

A Study On A Solution Approach To Fuzzy Linear Frctional Programming Problems

Mohamed Assarudeen SN¹ and Devi M²

Received: 25 April 2023/ Accepted: 25 May 2023/ Published online: 19 June 2023

©Sacred Heart Research Publications 2017

Abstract

In this paper, we propose a method of solving the fully fuzzy linear fractional programming problems, Express all the parameters and variables are triangular fuzzy numbers. Convert all the triangular fuzzy numbers in their parametric form, we convert the fractional programming problem in to a single objective linear programming problem in parametric form. We put new fuzzy arithmetic and fuzzy ranking, we obtain the optimal solution the given fully fuzzy linear fractional programming problem without converting to its equivalent crisp linear programming problem. A numerical example is provided to illustrate the efficiency of the proposed method.

Key words: fractional programming, triangular fuzzy numbers, parametric form, fuzzy arithmetic, fuzzy ranking.

AMS classification: 03B52, 90C05, 246636

1 Introduction

Linear fractional programming is a ratio of two linear functions, is optimized. Its applications is used several fields such as production, financial, corporate, health care etc. Bellman and Zadeh proposed the concept of decision making in a fuzzy environment. Subsequently, Many authors discussed the FLFPP. e.g. Chakraborty and Gupta, Li , Mehlawat et.al, Mitlif et.al, Muruganandam et.al, Nachammai et.al, Pop et.al, Sanjay Jain et.al, Stanojevic et. al, Das et al. have proposed more methods for solving FLFPP. Safaei et.alsolving FLP problems with fuzzy goal and fuzzy constraints in two-dimensional space by geometric approach. Pop and Stancu Minasian represent the variables are triangular fuzzy numbers and solving FFLFP problems.

In this paper, we consider the fully fuzzy linear fractional programming problem (FFLFP). First, the FFLFP problem is reduced into a fully fuzzy linear

¹Assistant professor, Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi 635 752.
Email:assarudeen@gmail.com

²Research scholar, Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi 635 752.
Email:devimanivannan77@gmail.com

programming (FFLP) problem and then expressed in its parametric form. We propose a simplex type algorithm for the solution of fully fuzzy linear programming problem without changing to an equivalent crisp problem. In section 2, we give some basic definitions and notations along with some preliminary results to be used in subsequent sections. In section 3, we talk about fully fuzzy linear fractional programming problem and present details of the solutions method to solve it. In section 4, we present a numerical example to illustrate the efficiency of the solution approach developed in this paper.

2 Preliminaries

Definition 2.1 A fuzzy set a defined on the set of real numbers R is said to be a fuzzy number, if its membership function $\tilde{a} : R \rightarrow [0, 1]$ has the following characteristics:

- (i) \tilde{a} is convex, (i.e.) $\tilde{a}(\lambda x_1 + (1 - \lambda)x_2) \geq \min \{\tilde{a}(x_1), \tilde{a}(x_2)\}, \lambda \in [0, 1],$ for all $x_1, x_2 \in R$
- (ii) \tilde{a} is normal, (i.e.) there exists an $x \in R$ such that $\tilde{a}(x) = 1$
- (iii) \tilde{a} is piecewise continuous.

Definition 2.2 A fuzzy number \tilde{a} on R is a triangular fuzzy number if its membership function $\tilde{a} : R \rightarrow [0, 1]$ has the following characteristics:

$$\tilde{a}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

We denote this triangular fuzzy number as $\tilde{a} = (a_1, a_2, a_3)$. We use $F(R)$ to denote the set of all triangular fuzzy numbers defined on R .

Definition 2.3 A triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3) \in F(R)$ can also be represented as an pair $\tilde{a} = (\underline{a}, \bar{a})$ of functions $\underline{a}(r), \bar{a}(r)$, for $0 \leq r \leq 1$ which satisfies the following requirements:

- (i) $\underline{a}(r)$ is a bounded monotonic increasing left continuous function.
- (ii) $\bar{a}(r)$ is a bounded monotonic decreasing left continuous function.

