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A Study of Multiserver Retrial Queues With Different Stages of Homogeneous Service

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Abstract

We discuss a queuing system with retrial of customers. Two models are discussed. First, we investigate single server queues in parallel, when the customer going to search and join the shorter of the two queues and in the second model we introduce the multiserver queue to multiserver retrial queue system. Multiserver provides different stages of homogeneous service in succession.

Key words: Retrial Queue, Shortest queue, Multiserverqueue.

AMS classification: 60K25, 58B12

1 Introduction

In inter-arrival time, the arrival of the customers can be modeled according to different disciplines depending on each particular application. Arriving customers join the shorter of the two queues with ties broken in any plausible manner. No jockeying between the queues is allowed. This situation is like in the ticket counter for theater, bank and vehicles going through toll booths etc.., here we use inter-arrival time to obtain "bounds" for the probability distribution for the number of customer in the system, its expected value in equilibrium by linear programming techniques and obtained the value of lower and upper bounds, inform of equilibrium.

2 The Mathematical Description

A Poisson stream of customer with rate λarrives at a service which consists of two single server queues in parallel. The service time of the customers is independent and exponentially distributed with rate. Both servers serve at an equal rate.

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Each server has associated queues if their sizes are unequal, otherwise he joins any queue. Jockeying between the queues is not allowed. This system which is known as the "shortestqueue",has received a lot of attention in the literature because it (with its generalizations to many serves and general service times) modes many real-life situations such as vehicles going through tool booths, jobs scheduled on a multiprocessor system, etc.,. On the other hand, no simple method for analyzing this system is known. The problem was originally introduced by Haight. Kingman proved that an equilibrium distribution exists whenever $\frac{\lambda}{\mu} \leq 2$. Both Flatto and McKean treated the problem by applying techniques of complex function theory to obtain representation for the general functions of the state probability. The above mentioned papers derive some asymptotes approximations for the state probability. The above mentioned papers derive sum asymptotic approximation for the state probabilities for large number of customers in the system, and for any traffic. Connolly discussed the finite -waiting-room version of the problem. In current paper, we derive upper and lower bounds for the state probabilities, tail probabilities and mean number of customers in the is equilibrium.

The bounds are derived by considering a subset of the balance equation, which bounds for the tail programs which in turn produce bounds. We also derive lower bounds for the tail distribution by comparison with an M/M/2 system. These rather elementary techniques produce bound which are within ten of the true values for $1 \leq \frac{\lambda}{\mu} \leq 2$, and which are asymptotically tight in heavy trafic.

3 Preliminary Results

The state space consists of pairs (i,j) where i,j=0,1,2,... and $i \geq j$. we say that the system is in (i,j) if the number of customers in the longer queue is "i" and the number of customer in the shorter queue is "j" (number in queue includes the customers in service, if any) note that under this description servers are not associated with a particular component of the state vector and there is no need to specify what happens when a customer finds that both queues are of equal length. Clearly the system behaves as Markova chains on the state space with transition intensities as described in figure 1.1.

Let $a = \frac{\lambda}{\mu}$. We assume that $a \leq 2$, which implies that the system is stable. Let P_{ij} be the equilibrium probability of the state (i,j). Let $\pi_n = \sum_{i+j=n} P_{i,j}$, n=0,1,2,.... Thus π_n is the probability that there are exactly n customers in the system in equilibrium.

Denote by q_k the sum of probabilities in the k^{th}time is diagonal

$$q_k = \sum_{i=0}^{\infty} p_{i+ki}, \quad k = 0,1,2$$

and let $q^* = \sum_{k=1}^{\infty} q_k$ be the probability that the queues are unequal. Draw the diagram

3.1 Transition Intensities for the "Shortest Queue"

It is well known that for a Markov chain in equilibrium if the state space is divided into two disjoint subsets, then the "intensity flows" from one subset into the other, are equal for both subsets.

First, for a fixed i, we divide the states into those with first component less than i, and those with first component greater than or equal to i. We get

$$\lambda P_{i-1,i-1} = \mu \sum_{j=0}^{i-1} P_{ij}, i = 1,2,3 \tag{1}$$

Summing (8) over 'i' we get

$$\lambda q_0 = \mu q^* \tag{2}$$

Since, we have

$$q_0 + q^* = 1 \tag{3}$$

Next, w define the 'diagonals'

$$D_k = \bigcup_{i=0}^{\infty} (i+k,i), \quad k = 0,1,2 \tag{4}$$

and let us seperate the diagonals $D_0, D_1, D_2, \cdots, D_k$ from $D_{k+1}, D_{k+2}, \cdots$
The resulting cut yield

