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A Study On Different Types Of Product Of Anti Fuzzy Graphs

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Abstract

In this article, we consider to obtain new anti fuzzy graph from given anti fuzzy graphs. Product of anti fuzzy graphs is an operation on anti fuzzy graphs that produce new anti fuzzy graph. We consider tensor, normal, modular anti fuzzy graphs products, which are adapted from fuzzy graphs products. In general, product of any two anti fuzzy graphs is an anti fuzzy graph. Product of any two strong anti fuzzy graphs is an anti fuzzy graph strong. Normal product of two anti fuzzy graph complete is a complete anti fuzzy graph. Other than that, different of anti fuzzy products are also discussed.

Key words: Anti fuzzy graph, tensor product, normal product, normal product

AMS classification: 46A45, 46B45.

1 Introduction

Fuzzy graphs are extensions of classical graphs that are developed based on concepts on fuzzy logic and fuzzy relations in fuzzy set theory. If in the classic graph each point and edges have a membership value of one or zero, then in the fuzzy graph, each point and its edges have a membership value at interval [0,1]. The membership value of each of these elements states the degree of membership of the element in a fuzzy graph. Since fuzzy graph was introduced, the researchers began to generalize and develop several studies in classical graphs into fuzzy graphs both in theory and application. In application, fuzzy graph can be said as a topic that can be applied in modern science and technology, especially in the fields of information theory, neural networks, cluster analysis, medical diagnosis, and control theory. Dey & Pal have applied fuzzy graphs to solve traffic light problems, and Swaminathan

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has applied fuzzy graphs to work allocation problems. Previously, Zadeh introduced several concepts related to fuzzy graphs such as the concept of connectedness. As results, more in-depth theoretical results about fuzzy graphs are resulted, including who introduced the concepts of operations on fuzzy graphs, including joint operations, intersection, cartesian results, and compositions of two fuzzy graphs. Next, Nirmala & Vijaya determined a degree of point in cartesian product of two fuzzy graphs, tensor, normal product, and composition of two fuzzy graphs.

2 Preliminaries

Definition 2.1 ***(anti fuzzy graph)***. A anti fuzzy graph $G=(V,\sigma,\mu)$ where $\sigma: V\to[0,1]$ is fuzzy set in V and $\mu: V\times V\to[0,1]$ is anti fuzzy relation on E such that $\mu(uv)\geq\sigma(u)\vee\sigma(v)$ for all $uv\in E$. We note that μ is symmetric relation.

Definition 2.2 ***Fuzzy graph strong***. A fuzzy graph $G=(\sigma,\mu)$ is called fuzzy graph strong, when $\mu\left(v_iv_j\right)=\sigma\left(v_i\right)\wedge\sigma\left(v_j\right)$ for all $\left(v_i,v_j\right)\in E$ with $E\subseteq V\times V$.

Definition 2.3 ***Anti fuzzy graph strong***. A fuzzy graph $G=(\sigma,\mu)$ is called anti fuzzy graph strong, when $\mu\left(v_iv_j\right)=\sigma\left(v_i\right)\vee\sigma\left(v_j\right)$ for all $\left(v_i,v_j\right)\in E$ with $E\subseteq V\times V$.

Definition 2.4 ***Fuzzy graph complete***. A fuzzy graph $G=(\sigma,\mu)$ is called fuzzy graph complete, when $\mu\left(v_iv_j\right)=\sigma\left(v_i\right)\vee\sigma\left(v_j\right)$ for all $\left(v_i,v_j\right)\in V\times V$.

Definition 2.5 ***Anti fuzzy graph complete***. A fuzzy graph $G=(\sigma,\mu)$ is called anti fuzzy graph complete, when $\mu\left(v_iv_j\right)=\sigma\left(v_i\right)\vee\sigma\left(v_j\right)$ for all $\left(v_i,v_j\right)\in V\times V$.

Definition 2.6 ***Anti fuzzy graph complement.*** Let $G=(\sigma,\mu)$ is an anti fuzzy graph. Anti fuzzy graph complement of $G=(\sigma,\mu)$ is $G^c=(\sigma^c,\mu^c)$, where $\sigma^c=\sigma$ and $\mu^c\left(v_iv_j\right)=\mu\left(v_iv_j\right)-\sigma\left(v_i\right)\vee\sigma\left(v_j\right)$ for all $v_i,v_j\in V$.