(iii) $\underline{a}(r) \leq \bar{a}(r), 0 \leq r \leq 1$

It is also represented by $\tilde{a} = (a_0, a_*, a^*)$ where $a_* = (a_0 - \underline{a}), a^* = (\bar{a} - a_0)$ are called the left fuzziness index function and the right fuzziness index function respectively. For an arbitrary triangular fuzzy number $\tilde{a} = (\underline{a}, \bar{a})$ the number $a_0 = \left(\frac{a(1) + \bar{a}(1)}{2} \right)$ is said to be a location index number of \tilde{a} .

2.1 Ranking of Triangular Fuzzy Numbers

Several approaches for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for comparing the fuzzy numbers is by the use of a ranking function based on their graded means. We define the magnitude of the triangular fuzzy number \tilde{a} by

$$R(\tilde{a}) = \left(\frac{a^* + 4a_0 - a_*}{4} \right) = \left(\frac{\underline{a} + \bar{a} + a_0}{4} \right)$$

Consider any two triangular fuzzy numbers

$$\tilde{a} = (a_1, a_2, a_3) = (a_0, a_*, a^*) \text{ and } \tilde{b} = (b_1, b_2, b_3) = (b_0, b_*, b^*) \text{ in } F(\mathbb{R}) \text{ we have}$$

- (i)** $\tilde{a} \succeq \tilde{b}$ if and only if $R(\tilde{a}) \geq R(\tilde{b})$
- (ii)** $\tilde{a} \preceq \tilde{b}$ if and only if $R(\tilde{a}) \leq R(\tilde{b})$
- (iii)** $\tilde{a} \approx \tilde{b}$ if and only if $R(\tilde{a}) = R(\tilde{b})$

2.2 Arithmetic Operations of TrFN

Ming Ma et. al., analyzed fuzzy computation based on location index, fuzziness index functions. The location index number used ordinary computation and the fuzziness index functions used the lattice guidelines which is least upper bound in the lattice L. let $\tilde{a}, \tilde{b} \in L$, we derive $\tilde{a} \vee \tilde{b} = \max\{\tilde{a}, \tilde{b}\}$ and $\tilde{a} \wedge \tilde{b} = \min\{\tilde{a}, \tilde{b}\}$. If any two triangular fuzzy number $\tilde{a} = (a_0, a_*, a^*), \tilde{b} = (b_0, b_*, b^*)$ and $\bullet = \{+, -, \times, \div\}$, the computation operations on the fuzzy numbers then

$$\tilde{a} \bullet \tilde{b} = (a_0, a_*, a^*) \bullet (b_0, b_*, b^*) = (a_0 \bullet b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$$

3 Fully fuzzy linear fractional programming problem (FFLFPP)

A general FFLFPP is given by

$$\begin{aligned} \max \tilde{z} &= \frac{\sum \tilde{c}_j \tilde{x}_j + \tilde{\alpha}}{\sum \tilde{d}_j \tilde{x}_j + \tilde{\beta}} \\ \text{Subject to } &\sum_{j=1}^n \tilde{a}_{ij} \tilde{x}_j \preceq \tilde{b}_i, \quad i = 1, 2, 3, \dots, m \\ \text{and } &\tilde{x}_j \succeq \tilde{0}, \quad \text{for all } j = 1, 2, 3, \dots, n. \end{aligned}$$

We assume that, $\tilde{a}_{ij}, \tilde{b}_i, \tilde{c}_j$, and \tilde{x}_j are TrFN for each $i = 1, 2, 3, \dots, m$ and $j = 1, 2, 3, \dots, n$. The general fully fuzzy linear fractional programming problem is expressed in matrix form as

$$\begin{aligned} \max \tilde{z} &= \frac{\tilde{\mathbf{c}}\tilde{\mathbf{x}} + \tilde{\alpha}}{\tilde{\mathbf{d}}\tilde{\mathbf{x}} + \tilde{\beta}} \\ \text{Subject to } &\tilde{\mathbf{A}}\tilde{\mathbf{x}} \preceq \tilde{\mathbf{b}}, \\ \text{and } &\tilde{\mathbf{x}} \succeq \tilde{\mathbf{0}}, \end{aligned}$$

where $\tilde{\mathbf{A}} = (\tilde{a}_{ij})_{(m \times n)}$, $\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \dots, \tilde{x}_n)$, $\tilde{\mathbf{b}} = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \dots, \tilde{b}_m)$ and $\tilde{\mathbf{c}} = (\tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \dots, \tilde{c}_n)$, $\tilde{\mathbf{d}} = (\tilde{d}_1, \tilde{d}_2, \tilde{d}_3, \dots, \tilde{d}_n)$, $\tilde{\alpha}, \tilde{\beta} \in F(\mathbf{R})$

It is assumed that the denominator is positive for all feasible solution.

