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On the Crossing numbers of $TS_n \times P_m$ and $TS_n \times C_m$

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Abstract

The crossing numbers of graphs is the least number of edge crossings in all possible good drawings of graph G. Crossing number of graphs has many applications in day today life such as road and railways crossings, Networking, VLSI circuit development etc. Cartesian product and Join product of graph has received maximum attention in many research problems. In this paper we proved, $Cr_D(TS_n \times P_m) = \lfloor \frac{n}{2} \rfloor .(m-2)$ for $m \geq 3$ using the results proved in [6].

Key words: Crossing number, Cartesian product of graphs, Join of graphs, Double Triangular snake graph, path, cycle.

AMS classification: 68R10; 05C10; 05C62

1 Introduction

A drawing is said to be a good drawing if two edges which are incident with a common vertex do not have another point in common and no two edges have more than one point in common. Subdrawing of a good drawing is good. If two edges have a common point which is not a vertex, then that point is called a Crossing. The Crossing number Cr(G) of graph G is the least number of edge crossings among the drawings of G in the plane.

Union of two graph G and G' is the graph with vertex set $(V(G) \cup V(G'))$ and edge set $(E(G) \cup E(G'))$. Join product of two disjoint graph G and G' is obtained from $G \cup G'$ by joining every vertex of G to every vertex of G'.It is denoted by $G + G'$

The Cartesian product of two graphs G and H, denoted $G \times G'$ is the graph with vertex set $V(G) \times V(G)$ and having edages of the form (u, v)(x, y) where either u=x and $(v, y) \in E(G')$ or $v = y$ and $(u, x) \in E(G)$.

Let P_m denote the path on m vetices of length (m-1). There are few results with crossing number of cartesian product of some graphs with P_n.

The crossing number of the cartesian product of some special graphs with path have

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subject of investigation in [1],[2],[3],[4], [5], [7]. In this paper we give the exact value of crossings of $TS_n \times P_m$ and $TS_n \times C_m$, where TS_n is a Triangular snake graph on n vertices.

2 Crossing numbers of the Cartesian product of Triangular snake graphs TS_n with P_m and C_m

Definition 2.1 ***(Triangular Snake graph*** TS_n***)*** It is a graph having a vertex set $V = \{z_1, z_2, \cdots, z_n\}$ where n is odd and edge set $E = \{z_{2i-1}z_{2i+1}/1 \leq i \leq \frac{n-1}{2}\} \cup \{z_i z_{i+1}/1 \leq i \leq n-1\}$.

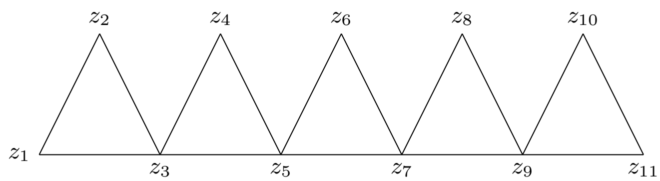

Figure 1: (**Triangular Snake graph** TS_{11})

We are listing some trival results.

(1) $Cr(TS_5 + 2K_1) = 2$

(2) $Cr(TS_n + mK_1) = Z(n, m) + \lfloor \frac{n}{2} \rfloor \lfloor \frac{m}{2} \rfloor$, $n \geq 5$ and $m \leq 6$. [6]

Remark 2.2 $Cr(TS_3 \times P_m) = 0$.

Lemma 2.3 Let α be a good drawing of $TS_n \times P_3$. Let β be a good drawing of graph S obtained by joining a vertex $x \notin V(TS_n)$ with all vertices of one copy of TS_n then $Cr_\beta(S) = \lfloor \frac{n}{2} \rfloor$.

Proof: Let β be a good drawing of S as shown fig. 2.

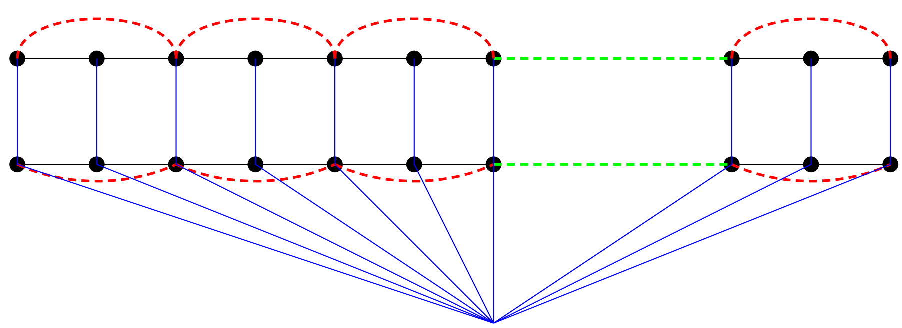

Figure 2: (**Drawing** β **of graph** S)

Since each copy of TS_n has $\lfloor \frac{n}{2} \rfloor$ three cycles. Thus from fig. 2, we can conclude that

$$Cr_\beta(S) \leq \lfloor \frac{n}{2} \rfloor \tag{1}$$

Now we have to prove otherway inequality. By contracting all vertives and edges of other copy of TS_n, we get drawing of $TS_n + 2K_1$ in β. Thus

$$Cr_\beta(S) \geq Cr(TS_n + 2K_1) = \lfloor \frac{n}{2} \rfloor \tag{2}$$

Hence Proved.

