

Bernoulli-Based Queueing Model Working Vacations, Interrupted Vacations and Subject to Catastrophes

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Abstract

In this study, we explore an M/M/1 queueing model with working vacation and a variation of multiple vacation that is vulnerable to catastrophe events. When the system is not empty, catastrophes happen and wipe out all of the system's current consumers. By employing the probability generating function approach, we create the steady state probabilities and resolve the differential equations.

Key words: Markovian Queueing System, Server Vacations, Bernoulli Schedule, Probability Generating Function, Catastrophes

AMS classification: 60K25, 60K30.

1 Introduction

In recent decades, numerous academics have effectively investigated queueing models with server vacation and successfully applied them to a variety of real-world issues, including the design and operation of telecommunication networks, the industrial sector, and other systems that are linked. The two fundamental queueing models for vacations are multiple vacation and single vacation. The impact of catastrophes, specifically birth and death models, has captured the queueing models' interest over the last 40 years. The catastrophes enter the system as unfavourable users, and one of their traits is the elimination of some or all of the system's normal users. A multi-server Markovian queueing system with heterogeneous servers and disasters is taken into consideration by Dharmaraja and Kumar[6]. The transient solution of an M/M/2/N queueing system with variable catastrophic intensity and restoration was found by Jain and Bura[7].

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2 Model for queue using bernoulli plan working vacation interrupting vacation and balking and system breakdown

We consider an M/M/1 queueing model with working vacation using Bernoulli based schedule and subject to catastrophes event. According to a Poisson stream with parameter λ , customers enter the system. The service time distribution has an exponential shape that is independently identical with parameter μ . Catastrophes occur according to a Poisson process with rate w when the system is not empty. Customers in the orbit repeatedly request services at a retrial rate δ that follows a Poisson distribution. The server stops running when the system is empty. The length of the vacation is distributed exponentially with ϕ . Customer service in the working vacation state is provided at a lower rate $\epsilon (< \mu)$ that is exponentially dispersed for the customer. In addition, when the system is not empty, the catastrophes may also happen at the service facility as a Poisson process with a rate of w . All of the system's customers are destroyed at the moment of a catastrophe, all of the servers are momentarily deactivated, and the servers quickly reactivate following the catastrophes.

Assume that $\widetilde{N}(t)$ represents the number of customers in the orbit (a free pool) at the moment 't' and $\widetilde{H}(t)$ represent the server's condition at the same time . The server state $\widetilde{H}(t)$ may take on any at the following values :

$$\widetilde{H}(t) = \begin{cases} 0, & \text{The server is open in its define H condition} \\ 1, & \text{The server is busy with operational state} \\ 2, & \text{The server is available when it is on vacation} \\ 3, & \text{The server is active and in a state working vacation} \\ 4, & \text{The server is in catastrophes} \end{cases}$$

A Markov process with state space is $\widetilde{N}(t), \widetilde{H}(t)$, then

$$\widetilde{S} = \{ \{(n, h), n \geq 0, h = 0, 1, 3, 4\} \} U(0, 3)$$

We take into account bernoulli schedule working vacation and vacation interruption policies in the current method. Therefore, the statement $(n, 2), n \geq 1$ are false.

3 Equations of Steady State

The present model is governed by the balancing equations shown below:

$$\lambda + \mathcal{P}_{0,0} = q\phi\mathcal{P}_{0,2} \tag{1}$$

$$(\lambda + n\epsilon)\mathcal{P}_{n,0} = \mu\mathcal{P}_{n,1} + \theta\mathcal{P}_{n,3} + \delta\mathcal{P}_{n,4} \quad n \geq 1 \quad (2)$$

$$(\lambda b + \mu)\mathcal{P}_{0,1} = \epsilon\mathcal{P}_{1,0} + \phi\mathcal{P}_{0,3} + \delta\mathcal{P}_{0,4} + \lambda\mathcal{P}_{0,0} \quad (3)$$

$$(\lambda b + \mu)\mathcal{P}_{n,1} = (n + 1) \epsilon \mathcal{P}_{n+1,0} + \phi\mathcal{P}_{n,3} + \delta\mathcal{P}_{n,4} + \lambda\mathcal{P}_{n,0} + \lambda b\mathcal{P}_{n-1,1} \quad n \geq 1 \quad (4)$$

$$(\lambda + \phi q)\mathcal{P}_{0,2} = \theta\mathcal{P}_{0,3} + \delta\mathcal{P}_{0,4} + \mu\mathcal{P}_{0,1} \quad (5)$$

$$(\lambda v + \theta + \phi)p_{0,3} = \lambda p_{0,2} \quad (6)$$

$$(\lambda v + \theta + \phi)\mathcal{P}_{n,3} = \lambda v\mathcal{P}_{n-1,3} \quad n \geq 1 \quad (7)$$

$$(\lambda v + \theta + \phi + \delta)\mathcal{P}_{0,4} = \lambda\mathcal{P}_{0,3} \quad (8)$$

$$(\lambda v + \theta + \phi + \delta)\mathcal{P}_{n,4} = \lambda v\mathcal{P}_{n-1,4} \quad n \geq 1 \quad (9)$$