Definition 2.7 ***Anti fuzzy graph null***. Let $G=(\sigma,\mu)$ be an anti fuzzy graph. If $d_G(u)=0$ for all $u\in V$, then $G=(\sigma,\mu)$ is called anti fuzzy graph null.

Lemma 2.8 Let $G=(\sigma,\mu)$ is an anti fuzzy graph complete, then $G^c=(\sigma^c,\mu^c)$ is anti fuzzy graph null.

Proof: It is sufficient to proof that $d_G c(u) = 0, \forall u \in V$. By definition of anti fuzzy graph null, then $\mu\left(v_i v_j\right) = \sigma\left(v_i\right) \vee \sigma\left(v_j\right)$ for all $v_i, v_j \in V$. Therefore $\mu^c\left(v_i, v_j\right) = \mu\left(v_i v_j\right) - \sigma\left(v_i\right) \vee \sigma\left(v_j\right) = \mu\left(v_i v_j\right) - \mu\left(v_i v_j\right) = 0$. Hence $d_{G_A c}(v) = \sum \mu^c\left(v_i, v_j\right) = 0$

Lemma 2.9 Let $G = (\sigma, \mu)$ is an anti fuzzy graph strong, then $G^c = (\sigma^c, \mu^c)$ is anti fuzzy graph strong.
Proof: By definition of anti fuzzy graph strong, then there is exist $v_p v_q \in E^c$ and $v_p v_q \notin E$ for some $v_p, v_q \in V$. Hence $E^c \neq \emptyset$ and $E^c \subseteq V \times V$.

3 Tensor Product of Anti Fuzzy Graph

Definition 3.1 ***Tensor product of anti fuzzy graph.*** Let $G_1 = (\sigma_1, \mu_1)$ dan $G_2 = (\sigma_2, \mu_2)$ be two anti fuzzy graphs with underlying vertex sets V_1 and V_2 and edge sets $\mathscr{E}_1$ and $\mathscr{E}_2$ respectively. Then tensor product of G_1 and G_2 (denote by $\mathrm{G}_1 \otimes \mathrm{G}_2$) is a pair of functions $(\sigma_1 \otimes \sigma_2, \mu_1 \otimes \mu_2)$ with underlying vertex set $V_1 \otimes V_2 = \left\{\left(u_i, v_j\right) \mid u_i \in V_1, v_j \in V_2\right\}$ with

$$(\sigma_1 \otimes \sigma_2)\left(u_i, v_j\right) = \sigma_1\left(u_i\right) \vee \sigma_2\left(v_j\right)$$

where $u_i \in V_1$ and $v_j \in V_2$. And underlying edge set $\varepsilon_1 \otimes \varepsilon_2 = \left\{\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right) \mid u_i u_k \in \varepsilon_1, v_j v_l \in \varepsilon_2\right\}$, with

$$(\mu_1 \otimes \mu_2)\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right) = \mu_1\left(u_i u_k\right) \vee \mu_2\left(v_j v_l\right) \text{ if } u_i u_k \in \varepsilon_1, v_j v_l \in \varepsilon_2.$$

Using above Definition, the following properties can be proved. We apply tensor product on an arbitrary anti fuzzy graphs in Theorem 3.2, on anti fuzzy graphs strong in Theorem 3.3, on anti fuzzy graphs complete in Theorem 3.4, and on anti fuzzy graphs complement in Theorem 3.5 and Theorem 3.6.

Theorem 3.2 Tensor product of two anti fuzzy graphs is an anti fuzzy graph.
Proof: Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ are two an arbitrary of anti fuzzy graph. Then by Definition, let $\left(u_i, v_j\right) \in V_1 \otimes V_2$ be an arbitrary, then

$$(\sigma_1 \otimes \sigma_2)\left(u_i, v_j\right) = \sigma_1\left(u_i\right) \vee \sigma_2\left(v_j\right)$$

Since $\sigma_1\left(u_i\right), \sigma_2\left(v_j\right) \in [0,1]$, then $\sigma_1\left(u_i\right) \vee \sigma_2\left(v_j\right) \in [0,1]$. Hence $(\sigma_1 \otimes \sigma_2)$ is fuzzy on $V_1 \otimes V_2$. Let $\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right) \in \mathscr{E}_1 \otimes \mathscr{E}_2$ with $u_i u_k \in \varepsilon_1, v_j v_l \in \mathscr{E}_2$. Then $\left(\mu_1 \otimes \mu_2\right)\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right)=\mu_1\left(u_i u_k\right) \vee \mu_2\left(v_j v_l\right)$.