Theorem 2.4 $Cr(TS_n \times P_3) = \lfloor \frac{n}{2} \rfloor$, $n \geq 5$.

Proof: Consider a drawing of $Cr(TS_n \times P_3)$ as shown in fig. 3.

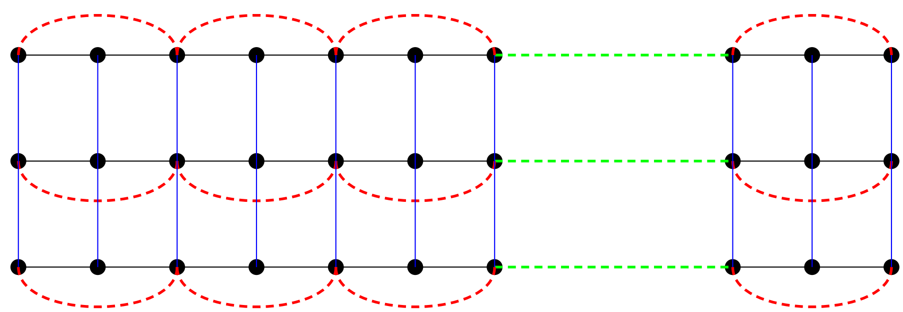

Figure 3: $(TS_n \times P_3)$

Since a copy of TS_n has $\lfloor \frac{n}{2} \rfloor$ number of three cycles. Thus,

$$Cr(TS_n \times P_3) \leq \lfloor \frac{n}{2} \rfloor \qquad n \geq 5 \tag{3}$$

Now we have to prove other way inequality. If we contract all vertices and edges of first and third copy of TS_n, we get drawing of $TS_n + 2K_1$. Thus,

$$Cr(TS_n \times P_3) \geq Cr(TS_n + 2K_1) = \lfloor \frac{n}{2} \rfloor \tag{4}$$

Thus from eq. (3) and eq. (4) we can conclude that,

$$Cr(TS_n \times P_3) = \lfloor \frac{n}{2} \rfloor \qquad n \geq 5$$

Theorem 2.5 $Cr(TS_n \times P_m) = \lfloor \frac{n}{2} \rfloor .(m-2)$ for $n \geq 5$.

Proof: We prove the result using method of induction on m.
By theorem 2.4 result holds for m=3. Let us assume that result holds for less than m. Now we have to prove it for m. By contracting all vertices and edges of m^{th} copy of TS_n we get subdrawing of union of $(TS_n \times P_{m-1}) \cup S$. Thus,

$$Cr(TS_n \times P_m) \geq Cr(TS_n \times P_{m-1}) \cup S)$$

$$\geq Cr(TS_n \times P_{m-1}) + Cr(S)$$
$$\geq \lfloor \frac{n}{2} \rfloor .(m-3) + \lfloor \frac{n}{2} \rfloor$$
$$\geq \lfloor \frac{n}{2} \rfloor .(m-2)$$
$$Cr(TS_n \times P_m) \geq \lfloor \frac{n}{2} \rfloor .(m-2) \tag{5}$$

Hence proved.

Theorem 2.6 $Cr(TS_n \times C_m) = \lfloor \frac{n}{2} \rfloor .(m-2)$ for $n \geq 5$.

3 Conclusion

In this paper we obtained the exact crossing numbers of $TS_n \times P_m$ and $TS_n \times C_m$ for $m \geq 3$.

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References

[1] Bokal D, On the crossing numbers of cartesian products with paths. Journal of Combinatorial Theory, Series B 97, 3,381–384(2007).

[2] Klesc M, " The crossing numbers of products of paths and stars with 4-vertex graphs." Journal of Graph Theory 18, 6, 605–614(1994).

[3] Klesc M, " The crossing numbers of cartesian products of paths with 5-vertex graphs". Discrete Mathematics 233, 1-3 , 353–359(2001).

[4] Ouyang Z, Wang, J., and Huang, Y. The crossing number of the cartesian product of paths with complete graphs. Discrete Mathematics 328 , 71–78(2014).

[5] Peng Y and Yiew Y, The crossing number of $p(3,1) \times P_n$. Discrete mathematics 306, 16 , 1941–1946(2006).

[6] Rajan B, Rajasingh I and Beulah PV, Crossing number of the join of a triangular snake with mK1, path, and cycle. International Journal of Computer Applications 975, 8887(2012).

[7] Yiew YC, Chia GL and Ong PH, Crossing number of cartesian product of prism and path. AKCE International Journal of Graphs and Combinatorics 0, 0 , 1–7(2020).