These are the Probability generating functions:

$$\tilde{F}_0(w) = \sum_{n=0}^{\infty} \mathcal{P}_{n,0} w^n \quad (10)$$

$$\tilde{F}_1(w) = \sum_{n=0}^{\infty} \mathcal{P}_{n,1} w^n \quad (11)$$

$$\tilde{F}_3(w) = \sum_{n=0}^{\infty} \mathcal{P}_{n,3} w^n \quad (12)$$

$$\tilde{F}_4(w) = \sum_{n=0}^{\infty} \mathcal{P}_{n,4} w^n \quad (13)$$

On dividing equation (2) by w^n , adding $n = 0$ to ∞ and summing the result equation (1)

$$(\lambda + n\epsilon)\mathcal{P}_{n,0} w^n = \mu\mathcal{P}_{n,1} + \theta p_{n,3} w^n + \delta\mathcal{P}_{n,4}$$

$$\sum_{n=0}^{\infty} (\lambda + n\epsilon)\mathcal{P}_{n,0} w^n = \sum_{n=0}^{\infty} [\mu\mathcal{P}_{n,1} + \theta\mathcal{P}_{n,3} + \delta\mathcal{P}_{n,4}] w^n$$

$$\sum_{n=0}^{\infty} \lambda\mathcal{P}_{n,0} w^n + n\epsilon \sum_{n=0}^{\infty} \mathcal{P}_{n,0} w^n = \sum_{n=0}^{\infty} \mu\mathcal{P}_{n,1} w^n + \sum_{n=0}^{\infty} \theta\mathcal{P}_{n,3} w^n + \sum_{n=0}^{\infty} \delta\mathcal{P}_{n,4} w^n$$

$$\sum_{n=0}^{\infty} \lambda\mathcal{P}_{n,0} w^n + \sum_{n=0}^{\infty} n\epsilon\mathcal{P}_{n,0} w^n - \sum_{n=0}^{\infty} \mu\mathcal{P}_{n,1} w^n - \sum_{n=0}^{\infty} \theta\mathcal{P}_{n,3} w^n - \sum_{n=0}^{\infty} \delta\mathcal{P}_{n,4} w^n = 0$$

$$\left[\begin{aligned} \epsilon w \tilde{F}'_0(w) &= \epsilon w n \sum_{n=0}^{\infty} \mathcal{P}_{n,3} w^{n-1} = \epsilon n \sum_{n=0}^{\infty} \mathcal{P}_{n,3} w^n = n \epsilon \tilde{F}_0(w) \\ \lambda \tilde{F}_0(w) + w \epsilon \tilde{F}'_0(w) - \mu \tilde{F}_3(w) - \theta \tilde{F}_3(w) - \delta \tilde{F}_4(w) &= 0 \end{aligned} \right]$$

Equation of addition (1)

$$\begin{aligned} \lambda \tilde{F}_0(w) + w \epsilon \tilde{F}'_0(w) - \mu \tilde{F}_3(w) - \theta \tilde{F}_3(w) - \delta \tilde{F}_4(w) + \lambda \mathcal{P}_{0,0} &= q \phi \mathcal{P}_{0,2} \\ \lambda \left(\frac{\lambda}{q\phi} \right) \mathcal{P}_{0,0} + \lambda \tilde{F}_0(w) + w \epsilon \tilde{F}'_0(w) - \mu \tilde{F}_3(w) - \theta \tilde{F}_3(w) - \delta \tilde{F}_4(w) &= 0 \\ \left(\frac{\lambda^2}{q\phi} \right) \mathcal{P}_{0,0} + \lambda \tilde{F}_0(w) + w \epsilon \tilde{F}'_0(w) - \mu \tilde{F}_{+3}(w) - \theta \tilde{F}_3(w) - \delta \tilde{F}_4(w) &= 0 \\ \lambda \tilde{F}_0(w) + w \epsilon \tilde{F}'_0(w) - \mu \tilde{F}_1(w) - \theta \tilde{F}_3(w) - \delta \tilde{F}_4(w) &= \frac{-\lambda^2}{q\phi} \mathcal{P}_{0,0} \end{aligned} \quad (14)$$