3.1 Conversion of fuzzy linear fractional programming problem into fuzzy linear programming problem

We assume that the feasible region $\tilde{\mathbf{S}} = \{\tilde{\mathbf{x}} \in F^n(\mathbf{R}) : \tilde{\mathbf{A}}\tilde{\mathbf{x}} \preceq \tilde{\mathbf{b}}, \tilde{\mathbf{x}} \succeq \tilde{\mathbf{0}}\}$ is nonempty and bounded and the denominator $(\tilde{\mathbf{d}}\tilde{\mathbf{x}} + \tilde{\beta}) \succ \tilde{0}$.

If $(\tilde{\mathbf{d}}\tilde{\mathbf{x}} + \tilde{\beta}) \prec \tilde{0}$, then the condition $\frac{\tilde{\beta}(\tilde{\mathbf{A}}\tilde{\mathbf{x}} - \tilde{\mathbf{b}})}{\tilde{\beta}(\tilde{\mathbf{d}}\tilde{\mathbf{x}} + \tilde{\beta})} \preceq \tilde{0}$ will not hold. As a result solution to the LFP cannot be found.

Now we can convert the above LFP into an LP in the following way assuming that $\tilde{\beta} \neq \tilde{0}$.

3.2 Transformation of the objective function

Multiplying both the denominator and the numerator of $\max \tilde{z} = \frac{\tilde{\mathbf{c}}\tilde{\mathbf{x}} + \tilde{\alpha}}{\tilde{\mathbf{d}}\tilde{\mathbf{x}} + \tilde{\beta}}$ by $\tilde{\beta}$, where $\tilde{\beta} \neq \tilde{0}$.

We have

$$\begin{aligned} \tilde{z} &= \frac{\tilde{c}\tilde{x}\tilde{\beta} + \tilde{\alpha}\tilde{\beta}}{\tilde{d}\tilde{x}\tilde{\beta} + \tilde{\beta}\tilde{\beta}} = \frac{\tilde{c}\tilde{x}\tilde{\beta} + \tilde{\alpha}\tilde{\beta}}{\tilde{\beta}(\tilde{d}\tilde{x} + \tilde{\beta})} = \frac{\tilde{c}\tilde{x}\tilde{\beta} - \tilde{d}\tilde{x}\tilde{\alpha} + \tilde{d}\tilde{x}\tilde{\alpha} + \tilde{\alpha}\tilde{\beta}}{\tilde{\beta}(\tilde{d}\tilde{x} + \tilde{\beta})} \\ &= \frac{(\tilde{c}\tilde{\beta} - \tilde{d}\tilde{\alpha})\tilde{x} + (\tilde{d}\tilde{x} + \tilde{\beta})\tilde{\alpha}}{\tilde{\beta}(\tilde{d}\tilde{x} + \tilde{\beta})} = \left(\tilde{c} - \tilde{d}\frac{\tilde{\alpha}}{\tilde{\beta}}\right) \frac{\tilde{x}}{(\tilde{d}\tilde{x} + \tilde{\beta})} + \frac{\tilde{\alpha}}{\tilde{\beta}} = \tilde{p}\tilde{y} + \tilde{g} \end{aligned}$$

where $\tilde{p} = \left(\tilde{c} - \tilde{d}\frac{\tilde{\alpha}}{\tilde{\beta}}\right)$, $\tilde{y} = \frac{\tilde{x}}{(\tilde{d}\tilde{x} + \tilde{\beta})}$ and $\tilde{g} = \frac{\tilde{\alpha}}{\tilde{\beta}}$.

Hence $F(y) = \tilde{p}\tilde{y} + \tilde{g}$.

