2456-8686, vii(ii), 2023: 001-014
https://doi.org/10.26524/cm171

ORIGINAL RESEARCH Open Access

Conformable Fraction Order of SEIQJR Epidemic Model

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Received: 03 April 2023/ Accepted: 25 April 2023/ Published online: 31 July 2023

Abstract

In this paper, we have describe the variational iteration method and derived approximate solutions using the Conformable Fractional Differential transform method for the SEIQJR epidemic model. These techniques are based on conformable derivative, which has recently gained a lot of popularity. First, we have redefined the conformable differential transformation method (CDTM) and variational iteration method utilizing the α-derivative. We then used the SEIQJR epidemic model to illustrate the effectiveness and accuracy of the suggested strategies.

Keywords: Approximate solutions, Conformable transformation method, Variational iteration method, Coronavirus infection system, Numerical methods for difference equations.

AMS Classification: 30C70, 26A33, 49k15, 65Q20.

1 Introduction

From a medical engineering perspective, scientists and experts around the world are working to develop antibiotics or treatments for the COVID-19 pandemic and control the spread of the disease. All infectious diseases can be well-known and understood using mathematical models. The concept began in 1927. Since then, many different mathematical models have been developed for various diseases and ailments. We refer to [1-8] for some studies. [10] Change from SEIR data model, for example Poisson increases with daily usage time, base refresh amount of has value of 3.1. Tang et al. [11] proposed a compartmentalized model of decision-making for chronic disease development, human infection, and intervention. The authors found that protected against up to six pregnancies. 47 and Easy traceability and related technology such as isolation and isolation can be effective in preventing pregnancy and reducing the risk of infection. To determine the scale of the virus outbreak

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in Wuhan, Iman [12] made a computational model of the spread of the disease, focusing on human-to-human transmission. The findings showed that managements should be effective in controlling in more than 60 percentage of outbreaks. To identify and predict new disease outbreaks, Guo et al. [13] developed a deep learning algorithm. The two animals in which they found the disease are the bat and the mink. Most models show the important role of direct human-to-human transmission in the epidemic, as evidenced by the fact that many infected in the Wuhan are a remain unaffected and the number of cases is increasing rapidly. There is more. More than 20 people in all provinces of China [14]. Many infected people have a long incubation period, so they do not show symptoms and do not know they are infected for 10-days. Over time, the disease spreads easily from to other people through direct transmission. On the other hand, the environment allocation of COVID-19 transmission has not been determined in models published so far. Farman et al. studied the stability and regulation of the glucose-insulin-glucagonsystem in humans. The dynamic behavior of a cancer model with vaccine in general has been discussed by Farman et al. Gondim and Machado provide a good strategy for classifying the COVID-19 virus into separate age groups. Davis et al. examined the impact of age-relatedness on the prevalence of and the control of the spread of COVID-19. Youssef et al. adapted the SEIR model and applied it to real cases of COVID-19 in Saudi Arabia. Feng and Theme evaluated the SEIQR (sensitive exposure-infection-quarantine-recovery) model for the period of disease suspicion, including quarantine, and assumed that overall all individuals affected the quarantine level, and model dynamics were examined. Jumpen 1 et al. proposed an epidemic SEIQR model and analyzed the product model and introduced a differential transformation (DE) algorithm to determine the importance of parameters in the model. Gerberry and Milner used the SEIQR model for childhood diseases.

2 Preliminaries of Conformable Fractional Derivative

In this section we recall some of the properties of conformable fractional derivatives.

