ISSN: 2456-8686, Volume 2, Issue 1, 2018:75-79
DOI : http://doi.org/10.26524/cm29

Journal of Computational Mathematica

Open Access

Skolem Mean Labeling of Four Star Graphs $K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $|b-(a_1+a_2+a_3)| = 4$

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Abstract

In this paper, we prove that four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$ is a skolem mean graph if $|b-(a_1+a_2+a_3)| = 4$.

Key words: Skolem mean graph, skolem mean labeling, star graphs.

AMS classification: 39A10, 39A11, 39A13, 39A70, 49M.

1. Introduction

In this paper all graphs are finite, simple and undirected. Terms and notations are used in the sense of Harary [3]. Much work is done by many researchers on skolem mean labelling [1], [2] and [3]. In [5], [6] and [7] some results are proved in four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ on skolem mean labelling. In this paper, we prove that four star graph $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$ is a skolem mean graph if $|b-(a_1+a_2+a_3)| = 4$. That is when $b = (a_1+a_2+a_3)+4$ and $b = (a_1+a_2+a_3)-4$.

Definition 1.1 A graph $G = (V,E)$ with p vertices and q edges is said to be a skolem mean graph if there exists a function f from the vertex set of G to $1, 2, \cdots, p$ such that the induced map f^* from the edge set of G to $2, 3, \cdots, p$ defined by

$$f^*(e=uv) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if} \quad f(u)+f(v) \quad \text{is even} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if} \quad f(u)+f(v) \quad \text{is odd} \end{cases}$$

the resulting edges get unique labels from the set $2, 3, \cdots, p$.

Theorem 1.2 The four star $G = K_{1,a_1} \cup K_{1,a_2} \cup K_{1,a_3} \cup K_{1,b}$ where $a_1 \le a_2 \le a_3$

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is a skolem mean graph if $|b-(a_1+a_2+a_3)|=4$. Proof: Let $A_i=\sum_{k=1}^{i}a_k$. That is, $A_1=a_1$; $A_2=a_1+a_2$ and $A_3=a_1+a_2+a_3$. Consider the graph $G=K_{1,a_1}\cup K_{1,a_2}\cup K_{1,a_3}\cup K_{1,b}$. Let $V=\bigcup_{k=1}^{4}V_k$ be the vertex set of G where $V_k=\{v_{k,i}:0\le i\le a_k\}$ for $1\le k\le 3$ and $V_4=\{v_{4,i}:0\le i\le b\}$. Let $E=\bigcup_{k=1}^{4}E_k$ be the edge set of G where $E_k=\{v_{k,0}v_{k,i}:0\le i\le a_k\}$ for $1\le k\le 3$ and $E_4=\{v_{4,0}v_{4,i}:0\le i\le b\}$.
The condition $|b-(a_1+a_2+a_3)|=4\Rightarrow b=A_3-4\,or\,b=A_3+4$.
That is, there are two cases viz. $b=A_3-4$ and $b=A_3+4$.
Let us prove in each of the two cases the graph G is a skolem mean graph.
Case 1: Let $b=A_3+4$
G has $A_3+b+4=2A_3+8$ vertices and $A_4+b=2A_3+4$ edges.
The vertex labeling
$f:V\rightarrow\{1,2,\cdots,A_3+b+4=2A_3+8\}$ is defined as follows:

$$
\begin{aligned}
f(v_{1,0})&=1;\quad f(v_{2,0})=2;\quad f(v_{3,0})=4;\\
f(v_{4,0})&=A_3+b+3=2A_3+7\\
f(v_{1,i})&=2i+4 && 1\le i\le a_1\\
f(v_{2,i})&=2A_1+2i+4 && 1\le i\le a_2\\
f(v_{3,i})&=2A_2+2i+4 && 1\le i\le a_3\\
f(v_{4,i})&=2i+11\le i\le b-2=A_3+2\\
f(v_{4,b-1})&=A_3+b+2=2A_3+6\\
f(v_{4,b})&=A_3+b+4=2A_3+8
\end{aligned}
$$

The corresponding edge labels are as follows:

The edge label of $v_{1,0}v_{1,i}$ is $3+i$ for $1\le i\le a_1$ (edge labels are $4,5,\cdots,a_1+3=A_1+3$), $v_{2,0}v_{2,i}$ is A_1+3+i for $1\le i\le a_2$ (edge labels are $A_1+4,A_1+5,\cdots,A_2+3$), $v_{3,0}v_{3,i}$ is A_2+4+i for $1\le i\le a_3$ (edge labels are $A_2+5,A_2+6,\cdots,A_3+4$), $v_{4,0}v_{4,i}$ is A_3+4+i for $1\le i\le b-2=A_3+2$ (edge labels are $A_3+5,A_3+6,\cdots,2A_3+6$), $v_{4,0}v_{b-1}$ is $2A_3+7$ and $v_{4,0}v_{4,b}$ is $2A_3+8$.