3.3 Transformation of the constraints

From the constraint $\tilde{A}\tilde{x} \preceq \tilde{b}$, we have,

$$\begin{aligned} \tilde{A}\tilde{x} - \tilde{b} \preceq \tilde{0} &\Rightarrow \frac{\tilde{\beta}(\tilde{A}\tilde{x} - \tilde{b})}{\tilde{\beta}(\tilde{d}\tilde{x} + \tilde{\beta})} \preceq \tilde{0} \\ &\Rightarrow \frac{(\tilde{A}\tilde{\beta}\tilde{x} - \tilde{b}\tilde{\beta})}{\tilde{\beta}(\tilde{d}\tilde{x} + \tilde{\beta})} \preceq \tilde{0} \Rightarrow \frac{(\tilde{A}\tilde{\beta}\tilde{x} + \tilde{b}\tilde{d}\tilde{x} - \tilde{b}\tilde{d}\tilde{x} - \tilde{b}\tilde{\beta})}{\tilde{\beta}(\tilde{d}\tilde{x} + \tilde{\beta})} \preceq \tilde{0} \\ &\Rightarrow \frac{(\tilde{A}\tilde{\beta}\tilde{x} + \tilde{b}\tilde{d}\tilde{x})}{\tilde{\beta}(\tilde{d}\tilde{x} + \tilde{\beta})} - \frac{(\tilde{b}\tilde{d}\tilde{x} + \tilde{b}\tilde{\beta})}{\tilde{\beta}(\tilde{d}\tilde{x} + \tilde{\beta})} \preceq \tilde{0} \\ &\Rightarrow \frac{\tilde{\beta} \left(\tilde{A} + \frac{\tilde{b}}{\tilde{\beta}}\tilde{d}\right) \tilde{x}}{\tilde{\beta}(\tilde{d}\tilde{x} + \tilde{\beta})} - \frac{\tilde{b}(\tilde{d}\tilde{x} + \tilde{\beta})}{\tilde{\beta}(\tilde{d}\tilde{x} + \tilde{\beta})} \preceq \tilde{0} \Rightarrow \left(\tilde{A} + \frac{\tilde{b}}{\tilde{\beta}}\tilde{d}\right) \frac{\tilde{x}}{(\tilde{d}\tilde{x} + \tilde{\beta})} - \frac{\tilde{b}}{\tilde{\beta}} \preceq \tilde{0} \\ &\Rightarrow \left(\tilde{A} + \frac{\tilde{b}}{\tilde{\beta}}\tilde{d}\right) \frac{\tilde{x}}{(\tilde{d}\tilde{x} + \tilde{\beta})} \preceq \frac{\tilde{b}}{\tilde{\beta}} \\ &\Rightarrow \tilde{G}\tilde{y} \preceq \tilde{h} \text{ where } \tilde{G} = \left(\tilde{A} + \frac{\tilde{b}}{\tilde{\beta}}\tilde{d}\right), \tilde{y} = \frac{\tilde{x}}{(\tilde{d}\tilde{x} + \tilde{\beta})} \text{ and } \tilde{h} = \frac{\tilde{b}}{\tilde{\beta}}. \end{aligned}$$

From the above equations we finally obtain the new LP form of the given LFP as follows:

$$\begin{aligned} \max F(y) &= \tilde{p}\tilde{y} + \tilde{g} \\ \text{subject to } \tilde{G}\tilde{y} &\preceq \tilde{h} \\ \text{and } \tilde{y} &\succeq \tilde{0}. \end{aligned}$$

4 Numerical Example

A company makes two products A and B. Added a permanent rate of around 1 rupee to the rate function.

Products	Profit(around)/ per unit	Rate (around) per unit	Row material(pound)	Man-hours(daily)
A	5	5	3	5 hours
B	3	2	5	2 hours
Available			15	10 hours

Compute how many products A and B should be produced in order to maximize the total profit.

