_2}{\Delta^{-1}} u(q_2e^k)+\gamma_1 \underset{(q_2)\gamma_2}{\Delta^{-1}} u\left(\frac{q_2e^k}{q_1}\right)+\cdots+\gamma_1^m \underset{(q_2)\gamma_2}{\Delta^{-1}} u\left(\frac{q_2e^k}{q_1^m}\right)$$
$$= \underset{(q_1)\gamma_1}{\Delta^{-1}} \underset{(q_2)\gamma_2}{\Delta^{-1}} u(q_1q_2e^k)-\gamma_1^{m+1} \underset{(q_1)\gamma_1}{\Delta^{-1}} \underset{(q_2)\gamma_2}{\Delta^{-1}} u\left(\frac{q_2e^k}{q_1^m}\right) \tag{12}$$

changing the subscripts (1) to (2) in (11) we find,

$$u(e^k)+\gamma_2 u\left(\frac{e^k}{q_2}\right)+\cdots+\gamma_2^m u\left(\frac{e^k}{q_2^m}\right) = \underset{(q_2)\gamma_2}{\Delta^{-1}} u(q_2e^k)-\gamma_2^{m+1} \underset{(q_2)\gamma_2}{\Delta^{-1}} u\left(\frac{e^k}{q_2^m}\right) \tag{13}$$

Replacing e^k by $\frac{e^k}{q_1^r}$ and multiplying by γ_1^r for $r=1,2,\cdots m$ in (13) we obtain.

$$\gamma_1^r\left\{u\left(\frac{e^k}{q_1^r}\right)+\gamma_2 u\left(\frac{e^k}{q_1^r q_2}\right)+\gamma_2^2 u\left(\frac{e^k}{q_1^r q_2^2}\right)+\cdots+\gamma_2^m u\left(\frac{e^k}{q_1^r q_2^m}\right)\right\}$$
$$=\gamma_1^r\left\{\underset{(q_2)\gamma_2}{\Delta^{-1}} u\left(\frac{q_2e^k}{q_1^r}\right)-\gamma_2^{m+1} \underset{(q_2)\gamma_2}{\Delta^{-1}} u\left(\frac{e^k}{q_1^r q_2^m}\right)\right\} \tag{14}$$

Adding equation (13) and (14) for $r=1,2\cdots m$ and then applying (12), we arrive.

$$\sum_{r_1=0}^{m}\sum_{r_2=0}^{m}\gamma_1^{r_1}\gamma_2^{r_2} u\left(\frac{e^k}{q_1^{r_1}q_2^{r_2}}\right) = \underset{(q_1)\gamma_1}{\Delta^{-1}} \underset{(q_2)\gamma_2}{\Delta^{-1}} u(e^kq_1q_2)-\gamma_1^{m+1} \underset{(q_1)\gamma_1}{\Delta^{-1}} \underset{(q_2)\gamma_2}{\Delta^{-1}} u\left(\frac{e^kq_2}{q_1^m}\right)$$
$$-\gamma_2^{m+1}\sum_{(r_1=0)}^{m}\gamma_1^{r_1} \underset{(q_2)\gamma_2}{\Delta^{-1}} u\left(\frac{e^k}{q_1^{r_1}q_2^m}\right) \tag{15}$$

Changing subscripts (1) to (2) and (2) and (3) in (15), we get.

$$\sum_{r_2=0}^{m}\sum_{r_3=0}^{m}\gamma_2^{r_2}\gamma_3^{r_3} u\left(\frac{e^k}{q_2^{r_2}q_3^{r_3}}\right) = \underset{(q_2)\gamma_2}{\Delta^{-1}} \underset{(q_3)\gamma_3}{\Delta^{-1}} u(e^kq_2q_3)-\gamma_1^{m+1} \underset{(q_2)\gamma_2}{\Delta^{-1}} \underset{(q_3)\gamma_3}{\Delta^{-1}} u\left(\frac{e^kq_3}{q_2^m}\right)$$
$$-\gamma_3^{m+1}\sum_{(r_2=0)}^{m}\gamma_2^{r_2} \underset{(q_3)\gamma_3}{\Delta^{-1}} u\left(\frac{e^k}{q_2^{r_2}q_3^m}\right) \tag{16}$$

Replacing e^k by $\frac{e^k}{q_1^r}$ and multiplying both sides by γ_1^r in (16) and then adding the corresponding expressions for $r = 0,1,2\cdots m$ we arrive.