$$(1+a)q_0 = (2+a)q_0 - 2p_{00}, \quad for \;\; k = 0 \tag{5}$$

$$(1+a)q_{k+1} = q_k - p_{k0}, \quad for \;\; k = 1,2,3 \tag{6}$$

Summing (5) and (6) over k we get

$$(1+a)q^* = (2+a)q_0 + q^* - (2p_{00} + \sum_{k=1}^{\infty} p_{k0}),$$

and substracting the values of q_0 and q^*, we get the following result

$$2p_{00}+\sum_{k=1}^{\infty}p_{k0}=2-a, \quad for \;\; 0\geq a\geq 2 \tag{7}$$

It is introduced to note that $q_0, q_1, \cdots$ Exist even when $a \geq 2$, if we interpret q_k as the limit of the probability of being in D_k at time t, when $t \to \infty$ independent of the initial conditional. To see that. when $a \geq 2$ the probability that any queue is empty at time t converges to 0 for $t \to \infty$, thus the process behave asymptotically as birth-and-death process on the diagonals D_k, with the birth rate being and the death rate being $\lambda+\mu$ for$k \geq 0$, and for k=0, the birth rate being $\lambda+2\mu$ and the death rate 0. Thus a limiting distribution exists, independent of the initial conditions, and it can be calculated explicity as follows.

4 Analysis of the system

4.1 Service in II when the system is not blocked

Let T_1 and T_2 be two successive epochs of arrival and the system be unlocked at time T_1+0. The system is then necessarily unblocked during the entire interval (T_1, T_2). We will then assume that in the interval (T_1, T_2) the departure process from II is a Markovian death process with possibly state-dependent death rates.

Specifically, let $(T_1 < t < t+dt < T_2)$ and let there be j II-customers in the system at time t, then the event that a customer leaves in $(t, t+dt)$ depends only on j and has probablity $\sigma_i dt$ with $\sigma_0 = 0$ and $\sigma_j > 0$ for j=1,2,3,..,k.

4.2 Service in II when the system is blocked

Let T be any epoch in which the number of II-customers in the system reaches k+1. T is neccessarily the time of a service completion and we refer to the corresponding customers as the blocking customer. The system remains blocked until a later time $T^{'}$ when the blocking customer is released. We assume that the duration $T^{'}-T$ of the blocked time is stochastically independent of the arrival process, the service process in I is conditionally independent of the service process before time T. The probability that $T^{'}-T$ is at most x and that the number of II-customer at $T^{'}+0$ is equal to j will be denoted by $H_j(x)$ with $\sum_{j=1}^{n} H_j(x) = \overline{H(x)}$.

4.3 The Simple Death (Departure) Process

Let $F_\sigma(.)$ denotes the negative exponential distribution of mean $\sigma^{(-1)}$. Suppose that we have a simple Markovian death process with I individuals at time $t = 0, 0 < i < k$ .If at any time t there are ν individuals then the probability that a death, here a departure from II occurs in $(t, t+dt)$ is given by $\sigma_{nu}dt + \sigma(dt)$. We denote by $p_{ij}(t)$ the conditional probability that there are j individuals at time t, given that there are i at t= 0.
The probability $p_{ij}(t)$ are given by
$p_{ij}(t) = F_{(\sigma t)} * F_{(\sigma i-1)} * F_{(\sigma i-2)} * \cdots * F_{\sigma(1)}, p_{ij} = 0, j > i > 0,$
$p_{ij}(t) = F_{\sigma i} * \cdots * F_{\sigma i+1} - F_{\sigma i} * \cdots * F_{\sigma i}, i > j > 0,$

$$p_{ij}(t) = 1 - F_{\sigma i(t)} = e^{\sigma_{it}}, 1 \leq i \leq k, \tag{8}$$

These expressions are elementary and follow from the independence of the times between successive deaths. This is a consequence of the Markov assumption. $A(k+1)$-state Markov renewal process is, related to the service process.

As long as there is a steady supply of customers, it is possible to describe the behaviour of the system in terms of a finite Markov renewal process. This will be made precise later, but at this state it is worthwhile to make the following heuristic consideration. Consider any instant of time in which there are $i \geq j$ Customers in I, one of which has just begun service.

If we disregard for the time being and new arrivals to I, then let $\tau_0, \tau_1, \tau_2, \tau_3, \cdots, \tau_{(i-1)}$be the epochs in which these I customers gin service and let τ_1 be the time of the i^{th} service. Let $\zeta_n, n = 0, 1, 2, 3, \cdots$ be the number at $\tau_{n+0}, n = 0, 1, 2, 3, \cdots, i$.If then follows readily from the assumptions on the system that the random variables $\zeta_n, \tau_0 = 0, \zeta_1, \tau_1 - \tau_0, \cdots, \zeta_i, \tau_i - \tau_{(i-1)}$ may be regarded as the first i+1 states and sorjourn times in a Markov renewal process with k+1 states $1, 2, 3, \cdots, k+1$.

4 Conclusion

The approach of the probability to its heavy traffic limit is very steep, so that this limit, which is easily computable, will not be a good approximation for most values of the load. Asymmetry in the service rates tends to extend to increase this probability in light traffic, but to decrease it in moderate to heavy traffic.

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