Solution: The mathematical formulation of the FFLFPP is:

$$\begin{aligned} \max \tilde{z} &= \frac{5\tilde{x}_1 + 3\tilde{x}_2}{5\tilde{x}_1 + 2\tilde{x}_2 + \tilde{1}} \\ \text{subject to } &3\tilde{x}_1 + 5\tilde{x}_2 \leq \tilde{15} \\ &5\tilde{x}_1 + 2\tilde{x}_2 \leq \tilde{10} \\ &\tilde{x}_1, \tilde{x}_2 \geq \tilde{0}. \end{aligned}$$

Applying the proposed algorithm, the FFLFPP is transformed into an equivalent FFLPP as

$$\begin{aligned} \max \tilde{z} &= 5\tilde{y}_1 + 3\tilde{y}_2 \\ \text{subject to } &\tilde{78}\tilde{y}_1 + \tilde{35}\tilde{y}_2 \leq \tilde{15} \\ &\tilde{55}\tilde{y}_1 + \tilde{22}\tilde{y}_2 \leq \tilde{10} \\ &\tilde{y}_1, \tilde{y}_2 \geq \tilde{0}. \end{aligned}$$

That is $\max \tilde{z} = (3, 5, 7)\tilde{y}_1 + (2, 3, 4)\tilde{y}_2$
 subject to $(76, 78, 80)\tilde{y}_1 + (34, 35, 36)\tilde{y}_2 \leq (11, 15, 19)$
 $(53, 55, 57)\tilde{y}_1 + (21, 22, 23)\tilde{y}_2 \leq (9, 10, 11)$
 and $\tilde{y}_1, \tilde{y}_2 \geq \tilde{0}$

Express this FFLP in its parametric form as

$$\begin{aligned} \text{Max } \tilde{z} &= (5, 2 - 2r, 2 - 2r)\tilde{y}_1 + (3, 1 - r, 1 - r)\tilde{y}_2 \\ \text{subject to } &(78, 2 - 2r, 2 - 2r)\tilde{y}_1 + (35, 1 - r, 1 - r)\tilde{y}_2 \leq (15, 4 - 4r, 4 - 4r) \\ &(55, 2 - 2r, 2 - 2r)\tilde{y}_1 + (22, 1 - r, 1 - r)\tilde{y}_2 \leq (10, 1 - r, 1 - r) \\ &\text{and } \tilde{y}_1, \tilde{y}_2 \geq \tilde{0}. \end{aligned}$$

The standard fully fuzzy linear programming (FFLP) problem into its parametric form is $\text{Max } \tilde{z} = (5, 2 - 2r, 2 - 2r)\tilde{y}_1 + (3, 1 - r, 1 - r)\tilde{y}_2 + (1, 0, 0)\tilde{s}_1 + (1, 0, 0)\tilde{s}_2$

$$\text{subject to } (78, 2 - 2r, 2 - 2r)\tilde{y}_1 + (35, 1 - r, 1 - r)\tilde{y}_2 + (1, 0, 0)\tilde{s}_1 = (15, 4 - 4r, 4 - 4r)$$

$$(55, 2 - 2r, 2 - 2r)\tilde{y}_1 + (22, 1 - r, 1 - r)\tilde{y}_2 + (1, 0, 0)\tilde{s}_2 = (10, 1 - r, 1 - r)$$

$$\text{and } \tilde{y}_1, \tilde{y}_2, \tilde{s}_1, \tilde{s}_2 \geq \tilde{0}.$$

The initial fuzzy basic feasible solution is

$$\tilde{s}_1 = (15, 4 - 4r, 4 - 4r), \tilde{s}_2 = (10, 1 - r, 1 - r).$$

		\tilde{c}_j	$(5, 2 - 2r, 2 - 2r)$	$(3, 1 - r, 1 - r)$	$(0, 0, 0)$	$(0, 0, 0)$
\tilde{c}_B	\tilde{y}_B	\tilde{x}_B	\tilde{y}_1	\tilde{y}_2	\tilde{s}_1	\tilde{s}_2
$(0, 0, 0)$	\tilde{s}_1	$(15, 4 - 4r, 4 - 4r)$	$(78, 2 - 2r, 2 - 2r)$	$(35, 1 - r, 1 - r)$	$(1, 0, 0)$	$(0, 0, 0)$
$(0, 0, 0)$	\tilde{s}_2	$(10, 1 - r, 1 - r)$	$(55, 2 - 2r, 2 - 2r)$	$(22, 1 - r, 1 - r)$	$(0, 0, 0)$	$(1, 0, 0)$
		\tilde{z}_j	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
		$(\tilde{z}_i - \tilde{c}_i)$	$(-5, 2 - 2r, 2 - 2r)$	$(-3, 1 - r, 1 - r)$	$(0, 0, 0)$	$(0, 0, 0)$

Initial iteration

Here leaving variable is \tilde{s}_2 and entering variable is \tilde{y}_1 .