Similarly, we obtain using equations (3) and (4) we get

$$\begin{aligned} (\lambda b + \mu) \mathcal{P}_{n,1} &= (n + 1) \epsilon p_{n+1,0} + \phi \mathcal{P}_{n,3} + \delta \mathcal{P}_{n,4} + \lambda \mathcal{P}_{n,0} + \lambda b \mathcal{P}_{n-1,1} \\ \sum_{n=0}^{\infty} (\lambda b + \mu) \mathcal{P}_{n,1} w^n &= \sum_{n=0}^{\infty} n \epsilon \mathcal{P}_{n+1,0} w^n + \sum_{n=0}^{\infty} \epsilon \mathcal{P}_{n+1,0} w^n + \sum_{n=0}^{\infty} \phi \mathcal{P}_{n,3} w^n \\ &\quad + \sum_{n=0}^{\infty} \delta \mathcal{P}_{n,4} w^n + \sum_{n=0}^{\infty} \lambda \mathcal{P}_{n,0} w^n + \sum_{n=0}^{\infty} \lambda b \mathcal{P}_{n-1,1} w^n \\ \implies \lambda b \sum_{n=0}^{\infty} \mathcal{P}_{n,1} w^n + \mu \sum_{n=0}^{\infty} \mathcal{P}_{n,1} w^n - n \epsilon \sum_{n=0}^{\infty} \mathcal{P}_{n+1,0} w^n - \epsilon \sum_{n=0}^{\infty} \mathcal{P}_{n+1,0} w^n \\ &\quad - \phi \sum_{n=0}^{\infty} \mathcal{P}_{n,3} w^n - \delta \sum_{n=0}^{\infty} \mathcal{P}_{n,4} w^n - \sum_{n=0}^{\infty} \lambda \mathcal{P}_{n,0} w^n - \lambda \sum_{n=0}^{\infty} b \mathcal{P}_{n-1,1} w^n = 0 \end{aligned}$$

Equation (6) and (7) give us

$$(\lambda b + \mu - \lambda b w) \tilde{F}_1(w) - \epsilon \tilde{F}'_0(w) - \lambda \tilde{F}_0(w) - \phi \tilde{F}_3(w) - \delta \tilde{F}_4(w) = 0 \quad (15)$$

From equation (6) and (7) we have

$$(\lambda v + \theta + \phi) \mathcal{P}_{n,3} = \lambda v \mathcal{P}_{n-1,3}$$

$$\begin{aligned}
 \sum_{n=0}^{\infty} (\lambda v + \theta + \phi) \mathcal{P}_{n,3} w^n &= \lambda v \sum_{n=0}^{\infty} \mathcal{P}_{n-1,3} w^n \\
 \sum_{n=0}^{\infty} (\lambda v + \theta + \phi) \mathcal{P}_{n,3} w^n - \lambda v \sum_{n=0}^{\infty} \mathcal{P}_{n,3} w^{n+1} &= 0 \\
 (\lambda v + \theta + \phi) \tilde{F}_3(w) - \lambda v w \tilde{F}_3(w) &= 0 \\
 \implies (\lambda v + \theta + \phi - \lambda v w) \tilde{F}_3(w) &= \frac{\lambda^2}{q\phi} \mathcal{P}_{0,0} \tag{16}
 \end{aligned}$$

From equation (8) and (9) we have

$$\begin{aligned}
 \sum_{n=0}^{\infty} (\lambda v + \theta + \phi + \delta) \mathcal{P}_{n,4} w^n &= \lambda v \sum_{n=0}^{\infty} \mathcal{P}_{n-1,4} w^n \\
 \implies (\lambda v + \theta + \phi + \delta - \lambda v w) \tilde{F}_4(w) &= \left(\frac{\lambda^3}{q\phi(\lambda v + \theta + \phi)} \right) \mathcal{P}_{0,0} \tag{17}
 \end{aligned}$$

Using equation (18) $\tilde{F}_1(w)$ and the value of $\tilde{F}_1(w)$ from equation (16) and using equation(13)

$$\begin{aligned}
 \lambda \tilde{F}_0(w) + w \epsilon \tilde{F}_0'(w) - \mu \tilde{F}_1(w) - \theta \tilde{F}_3(w) - \delta \tilde{F}_4(w) &= \frac{\lambda^2}{q\phi} \mathcal{P}_{0,0} \\
 \implies \tilde{F}_1(w) &= \frac{\lambda}{\mu} \tilde{F}_0(w) + \frac{w \epsilon}{\mu} \tilde{F}_0'(w) - \frac{\theta}{\mu} \tilde{F}_3(w) - \frac{\delta}{\mu} \tilde{F}_4(w) + \frac{\lambda^2}{q\phi \mu} \mathcal{P}_{0,0} \\
 (\lambda b + \mu - \lambda b w) \left(\frac{\lambda}{\mu} \tilde{F}_0(w) + \frac{w \epsilon}{\mu} \tilde{F}_0'(w) - \frac{\theta}{\mu} \tilde{F}_3(w) - \frac{\delta}{\mu} \tilde{F}_4(w) + \frac{\lambda^2}{q\phi \mu} \mathcal{P}_{0,0} \right) - \epsilon \tilde{F}_0'(w) - \lambda \tilde{F}_0(w) - \phi \tilde{F}_3(w) &= 0 \\
 - \left(\lambda^2 b w F_0(w) + \lambda b w^2 \epsilon \tilde{F}_0' - \theta \lambda b w F_3 + \frac{\lambda^3 b w}{q\phi} \mathcal{P}_{0,0} \right) - \epsilon \tilde{F}_0'(w) - \lambda \tilde{F}_0(w) - \phi \tilde{F}_3(w) - \delta \tilde{F}_4(w) &= 0 \\
 \implies \left[\frac{w \lambda b}{\mu} \epsilon + w \epsilon - \frac{\lambda}{\mu} b w^2 \epsilon - \epsilon \right] \tilde{F}_0'(w) &= 0 \implies \left[\frac{(1-w) \epsilon (\mu - \lambda b w)}{\mu} \right] \tilde{F}_0'(w) = 0
 \end{aligned}$$