Since

$$\mu_1\left(u_i u_k\right) \geq \sigma_1\left(u_i\right) \vee \sigma_1\left(u_k\right)$$

and

$$\mu_2\left(v_j u_l\right) \geq \sigma_2\left(u_j\right) \vee \sigma_2\left(u_l\right)$$

Then

$$\begin{aligned}\left(\mu_1 \otimes \mu_2\right)\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right) & \geq\left(\sigma_1\left(u_i\right) \vee \sigma_1\left(u_k\right)\right) \vee\left(\sigma_2\left(u_j\right) \vee \sigma_2\left(u_l\right)\right) \\ & =\left(\sigma_1\left(u_i\right) \vee \sigma_2\left(u_j\right)\right) \vee\left(\sigma_1\left(u_k\right) \vee \sigma_2\left(u_l\right)\right)\end{aligned}$$

or

$$\left(\mu_1 \otimes \mu_2\right)\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right) \geq\left(\sigma_1 \otimes \sigma_2\right)\left(u_i, v_j\right) \vee\left(\sigma_1 \otimes \sigma_2\right)\left(u_k, v_l\right)$$

Theorem 3.3 Tensor product of two anti fuzzy graphs strong is an anti fuzzy graph strong.

Proof: Let $G_1=\left(\sigma_1, \mu_1\right)$ and $G_2=\left(\sigma_2, \mu_2\right)$ are two an arbitrary of anti fuzzy graphs strong. Then by Definition, let $\left(u_i, v_j\right) \in V_1 \otimes V_2$ be an arbitrary, then $\left(\sigma_1 \otimes \sigma_2\right)\left(u_i, v_j\right)=\sigma_1\left(u_i\right) \vee \sigma_2\left(v_j\right)$. Since $\sigma_1\left(u_i\right), \sigma_2\left(v_j\right) \in[0,1]$, then $\sigma_1\left(u_i\right) \vee \sigma_2\left(v_j\right) \in[0,1]$. Hence $\left(\sigma_1 \otimes \sigma_2\right)$ is fuzzy on $V_1 \otimes V_2$. Let $\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right) \in \mathscr{E}_1 \otimes \mathscr{E}_2$ with $u_i u_k \in \varepsilon_1, v_j v_l \in \varepsilon_2$. Then $\left(\mu_1 \otimes \mu_2\right)\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right)=\mu_1\left(u_i u_k\right) \vee \mu_2\left(v_j v_l\right)$, by Definition.

Since $G_1=\left(\sigma_1, \mu_1\right)$ and $G_2=\left(\sigma_2, \mu_2\right)$ are anti fuzzy graphs strong, then $\mu_1\left(u_i u_k\right)=\sigma_1\left(u_i\right) \vee \sigma_1\left(u_k\right)$ and $\mu_2\left(v_j u_l\right)=\sigma_2\left(u_j\right) \vee \sigma_2\left(u_l\right)$. Therefore

$$\begin{aligned}\left(\mu_1 \otimes \mu_2\right)\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right) & =\left(\sigma_1\left(u_i\right) \vee \sigma_1\left(u_k\right)\right) \vee\left(\sigma_2\left(u_j\right) \vee \sigma_2\left(u_l\right)\right) \\ & =\left(\sigma_1\left(u_i\right) \vee \sigma_2\left(u_j\right)\right) \vee\left(\sigma_1\left(u_k\right) \vee \sigma_2\left(u_l\right)\right)\end{aligned}$$

or

$$\left(\mu_1 \otimes \mu_2\right)\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right)=\left(\sigma_1 \otimes \sigma_2\right)\left(u_i, v_j\right) \vee\left(\sigma_1 \otimes \sigma_2\right)\left(u_k, v_l\right)$$

Theorem 3.4 Tensor product of two anti fuzzy graphs complete is an anti fuzzy graph strong.