		\tilde{c}_j	$(5, 2 - 2r, 2 - 2r)$	$(3, 1 - r, 1 - r)$	$(0, 0, 0)$	$(0, 0, 0)$
\tilde{c}_B	\tilde{y}_B	\tilde{x}_B	\tilde{y}_1	\tilde{y}_2	\tilde{s}_2	\tilde{s}_2
$(3, 1 - r, 1 - r)$	\tilde{y}_2	$(0.43, 4 - 4r, 4 - 4r)$	$(2.23, 4 - 4r, 4 - 4r)$	$(1, 4 - 4r, 4 - 4r)$	$(0.03, 4 - 4r, 4 - 4r)$	$(0, 4 - 4r, 4 - 4r)$
$(0, 0, 0)$	\tilde{s}_2	$(0.57, 4 - 4r, 4 - 4r)$	$(5.97, 4 - 4r, 4 - 4r)$	$(0, 4 - 4r, 4 - 4r)$	$(-0.63, 4 - 4r, 4 - 4r)$	$(1, 4 - 4r, 4 - 4r)$
		\tilde{z}_j	$(6.69, 4 - 4r, 4 - 4r)$	$(3, 4 - 4r, 4 - 4r)$	$(0.09, 4 - 4r, 4 - 4r)$	$(0, 4 - 4r, 4 - 4r)$
		$(\tilde{z}_j - \tilde{c}_j)$	$(1.69, 4 - 4r, 4 - 4r)$	$(0, 4 - 4r, 4 - 4r)$	$(0.09, 4 - 4r, 4 - 4r)$	$(0, 4 - 4r, 4 - 4r)$

optimal iteration

Since all the $(\tilde{z}_j - \tilde{c}_j) \geq \tilde{0}$, current solution is optimal.

Hence we get $\tilde{y}_1 \approx \tilde{0}$ and $\tilde{y}_2 \approx (0.43, 4 - 4r, 4 - 4r)$

Next, by the proposed algorithm we find the values of \tilde{x}_1, \tilde{x}_2 using the transformation

$$(\tilde{x}_1, \tilde{x}_2) = \frac{(\tilde{y}_1, \tilde{y}_2)\tilde{\beta}}{1 - \tilde{d}(\tilde{y}_1, \tilde{y}_2)}$$

$$\text{That is } (\tilde{x}_1, \tilde{x}_2) = \frac{[(0, 0, 0), (0.43, 4 - 4r, 4 - 4r)](3, 1 - r, 1 - r)}{(0, 0, 0) - [(5, 1 - r, 1 - r)(2, 1 - r, 1 - r)][(0, 0, 0), (0.43, 4 - 4r, 4 - 4r)]}$$

$$\Rightarrow (\tilde{x}_1, \tilde{x}_2) = ((0, 4 - 4r, 4 - 4r), (3.07, 4 - 4r, 4 - 4r))$$

$$\Rightarrow \tilde{x}_1 = (0, 4 - 4r, 4 - 4r) \text{ and } \tilde{x}_2 = (3.07, 4 - 4r, 4 - 4r)$$

Substituting these values in the original linear fractional objective function, we have

$$\begin{aligned} \max \tilde{Z} &= \frac{\tilde{5}\tilde{x}_1 + \tilde{3}\tilde{x}_2}{\tilde{5}\tilde{x}_1 + 2\tilde{2}\tilde{x}_2 + \tilde{1}} = \frac{\tilde{5}(0, 4 - 4r, 4 - 4r) + \tilde{3}(3.07, 4 - 4r, 4 - 4r)}{\tilde{5}(0, 4 - 4r, 4 - 4r) + \tilde{2}(3.07, 4 - 4r, 4 - 4r) + \tilde{1}} = (1.29, 4 - 4r, 4 - 4r) \\ &\Rightarrow \max \tilde{Z} = (1.29, 4 - 4r, 4 - 4r) \end{aligned}$$