The $\tilde{F}_0(w)$ comes next

$$\left[\frac{\lambda^2 b}{\mu} + \lambda - \frac{\lambda^2 b w}{\mu} - \lambda w \right] \tilde{w}_0(w) = 0 \implies \left[-\frac{\lambda^2 b (1-w)}{\mu} \right] \tilde{F}_0(w) = 0$$

$$\frac{1}{\mu} \left[(-\theta\lambda b - \theta\mu + \theta\lambda bw - \phi\mu - \delta\mu)\tilde{F}_4(w)\tilde{F}_3(w) \right] + \frac{1}{\mu} \left[\frac{\lambda^3 b}{q\phi}\mathcal{P}_{0,0} + \frac{\lambda^2\mu}{q\phi}p_{0,0} - \frac{\lambda^3 bw}{q\phi}\mathcal{P}_{0,0} \right] = 0$$

Using an equation (16)

$$\begin{aligned} \frac{1}{\mu} \left[(-\theta\lambda b - \theta\mu + \theta\lambda bw - \phi\mu - \delta\mu) \left(\frac{\lambda^3}{q\phi(\lambda v + \theta + \phi)} \right) \mathcal{P}_{0,0} \frac{\lambda^2}{q\phi(\lambda v + \theta + \phi - \lambda v w)} \mathcal{P}_{0,0} \right] \\ + \frac{1}{\mu} \left[\frac{\lambda^3 b}{q\phi} q\phi \mathcal{P}_{0,2} + \frac{\lambda^2\mu}{q\phi} q\phi \mathcal{P}_{0,2} - \frac{\lambda^3 bw}{q\phi} q\phi \mathcal{P}_{0,2} \right] = 0 \\ \implies \frac{(1-w)\epsilon(\mu - \lambda bw)}{\mu} \tilde{F}'_0(w) - \frac{\lambda^2 b(1-w)}{\mu} \tilde{F}_{0(w)} \\ + \frac{(\theta\lambda^2 b + \theta\mu - \theta\lambda^2 bw + \lambda\phi\mu + \lambda\delta\mu)}{\mu(\lambda v + \theta + \phi - \lambda v w)} \mathcal{P}_{0,2} - \frac{\lambda(\mu + \lambda b - \lambda bw)}{\mu} \mathcal{P}_{0,2} = 0 \end{aligned}$$

Now, multiplying μ and dividing $(1-w)\epsilon(\mu - \lambda bw)$, we have

$$\begin{aligned} \tilde{F}'_0(w) - \frac{\lambda^2 b}{\epsilon(\mu - \lambda bw)} \tilde{F}_0(w) = \frac{\lambda(\mu + \lambda b - \lambda bw)}{(1-w)\epsilon(\mu - \lambda bw)} \mathcal{P}_{0,2} \\ - \frac{\lambda\phi\mu + \lambda\delta\mu + \lambda\theta(\mu + \lambda b - \lambda bw)}{(\lambda v + \theta + \phi + \delta - \lambda v w)(1-w)\epsilon(\mu - \lambda bw)} \mathcal{P}_{0,2} \quad (18) \end{aligned}$$

The result of solving differential equation (18) is

$$\begin{aligned} \tilde{F}'_0(w) - \frac{\lambda^2 b}{\epsilon(\mu - \lambda bw)} \tilde{F}_0(w) = \frac{\lambda(\mu + \lambda b - \lambda bw)}{(1-w)\epsilon(\mu - \lambda bw)} \mathcal{P}_{0,2} \\ - \frac{\lambda\phi\mu + \lambda\delta\mu}{(\lambda v + \theta + \phi + \delta - \lambda v w)(1-w)\epsilon(\mu - \lambda bw)} \mathcal{P}_{0,2} \\ - \frac{\lambda\theta(\mu + \lambda b - \lambda bw)}{(\lambda v + \theta + \phi + \delta - \lambda v w)(1-w)\epsilon(\mu - \lambda bw)} \end{aligned}$$