These induced edge labels of graph G are unique.

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Hence G is a skolem mean graph.

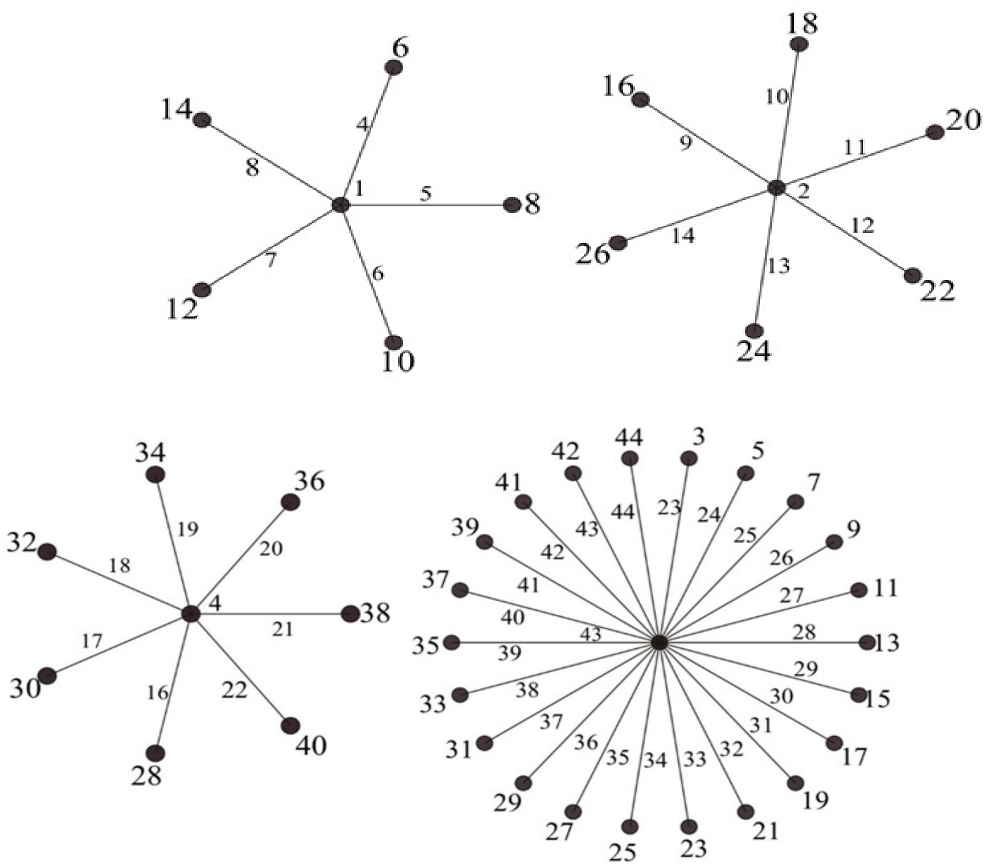

Figure 1: $K_{1,5} \cup K_{1,6} \cup K_{1,7} \cup K_{1,22}$

Case 2: Let $b = A_3 - 4$
G has $A_3 + b + 4 = 2A_3$ vertices and $A_3 + b = 2A_3 - 4$ edges.
The vertex labeling $f : V \rightarrow \{1, 2, \cdots, A_3 + b + 4 = 2A_3\}$ is defined as follows:

$$\begin{array}{ll}
f(v_{1,0}) = 2; \quad f(v_{2,0}) = 4; \quad f(v_{3,0}) = 6; & \\
f(v_{4,0}) = A_3 + b + 4 = 2A_3 & \\
f(v_{1,i}) = 2i - 1 & 1 \le i \le a_1 \\
f(v_{2,i}) = 2A_1 + 2i - 1 & 1 \le i \le a_2 \\
f(v_{3,i}) = 2A_2 + 2i - 1 & 1 \le i \le a_3 \\
f(v_{4,i}) = 2i + 6 & 1 \le i \le b
\end{array}$$