We get the optimal solution of the given problem as $\max \tilde{z} = (1.29, 4 - 4r, 4 - 4r)$ with $\tilde{X}_1 = (0, 4 - 4r, 4 - 4r)$ and $\tilde{x}_2 = (3.07, 4 - 4r, 4 - 4r)$

Value of r	max \tilde{z}	Value of r	max \tilde{z}	Value of r	max \tilde{z}
r = 0	(-2.71, 1.29, 5.29)	r = 0.72	(0.17, 1.29, 2.41)	r = 0.82	(0.57, 1.29, 2.01)
r = 0.25	(-1.71, 1.29, 4.29)	r = 0.74	(0.25, 1.29, 2.33)	r = 0.85	(0.69, 1.29, 1.89)
r = 0.5	(-0.71, 1.29, 3.29)	r = 0.75	(0.29, 1.29, 2.29)	r = 0.90	(0.89, 1.29, 1.69)
r = 0.68	(0.01, 1.29, 2.57)	r = 0.78	(0.41, 1.29, 2.17)	r = 0.95	(1.09, 1.29, 1.49)
r = 0.70	(0.09, 1.29, 2.49)	r = 0.80	(0.49, 1.29, 2.09)	r = 1	(1.29, 1.29, 1.29)

Optimal solution for different values of " r "

Methods	max \tilde{z}
veeramani et al.	(0.53, 1.27, 9)
Pop et al.	(0.4, 1.28, 11)
Stanojevic-Stancu Minasian method	(0.4, 1.3, 11)
Safaei method	(0.4, 1.28, 11)
Proposed Method	max \tilde{z} for $r \in [0.68, 1]$

Comparison with other methods

5 Conclusion

We have proposed a new method for the solution of fully fuzzy linear FPP without reducing to its equivalent crisp form. From table 3 , it is evident that the proposed method gives flexibility to the decision maker to choose his preferred solution whereas the other method does not give such facility to the decision maker. Also from table 4, we see that the proposed method gives vagueness reduced results comparing with other methods.

References

- [1] Bellman RE and Zadeh LA, Decision-making in a fuzzy environment. Management Science, 17(4), B-141,(1970).
- [2] Borza M,Rambely AS and Saraj M, Solving linear fractional programming with intervals coefficients in the objective function-A new approach , Applied Mathematical sciences, 6(69),3443 – 3452, (2012).
- [3] Chakraborty M and Gupta S, Fuzzy mathematical programming for multi-objective linear fractional programming problem. Fuzzy sets and systems, 125(3), 335-342, (2002).

- [4] Das SK and Edalatpanah SA, A general form of fuzzy linear fractional programs with trapezoidal fuzzy numbers. *International journal of data envelopment analysis and operations research*, 2(1), 16–19, (2016).
- [5] Das SK, Mandal T and Edalatpanah SA, A new approach for solving fully fuzzy linear fractional programming problems using the multi-objective linear programming. *RAIRO Operations Research* 51 (1), 285-297, (2017).
- [6] Das SK, Edalatpanah SA and Mandal T, A proposed model for solving fuzzy linear fractional programming problem: numerical point of view, *Journal of Computational Science*, 25, 367-375, (2018).
- [7] Das SK, Mandal T, A new model for solving fuzzy linear fractional programming problem with ranking function, *Journal of applied research on industrial engineering*, 4(2), 89-96, 2017.
- [8] Ganesan K and Veeramani V , Fuzzy linear programs with trapezoidal fuzzy numbers. *Ann. Oper. Res.* 143, 305-315, (2006).
- [9] Li DF and Chen S, A fuzzy programming approach to fuzzy linear fractional programming with fuzzy coefficients, *J. Fuzzy Math.* 4, 829-834, (1996).
- [10] Mehlawat MK and Kumar S, A solution procedure for a linear fractional programming problem with fuzzy numbers, *Adv. Intell. Soft Comput.* 130, 1037-1049, (2012).