$$\begin{aligned} \int_0^w \tilde{F}_0(x) dx = \frac{-\lambda}{\epsilon} \mathcal{P}_{0,2} \int_0^w \frac{(\mu + \lambda b - \lambda bw)}{(1-x)\epsilon(\mu - \lambda bw)} dx + \int_0^w \frac{\phi\mu + \delta\mu}{(\lambda v + \theta - \lambda v x)(1-x)(\mu - \lambda bx)} \\ + \int_0^w \frac{\theta(\mu - \lambda b - \lambda bx)}{(\lambda v + \theta + \phi + \delta - \lambda v x)(1-x)(\mu - \lambda bx)} \end{aligned}$$

$$\implies \tilde{F}_0(w) = -(\mu - \lambda bw) \frac{-\lambda}{\epsilon} (\phi \mu I_1(w) + \theta I_2(w) - I_3(w) - \delta \mu I_4(w)) \frac{\lambda^2 \mathcal{P}_{0,0}}{q\phi\epsilon} \quad (19)$$

Where $I = \frac{\lambda^2}{q\phi\epsilon} (\phi \epsilon I_1(w) + \theta I_2(w) - I_3(w) - \delta \mu I_4(w))$

From $\lambda \tilde{F}_0(w) + w \epsilon \tilde{F}_0^{(1)}(w) - \mu \tilde{F}_1(w) - \theta \tilde{F}_3(w) = \frac{\lambda^2}{q\phi} \mathcal{P}_{0,0}$, we have

$$I_1(w) = \int_0^w (1-x)^{-1} (\mu - \lambda bx)^{\frac{\lambda}{\epsilon}-1} (\lambda v + \theta + \phi - \lambda vx)^{-1} dx \quad (20)$$

$$I_2(w) = \int_0^w (1-x)^{-1} (\mu - \lambda bx)^{\frac{\lambda}{\epsilon}-1} (\lambda v + \theta + \phi - \lambda vx)^{-1} (\mu + \lambda b - \lambda bx) dx \quad (21)$$

$$I_3(w) = \int_0^w (1-x)^{-1} (\mu - \lambda bx)^{\frac{\lambda}{\epsilon}-1} (\mu + \lambda b - \lambda bx) dx \quad (22)$$

$$I_4(w) = \int_0^w (1-x)^{-1} (\mu - \lambda bx)^{\frac{\lambda}{\epsilon}-1} (\lambda v + \theta + \phi + \delta - \lambda vw) dx \quad (23)$$

Where we take $\tilde{F}_0(w)$ out of equation (16) and (23), we get

$$\begin{aligned} \lambda \tilde{F}_0(w) + w \epsilon \tilde{F}_0^{(1)}(w) - \mu \tilde{F}_1(w) - \theta \tilde{F}_3(w) &= \frac{-\lambda^2}{q\phi} \mathcal{P}_{0,0} \\ (\lambda - \lambda w) \tilde{F}_0(w) [w[\lambda bw + w\mu - \lambda bw^2 - \mu] - \tilde{F}_3(w)[w\phi + \theta]] &= \frac{-\lambda^2}{q\phi} \mathcal{P}_{0,0} \\ \tilde{F}_1(w) [(\lambda bw - \mu)(1-w)] &= \tilde{F}_3(w) [\theta + w\phi] - \lambda(1-w) \tilde{F}_0(w) - \frac{-\lambda^2}{q\phi} \mathcal{P}_{0,0} \\ \implies \tilde{F}_1(w) &= \frac{\tilde{F}_3(w) [\theta + w\phi] - \lambda(1-w) \tilde{F}_0(w) - \frac{-\lambda^2}{q\phi} \mathcal{P}_{0,0}}{(\lambda bw - \mu)(1-w)} \end{aligned} \quad (24)$$

Based on the equation (16)

$$\tilde{F}_3(w) = \frac{\lambda^2}{q\phi(\lambda v + \theta + \phi - \lambda vw)} \mathcal{P}_{0,0} \quad (25)$$

From the equation (16)

$$\tilde{F}_4(w) = \frac{\lambda^3}{q\phi(\lambda v + \theta + \phi + \delta - \lambda v w)} \mathcal{P}_{0,0} \quad (26)$$

We observe that $F_0(w), F_1(w)$ and $\mathcal{P}_{0,0}$, whose values may be obtained from the normalization condition, is used to express all of $F_3(w)$, from equation(19), (24)and(25) respectively,

3.1 Indicators of system performance

From the equation (25), we have

$$\tilde{F}_4(1) = \lim_{w \rightarrow 1} \tilde{F}_4(w) = \lim_{w \rightarrow 1} \frac{\lambda^2}{q\phi(\lambda v + \theta\phi + \delta - \lambda v w)} \mathcal{P}_{0,0} = \frac{\lambda^3}{q\phi(\theta + \phi + \delta)} \mathcal{P}_{0,0} \quad (27)$$