$$\sum_{r_1=0}^{m}\sum_{r_2=0}^{m}\sum_{r_3=0}^{m}\gamma_1^{r_1}\gamma_2^{r_2}\gamma_3^{r_3}u\left(\frac{e^k}{q_1^{r_1}q_2^{r_2}q_3^{r_3}}\right) = \underset{(q_1)\gamma_1}{\Delta^{-1}}\underset{(q_2)\gamma_2}{\Delta^{-1}}\underset{(q_3)\gamma_3}{\Delta^{-1}}u(e^k q_1q_2q_3)$$
$$-\gamma_1^{m+1}\underset{(q_1)\gamma_1}{\Delta^{-1}}\underset{(q_2)\gamma_2}{\Delta^{-1}}\underset{(q_3)\gamma_3}{\Delta^{-1}}u\left(\frac{e^kq_2q_3}{q_1^m}\right)$$
$$-\gamma_2^{m+1}\sum_{(r_1=0)}^{m}\gamma_1^{r_1}\underset{(q_2)\gamma_2}{\Delta^{-1}}\underset{(q_3)\gamma_3}{\Delta^{-1}}u\left(\frac{e^kq_3}{q_1^{r_1}q_2^m}\right)$$
$$-\gamma_3^{m+1}\sum_{(r_1=0)}^{m}\sum_{(r_2=0)}^{m}\gamma_1^{r_1}\gamma_2^{r_2}\underset{(q_3)}{\Delta^{-1}}\gamma_3 u\left(\frac{e^k}{q_1^{r_1}q_2^{r_2}q_3^m}\right) \tag{17}$$

Hence the proof

Corollary 3.3 For any real valued function $u(e^k)$ on $(-\infty,\infty)$ and $q_i \neq 0$, we have.

$$\sum_{(i=1)}^{t-1}\sum_{(r)_{1\to i}}^{m}\gamma^{m+1}\prod_{p=1}^{i}\gamma^{r_p}\underset{(q_i+1\to t)\gamma}{\Delta^{-1}}u\left(\frac{\prod_{p=i+2}^{t}q_pe^k}{\prod_{p=1}^{t}q_p^{r_p}q_{i+1}^m}\right)+\sum_{(r)_{1\to t}}^{m}\prod_{p=1}^{t}\gamma_p^r u\left(\frac{e^k}{\prod_{i=1}^{t}q_i^{r_i}}\right)$$
$$= \underset{(q_1\to t)\gamma}{\Delta^{-1}}u\left(\prod_{i=1}^{t}q_ie^k\right)-\gamma^{m+1}\underset{(q_1\to t)\gamma}{\Delta^{-1}}u\left(\prod_{p=2}^{t}\frac{q_pe^k}{q_1^m}\right)$$

proof:
The proof follows by putting $\gamma_1 = \gamma_2 = \cdots = \gamma_t = \gamma$ in (10)

Corollary 3.4 Let $0 \neq q_i \in (-\infty,\infty)$ and m be any positive integer then.

$$\sum_{(i=1)}^{t-1}\sum_{(r)_{1\to i}}^{m}\underset{(q_{i+1\to t})}{\Delta^{-1}}u\left(\frac{\prod_{p=i+2}^{t}q_pe^k}{\prod_{p=1}^{t}q_p^{r_p}q_{i+1}^m}\right)+\sum_{(r)_{1\to t}}^{m}u\left(\frac{e^k}{\prod_{i=1}^{t}q_i^{r_i}}\right)$$
$$= \underset{(q_1\to t)}{\Delta^{-1}}u\left(\prod_{i=1}^{t}q_ie^k\right)-\underset{(q_1\to t)\gamma}{\Delta^{-1}}u\left(\prod_{p=2}^{t}\frac{q_pe^k}{q_1^m}\right)$$

proof:
The proof follow by taking $\gamma = 1$ in corollary (3.3)

4 Conclusion

In this paper q-difference Δ_q and q-Gamma difference operator $\Delta_{(q)\gamma}$ are introduced and formula and multi series summation formula using inverse and q-Gamma difference equation. Also we derived some theorems and corollary.

References

[1] Abdel-Gawad H and Aldailami A , On q-dynamic equations modelling and complexity, Appl. Math. Mode, 34(3), 697-709(2010).

[2] Abreu L, Sampling theory associated with q-difference equation of the Sturn-Liouville type, J.Phy. A38(48), 10311-10319(2005).

[3] Adams CR, On the linear ordinary q-difference equation, Am.Math. Ser.II, 30, 195-205(1929).

[4] Adams CR,On the linear ordinary q-difference equation, Am.Math. Ser.II, 30, 165-182(1928).

[5] Agarwal RP, Difference Equations and Inequalities, Marcel Dekker, New york, 2000.

[6] Ahamad B and Ntouyas SK,Existence of solutions for nonlinear fractional q-difference inclusions with nonlocal Robin (separted) condition, Mediterr.J.Math. 10,1333-1351(2013).