Proof: Let $G_1=\left(\sigma_1, \mu_1\right)$ and $G_2=\left(\sigma_2, \mu_2\right)$ are two an arbitrary of an anti fuzzy graphs complete. Then by Lemma, G_1 and G_1 are an anti fuzzy graphs strong. Therefore, by Theorem 3.3, $\mathrm{G}_1 \otimes \mathrm{G}_2$ is an anti graph strong.

Theorem 3.5 Tensor product of an anti fuzzy graphs complete and its complement is an anti fuzzy graph null.
Proof: Let $G=(\sigma,\mu)$ is an anti fuzzy graph complete, by Lemma $G^c=(\sigma^c,\mu^c)$ is anti fuzzy graph null. Next, by definition of anti fuzzy graph null, $E^C=\emptyset$. Consequently in the tensor product definition, we obtain $(\mu_1\otimes\mu_2)((u_1,v_1)(u_2,v_2))=0, u_1u_2\in E$ dan $v_1v_2\in E^c$.

Theorem 3.6 Tensor product of an anti fuzzy graphs strong and its complement is an anti fuzzy graph strong.
Proof: Let $G=(\sigma,\mu)$ is an anti fuzzy graph strong, then directly by Theorem 3.3.

4 Normal Product of Anti Fuzzy Graph

Definition 4.1 Normal product of anti fuzzy graph. Let $G_1=(\sigma_1,\mu_1)$ dan $G_2=(\sigma_2,\mu_2)$ be two anti fuzzy graphs with underlying vertex sets V_1 and V_2 and edge sets ε_1 and ε_2 respectively. Then normal product of G_1 and G_2 (denote by $\mathrm{G}_1\triangle\mathrm{G}_2$) is a pair of functions $(\sigma_1\triangle\sigma_2,\mu_1\triangle\mu_2)$ with underlying vertex set $V_1\triangle V_2=\left\{\left(u_i,v_j\right)\mid u_i\in V_1, v_j\in\ V_2\right\}$ with

$$(\sigma_1\triangle\sigma_2)\left(u_i,v_j\right)=\sigma_1\left(u_i\right)\vee\sigma_2\left(v_j\right)$$

where $u_i\in V_1$ and $v_j\in V_2$. And underlying edge set $\mathscr{E}_1\triangle\mathscr{E}_2=\left\{\left(\left(u_i,v_j\right)\left(u_k,v_l\right)\right)\mid u_i=u_k, v_jv_l\in\mathscr{E}_2 \text{ or } v_j=\ v_l, u_iu_k\in\mathscr{E}_1,\right\}$, with

$$(\mu_1\triangle\mu_2)\left(\left(u_i,v_j\right)\left(u_k,v_l\right)\right)=\begin{cases}\sigma_1\left(u_i\right)\vee\mu_2\left(v_jv_l\right), & \text{if } u_i=u_k \text{ and } v_jv_l\in\mathscr{E}_2\\ \mu_1\left(u_iu_k\right)\vee\sigma_2\left(v_j\right), & \text{if } u_iu_k\in\mathscr{E}_1 \text{ and } v_j=v_l\\ \mu_1\left(u_iu_k\right)\vee\mu_2\left(v_jv_l\right), & \text{if } u_iu_k\in\mathscr{E}_1 \text{ and } v_jv_l\in\mathscr{E}_2\end{cases}$$

Theorem 4.2 Normal product of two anti fuzzy graphs is an anti fuzzy graph.
Proof: Let $G_1=(\sigma_1,\mu_1)$ and $G_2=(\sigma_2,\mu_2)$ are two an arbitrary of anti fuzzy graph. Then by Definition, let $\left(u_i,v_j\right)\in V_1\triangle V_2$ be an arbitrary, then

$$(\sigma_1\triangle\sigma_2)\left(u_i,v_j\right)=\sigma_1\left(u_i\right)\vee\sigma_2\left(v_j\right)$$

Since $\sigma_1\left(u_i\right),\sigma_2\left(v_j\right)\in[0,1]$, then $\sigma_1\left(u_i\right)\vee\sigma_2\left(v_j\right)\in[0,1]$. Hence $(\sigma_1\triangle\sigma_2)$ is fuzzy on $V_1\triangle V_2$. Let $\left(\left(u_i,v_j\right)\left(u_k,v_l\right)\right)\in\mathscr{E}_1\triangle\mathscr{E}_2$