Using equation (25) differentiating, we obtain

$$\tilde{F}_3(1) = \lim_{w \rightarrow 1} \tilde{F}_3(w) = \lim_{w \rightarrow 1} \frac{\lambda^2}{q\phi(\lambda v + \theta\phi - \lambda v w)} \mathcal{P}_{0,0} = \frac{\lambda^2}{q\phi(\theta + \phi)} \mathcal{P}_{0,0} \quad (28)$$

On differentiating equation (25), we get

$$\tilde{F}_3'(w) = \frac{-\lambda^2 - \lambda v}{q\phi(\lambda v + \theta\phi - \lambda v w)^2} \mathcal{P}_{0,0} = \frac{\lambda^3 v}{q\phi(\lambda v + \theta\phi - \lambda v w)^2} \mathcal{P}_{0,0} \quad (29)$$

$$\tilde{F}_3'(1) = \lim_{w \rightarrow 1} \tilde{F}_3'(w) = \lim_{w \rightarrow 1} \frac{\lambda^3 v}{q\phi(\lambda v + \theta\phi - \lambda v w)^2} \mathcal{P}_{0,0} = \frac{\lambda^3 v}{q\phi(\theta + \phi)^2} \mathcal{P}_{0,0} \quad (30)$$

$$\tilde{F}_0(1) = \lim_{w \rightarrow 1} \tilde{F}_0(w) = \lim_{w \rightarrow 1} -(\mu - \lambda b w)^{-\frac{\lambda}{\epsilon}} (\phi \epsilon I_1(w) + \theta I_2(w) - I_3(w) - \delta I_4(w)) \frac{\lambda^2 \mathcal{P}_{0,0}}{q\phi \epsilon}$$

$$\tilde{F}_0(1) = -(\mu - \lambda b w)^{-\frac{\lambda}{\epsilon}} (\phi \epsilon I_1(w) + \theta I_2(w) - I_3(w)) \frac{\lambda^2 \mathcal{P}_{0,0}}{q\phi \epsilon} = -(\mu - \lambda b w)^{-\frac{\lambda}{\epsilon}} I \mathcal{P}_{0,0} \quad (31)$$

Where $I = \frac{\lambda^2}{q\phi \epsilon} (\phi \epsilon I_1(z) + \theta I_2(w) - I_3(w))$

From $\lambda \tilde{F}_0(w) + w \epsilon \tilde{F}_0^{(1)}(w) - \mu \tilde{F}_1(w) - \theta \tilde{F}_3(w) = \frac{\lambda^2}{q\phi} \mathcal{P}_{0,0}$

$$\tilde{F}_0'(w) = \frac{1}{w \epsilon} \left(\mu \tilde{F}_1(w) + \theta \tilde{F}_3(w) + \delta \tilde{F}_4(w) - \lambda \tilde{F}_0(w) - \frac{\lambda^2}{q\phi} \mathcal{P}_{0,0} \right) \quad (32)$$

$$\tilde{F}_0'(1) = \frac{1}{\epsilon} \left(\mu \tilde{F}_1(1) + \theta \tilde{F}_3(1) + \delta \tilde{F}_4(w) - \lambda \tilde{F}_0(1) - \frac{\lambda^2}{q\phi} \mathcal{P}_{0,0} \right) \quad (33)$$

Also we obtain

$$\tilde{F}_1(1) = \lim_{w \rightarrow 1} \tilde{F}_1(w) = \lim_{w \rightarrow 1} \frac{\theta \tilde{F}_3(w) + \phi w \tilde{F}_3(w) - \lambda \tilde{F}_0(w) + \lambda(w) \tilde{F}_0(w) - \frac{\lambda^2}{q\phi} \mathcal{P}_{0\ 0}}{(\lambda bw - \mu - \lambda bw^2 + \mu w)}$$

Applying L-Hopital rule,

$$\begin{aligned} \tilde{F}'_1(1) &= \lim_{w \rightarrow 1} \frac{\theta \tilde{F}'_3(w) + \phi F_3(w) + \phi w \tilde{F}'_3(w) - \lambda \tilde{F}'_0(w) + \lambda \tilde{F}'_0(w) + \lambda \tilde{F}_0(w)}{\lambda b - 2\lambda bw + \mu} \\ &= \frac{\theta \tilde{F}'_3(1) + \phi \tilde{F}_3(1) + \phi \tilde{F}'_3(1) - \lambda \tilde{F}'_0(1) + \lambda \tilde{F}'_0(1) + \lambda \tilde{F}_0(1)}{\lambda b - 2\lambda b + \mu} \\ \tilde{F}_1(1) &= \frac{\lambda \tilde{F}_0(1) + \phi \tilde{F}_3(1) + (\theta + \phi) \tilde{F}'_3(1)}{(\mu - \lambda b)} \tag{34} \\ &\Rightarrow \tilde{F}_1(w) = \frac{(\theta + \phi w) \tilde{F}_3(w) - \lambda(1-w) \tilde{F}_0(w) - \frac{\lambda^2}{q\phi} \mathcal{P}_{0\ 0}}{(1-w)(\lambda bw - \mu)} \\ \tilde{F}_1(w) &= \frac{\theta \tilde{F}_3(w) + \phi w \tilde{F}_3(w) - \lambda \tilde{F}_0(w) + w \lambda \tilde{F}_0(w) \frac{\lambda^2}{q\phi} \mathcal{P}_{0\ 0}}{\lambda bw - \mu - \lambda bw^2 + w\mu} \end{aligned}$$