The corresponding edge labels are as follows:

The edge label of $v_{1,0}v_{1,i}$ is $1 + i$ for $1 \le i \le a_1$ (edge labels are $2, 3, \cdots, a_1 + 1 = A_1 + 1$), $v_{2,0}v_{2,i}$ is $A_1 + 2 + i$ for $1 \le i \le a_2$ (edge labels are

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$A_1 + 3, A_1 + 4, \cdots, A_2 + 2)$, $v_{3,0}v_{3,i}$ is $A_2 + 3 + i$ for $1 \le i \le a_3$ (edge labels are $A_2 + 4, A_2 + 5, ..., A_2 + a_3 + 3 = A_3 + 3)$, $v_{4,0}v_{4,i}$ is $A_3 + 3 + i$ for $1 \le i \le b = A_3 - 4$ (edge labels are $A_3 + 4,\ A_3 + 5, \cdots, A_3 + 3 + b = A_3 + 3 + A_3 - 4 = 2A_3 - 1)$.
These induced edge labels of graph G are unique.
Hence G is a skolem mean graph.

Example 1.3

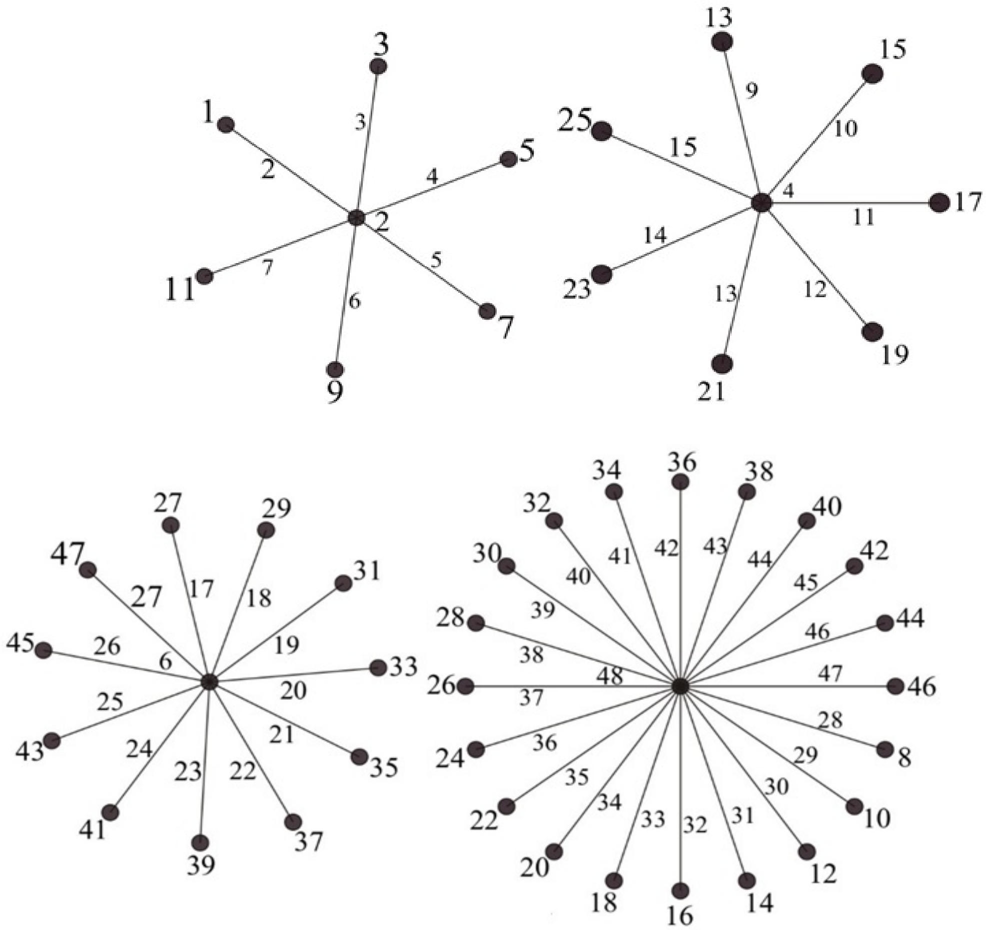

Figure 2: $K_{1,6} \cup K_{1,7} \cup K_{1,1} \cup K_{1,20}$

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