Case 1 if $u_i = u_k$ and $v_j v_l \in \mathscr{E}_2$. Then

$$\begin{aligned}(\mu_1 \triangle \mu_2)\left(\left(u_i, v_j\right)(u_k, v_l)\right) &= \sigma_1(u_i) \vee \mu_2\left(v_j v_l\right)\\ &= \sigma_1(u_i) \vee \sigma_1(u_k) \vee \mu_2\left(v_j v_l\right)\\ &\geq \sigma_1(u_i) \vee \sigma_1(u_k) \vee \sigma_2\left(v_j\right) \vee \sigma_2(v_l)\end{aligned}$$

Next by associative property of real number
$\sigma_1(u_i) \vee \sigma_1(u_k) \vee \sigma_2\left(v_j\right) \vee \sigma_2(v_l) = \left(\sigma_1(u_i) \vee \sigma_2\left(v_j\right)\right) \vee (\sigma_1(u_k) \vee \ \sigma_2(v_l))$.

Therefore $(\mu_1 \triangle \mu_2)\left(\left(u_i, v_j\right)(u_k, v_l)\right) \geq (\sigma_1 \triangle \sigma_2)\left(u_i, v_j\right) \vee (\sigma_1 \triangle \sigma_2)(u_k, v_l)$.
Case 2 if $u_i u_k \in \mathscr{E}_1$ and $v_j = v_l$. Then

$$\begin{aligned}(\mu_1 \triangle \mu_2)\left(\left(u_i, v_j\right)(u_k, v_l)\right) &= \sigma_2\left(v_j\right) \vee \mu_1(u_i u_k)\\ &= \sigma_2\left(v_j\right) \vee \sigma_2(v_l) \vee \mu_1(u_i u_k)\\ &\geq \sigma_2\left(v_j\right) \vee \sigma_2(v_l) \vee \sigma_1(u_i) \vee \sigma_1(u_k)\end{aligned}$$

Next by associative property of real number $\sigma_2\left(v_j\right) \vee \sigma_2(v_l) \vee \sigma_1(u_i) \vee \sigma_1(u_k) = \left(\sigma_1(u_i) \vee \sigma_2\left(v_j\right)\right) \vee (\sigma_1(u_k) \vee \ \sigma_2(v_l))$. Therefore $(\mu_1 \triangle \mu_2)\left(\left(u_i, v_j\right)(u_k, v_l)\right) \geq (\sigma_1 \triangle \sigma_2)\left(u_i, v_j\right) \vee (\sigma_1 \triangle \sigma_2)(u_k, v_l)$.
Case 3 if $u_i u_k \in \mathscr{E}_1$ and $v_j v_l \in \mathscr{E}_2$. Then

$$\begin{aligned}(\mu_1 \triangle \mu_2)\left(\left(u_i, v_j\right)(u_k, v_l)\right) &= \mu_1(u_i u_k) \vee \mu_2\left(v_j v_l\right)\\ &\geq \sigma_1(u_i) \vee \sigma_1(u_k) \vee \sigma_2\left(v_j\right) \vee \sigma_2(v_l)\\ &= \left(\sigma_1(u_i) \vee \sigma_2\left(v_j\right)\right) \vee (\sigma_1(u_k) \vee \sigma_2(v_l))\end{aligned}$$

Therefore $(\mu_1 \triangle \mu_2)\left(\left(u_i, v_j\right)(u_k, v_l)\right) \geq (\sigma_1 \triangle \sigma_2)\left(u_i, v_j\right) \vee (\sigma_1 \triangle \sigma_2)(u_k, v_l)$

Theorem 4.3 Normal product of two anti fuzzy graphs strong is an anti fuzzy graph strong.
Proof: Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ are two an arbitrary of anti fuzzy graph

strong. Then by Definition, let $\left(u_i, v_j\right) \in V_1 \triangle V_2$ be an arbitrary, then

$$(\sigma_1 \Delta \sigma_2)\left(u_i, v_j\right) = \sigma_1\left(u_i\right) \vee \sigma_2\left(v_j\right)$$