Using L-Hopital rule

$$\tilde{F}'_1(1) = \frac{(\mu - \lambda b)[(\theta + \phi) \tilde{F}'_3(1) + 2\phi \tilde{F}'_3(1) + 2\lambda \tilde{F}'_0(1)] + 2\lambda b[(\theta + \phi) \tilde{F}'_3(1) + \phi \tilde{F}_3(1) + \delta \tilde{F}_4(w) + \lambda \tilde{F}_0(1)]}{2(\mu - \lambda b)^2} \tag{35}$$

The normalize condition is

$$P_{02} + \tilde{F}_0(1) + \tilde{F}_1(1) + \tilde{F}_3(1) + \tilde{F}_4(1) = 1$$

$$\begin{aligned} \frac{\lambda}{q\phi} \mathcal{P}_{0\ 0} + \tilde{F}_0(1) + \frac{\lambda}{\mu - \lambda b} \tilde{F}_0(1) + \frac{\phi}{\mu - \lambda b} \tilde{F}_3(1) + \frac{\theta + \phi}{\mu - \lambda b} \tilde{F}'_3(1) + \tilde{F}_3(1) &= 1 \\ \frac{\lambda}{q\phi} \mathcal{P}_{0\ 0} + \tilde{F}_0(1) \left[1 + \frac{\lambda}{\mu - \lambda b} \right] + \tilde{F}_3(1) \left[\frac{\phi}{\mu - \lambda b} + 1 \right] + \left(\frac{\theta + \phi}{\mu - \lambda b} \right) \tilde{F}'_3(1) &= 1 \end{aligned}$$

$$\frac{\lambda}{q\phi} \mathcal{P}_{0,0} \left[1 + (\mu - \lambda b)^{-\frac{\lambda}{\epsilon}} I(1) + \frac{\lambda(\theta + \lambda v)}{(\theta + \phi)(\mu - \lambda b)} + \lambda(\mu - \lambda b)^{-\frac{\lambda}{\epsilon}-1} I(1) + \frac{\lambda}{\theta + \phi} \right] = 1$$

$$\mathcal{P}_{0,0} = \frac{q\phi}{\lambda} \left[1 + (\mu - \lambda b)^{-\frac{\lambda}{\epsilon}} (1 + \lambda(\mu - \lambda b)^{-1}) I(1) + \frac{\lambda(\theta + \lambda v) + \lambda(\mu - \lambda b)}{(\theta + \phi)(\mu - \lambda b)} \right]^{-1}$$

Let $E[L_k]$ be the expected number of customer in the orbit, where the states of the series are supposed to have values of $k = 0, 1, 2, 3$ and solution.

$$E[L_0] = \lim_{w \rightarrow 1} \tilde{F}'_1(w)$$

$$E[L_0] = \frac{1}{\epsilon} \left(\mu \tilde{F}_1(1) + \theta \tilde{F}_3(1) - \lambda \tilde{F}_0(1) - \frac{\lambda^2}{q\phi} \mathcal{P}_{0,0} \right) \quad (36)$$

similarly,

$$E[L_1] = \lim_{z \rightarrow 1} \tilde{F}'_1(w)$$

$$E[L_1] = \frac{(\mu - \lambda b)[(\theta + \phi)\tilde{F}_3^2(1) + 2\phi\tilde{F}_3^{(1)}(w) + 2\lambda\tilde{F}_0^1(1)] + 2\lambda b[(\theta + \phi)\tilde{F}_3^1(1) + \phi\tilde{F}_3(1) + \delta\tilde{F}_4(1) + \lambda\tilde{F}_0(1)]}{2(\mu - \lambda b)^2}$$

$$E[L_3] = \lim_{w \rightarrow 1} \tilde{F}_3^{(')}(w) = \frac{\lambda^3 v}{q\phi(\theta + \phi)^2} \mathcal{P}_{0,0}$$

$$E[L_4] = \lim_{w \rightarrow 1} \tilde{F}_4^{(')}(z) = \frac{\lambda^4 v}{q\phi(\theta + \phi + \delta)^2} \mathcal{P}_{0,0}$$

The predicted orbit length is provided by:

$$E[L_q] = E[L_0] + E[L_1] + E[L_3] + E[L_4]$$

$$\tilde{F}'_1(1) + \tilde{F}'_1(1) + \tilde{F}'_3(1) + \tilde{F}'_4(1)$$

Expected length of frame work is given by

$$E[L_s] = E[L_q] + \tilde{F}_1(1) + \tilde{F}_3(1) + \tilde{F}_4(1)$$

The likelihood of server being occupied state is $\mathcal{Pr}_B = \tilde{F}_1(1) + \tilde{F}_3(1) + \tilde{F}_4(1)$

The likelihood that a server is in a free state is:

$$\mathcal{Pr}_F = \tilde{F}_0(1) + \mathcal{P}_{0,2}$$

$$\mathcal{Pr}_F = 1 - \mathcal{Pr}_B$$

The likelihood of server being in a state if normal operation is

$$\mathcal{Pr}_N = \tilde{F}_0(1) + \tilde{F}_1(1)$$

The likelihood a server is an vacation while working is

$$\mathcal{P}r_v = \tilde{F}_3(1) + \mathcal{P}_{0,2}$$

$$\mathcal{P}r_v = 1 - \mathcal{P}r_N$$

The likelihood of server being in catastrophes state is

$$\mathcal{P}r_w = \tilde{F}_4(1) + \mathcal{P}_{0,2}$$

$$\mathcal{P}r_w = 1 - \mathcal{P}r_v$$

4 Conclusion

This study examines the Bernoulli-based queueing model, which is subject to catastrophes, interruption policies, and working vacations. By applying the probability generating function method, the explicit formulas for expected queue length and probabilities of various server statuses have been derived. The model can be used practically in a variety of circumstances that arise in the actual world, according to the derived numerical results. Numerous network and telecommunications systems can use this approach.

References

- [1] Anand Gnana Selvam S, Chandrasekar B, Moganraj D, Janci Rani J: Impact of Single Server Queue Having Machine Repair with Catastrophes Using Probability Generating Function Method, *Emerging Research in Data Engineering Systems and Computer Communications*, 1054 , 191-198(2020).
- [2] Arivudainambi D, Godhandarama P, Rajadurai P, Performance analysis of a single server retrial queue with working vacation, *OPSEARCH*. 51 , 434–462(2014).
- [3] Banik AD, Gupta UC, Pathak SS, On the GI/M/1/N queue with multiple working vacations—analytic analysis and computation, *Appl. Math. Model.* 31 , 1701–1710(2007).
- [4] Chandrasekar B, Monica T and Anand Gnana Selvam S, Impact And Steady State Analysis M/G/1 Queuing System With Random Revolution Subject To Catastrophics Event, *Advances in Mathematics: Scientific Journal* 9 , no.8, 6205–6212(2020).
- [5] Choudhury A, Medhi AP, Balking and reneging in multiserver Markovian queueing system, *Int. J. Math. Oper. Res.* 3 , 377-394(2011).
- [6] Dharmraja S, Kumar R, Transient solution of a Markovian queueing model with heterogeneous servers and catastrophes. *OPSEARCH*, 52(4),810-826(2015).

- [7] Jain NK, Bura, Gulab Singh: M/M/2/N queue subject to modified Binomially distributed catastrophic intensity with restoration time. *Journal of the Indian Statistical Association*, 49(2), 135-147(2011).
- [8] Keilson J, Servi LD, Oscillating random walk models for GI/G/1 vacation systems with Bernoulli schedule, *J. Appl. Probab.* 23(3) , 790-802(1986).
- [9] Laxmi PV, Goswami V, Jyothsna VK, Analysis of Finite Buffer Markovian Queue with Balking, Reneging and Working Vacations, *Int. J. Strat. Decision Sci.* 4 , 1–24(2013).
- [10] Li J, Tian N, The M/M/1 queue with working vacations and vacation interruptions, *J. Syst. Sci. Syst. Eng.* 16, 121–127(2007).
- [11] Kumar R, Sharma SK, Economic analysis of M/M/C/N queue with retention of impatient customers, *Int. J. Math. Oper. Res.* 5(6) , 709-720(2013).
- [12] Kumar R, Sharma S, Transient analysis of an M/M/c queuing system with balking and retention of reneging customers, *Commun. Stat.-Theory Meth.* 47, 1318–1327(2018).
- [13] Tao L, Wang Z, Liu Z, The GI/M/1 queue with Bernoulli-schedule-controlled vacation and vacation interruption, *Appl. Math. Model.* 37, 3724–3735(2013).
- [14] Wu DA, Takagi H, M/G/1 queue with multiple working vacations, *Perform. Eval.* 63, 654–681(2006).
- [15] Yue D, Yue DW, Xu G, Analysis of customers' impatience in an M/M/1 queue with working vacations, *J. Ind. Manage. Optim.* 8 , 895–908(2012).