Since $\sigma_1\left(u_i\right), \sigma_2\left(v_j\right) \in [0,1]$, then $\sigma_1\left(u_i\right) \vee \sigma_2\left(v_j\right) \in [0,1]$. Hence $(\sigma_1 \Delta \sigma_2)$ is fuzzy on $V_1 \triangle V_2$. Let $\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right) \in \mathscr{E}_1 \triangle \mathscr{E}_2$

Case 1 if $u_i = u_k$ and $v_j v_l \in \mathscr{E}_2$. Then

$$\begin{aligned}
(\mu_1 \triangle \mu_2)\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right) &= \sigma_1\left(u_i\right) \vee \mu_2\left(v_j v_l\right) \\
&= \sigma_1\left(u_i\right) \vee \sigma_1\left(u_k\right) \vee \mu_2\left(v_j v_l\right) \\
&= \sigma_1\left(u_i\right) \vee \sigma_1\left(u_k\right) \vee \sigma_2\left(v_j\right) \vee \sigma_2\left(v_l\right)
\end{aligned}$$

Next by associative property of real number $\sigma_1\left(u_i\right) \vee \sigma_1\left(u_k\right) \vee \sigma_2\left(v_j\right) \vee \sigma_2\left(v_l\right) = \left(\sigma_1\left(u_i\right) \vee \sigma_2\left(v_j\right)\right) \vee \left(\sigma_1\left(u_k\right) \vee \sigma_2\left(v_l\right)\right)$. Therefore $(\mu_1 \triangle \mu_2)\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right) = (\sigma_1 \triangle \sigma_2)\left(u_i, v_j\right) \vee (\sigma_1 \triangle \sigma_2)\left(u_k, v_l\right)$.

Case 2 if $u_i u_k \in \mathscr{E}_1$ and $v_j = v_l$. Then

$$\begin{aligned}
(\mu_1 \triangle \mu_2)\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right) &= \sigma_2\left(v_j\right) \vee \mu_1\left(u_i u_k\right) \\
&= \sigma_2\left(v_j\right) \vee \sigma_2\left(v_l\right) \vee \mu_1\left(u_i u_k\right) \\
&= \sigma_2\left(v_j\right) \vee \sigma_2\left(v_l\right) \vee \sigma_1\left(u_i\right) \vee \sigma_1\left(u_k\right)
\end{aligned}$$

Next by associative property of real number $\sigma_2\left(v_j\right) \vee \sigma_2\left(v_l\right) \vee \sigma_1\left(u_i\right) \vee \sigma_1\left(u_k\right) = \left(\sigma_1\left(u_i\right) \vee \sigma_2\left(v_j\right)\right) \vee \left(\sigma_1\left(u_k\right) \vee \sigma_2\left(v_l\right)\right)$. Therefore $(\mu_1 \triangle \mu_2)\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right) = (\sigma_1 \triangle \sigma_2)\left(u_i, v_j\right) \vee (\sigma_1 \triangle \sigma_2)\left(u_k, v_l\right)$.

Case 3 if $u_i u_k \in \varepsilon_1$ and $v_j v_l \in \varepsilon_2$. Then

$$\begin{aligned}
(\mu_1 \triangle \mu_2)\left(\left(u_i, v_j\right)\left(u_k, v_l\right)\right) &= \mu_1\left(u_i u_k\right) \vee \mu_2\left(v_j v_l\right) \\
&= \sigma_1\left(u_i\right) \vee \sigma_1\left(u_k\right) \vee \sigma_2\left(v_j\right) \vee \sigma_2\left(v_l\right) \\
&= \left(\sigma_1\left(u_i\right) \vee \sigma_2\left(v_j\right)\right) \vee \left(\sigma_1\left(u_k\right) \vee \sigma_2\left(v_l\right)\right)
\end{aligned}$$

Therefore $(\mu_1 \triangle \mu_2)\left(\left(u_i, v_j\right)(u_k, v_l)\right) = (\sigma_1 \triangle \sigma_2)\left(u_i, v_j\right) \vee (\sigma_1 \triangle \sigma_2)(u_k, v_l)$.

5 Conclusion

In this paper we have discussed about the fuzzy graphs and anti fuzzy graph. Also we have discussed about the tensor product of anti fuzzy graphs, normal product of anti fuzzy graphs.

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