Definition 2.1 Let $\upsilon \in 0, 1$ and f be υ differentiable at a point $t \succ 0$, and if f is differentiable, then

$$T_{\upsilon} f(t) = t^{1-\upsilon} \frac{d}{dt} f(t). \tag{1}$$

Definition 2.2 The fractional integral of order υ for conformable fractional derivative is defined by

$$I_\upsilon(f)(t) = I(t^{\upsilon-1}f) = \int_0^t s^{\upsilon-1}f(s)ds, \forall\ \upsilon \in (0,1). \tag{2}$$

2.1 The Conformable Fractional Differential Transformation

If f is an infinitely υ-differentiable function, then it can be expanded in a fractional power series expansion around the point $t = 0$ as

$$f(t) = \sum_{k=0}^{\infty} \frac{t^{\upsilon k}}{\upsilon^k k!}\left[(T_\upsilon f)^{(k)}\right]_{t=0}, 0 < t < R^{\frac{1}{\upsilon}}, R > 0. \tag{3}$$

Here, $\left[(T_\upsilon f)^{(k)}\right]_{t=0}$ stands for the fractional derivative's application to the k-times which is defined as

$$F_\upsilon(k) = \frac{1}{\upsilon^k k!}\left[(T_\upsilon f)^{(k)}\right]_{t=0}. \tag{4}$$

If equations (3) and (4) exists, then the following properties holds.

(i) $F_\upsilon(k) = U_\upsilon(k) \pm V_\upsilon(k)$ holds if $f(t) = u(t) \pm v(t)$ exists.
(ii) $F_\upsilon(k) = cU_\upsilon(k)$ holds if If $f(t) = cu(t), c \in \mathbb{R}$ exists.
(iii) $F_\upsilon(k) = \sum_{l=0}^{k} U_\upsilon(l)V_\upsilon(k-l)$ holds if $f(t) = u(t)v(t)$ exists.
(iv) $F_\upsilon(k) = \upsilon(k+1)U_\upsilon(k+1)$ holds if $f(t) = T_\upsilon u(t)$ exists.
(v) For $\delta(k) = \begin{cases} 1,\ if\ k = 0, \\ 0,\ if\ k \neq 0 \end{cases}$, we have $F_\upsilon(k) = \delta(k - (\frac{p}{\upsilon}))$ if $f(t) = (t - t_0)^p$.

3 Mathematical model

The SEIQJR model, an expanded version of SIR and SEIQR, simulates the interaction of humans in various conditions: susceptible (S), exposed (E), infected (I), quarantined (Q), hospitalized (J) and recovered (R). The SEIQJR model equations

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are defined as follows:

$$
\begin{cases}
(dS(t)/dt) = (-\beta S(t)/N)I_s(t) + P_1 I_a(t) + P_2 E_2(t) + P_3 Q(t) + P_4 J(t) \\
(dE_1(t)/dt) = (\beta S(t)/N)(I_s(t) + P_1 I_a(t) + P_2 E_2(t) + P_3 Q(t) + P_4 J(t)) - \mu E_1 \\
(dE_2(t)/dt) = \mu E_1(t) - \delta E_2 \\
(dI_a(t)/dt) = (1-\sigma)\delta E_2(t) - \eta I_a(t) \\
(dI_s(t)/dt) = \sigma\delta E_2(t) - (\rho_1 + \zeta_1 + \zeta_2 + \zeta_3) I_s(t) \\
(dQ(t)/dt) = \zeta I_s(t) - (\xi_1 + \xi_2) Q(t) \\
(dJ(t)/dt) = \zeta_3 I_s(t) + \xi_1 Q(t) - (\rho_2 + \upsilon) J(t) \\
(dR(t)/dt) = \eta I_a(t) + \zeta_2 I_s(t) + \xi_2 Q(t) + \upsilon J(t)
\end{cases}
\tag{5}
$$

where $S(t)$, $E(t)$, $I(t)$, $Q(t)$, $J(t)$ and $R(t)$ are respectively denoted as Susceptible, Exposed, Infected, Quarantined, Hospitalized and Recovered populations. Here the Total population $N = S(t) + E(t) + I(t) + Q(t) + J(t) + R(t)$.
The other parameters and Descriptions of equation (5) are as follows: β represents rate of infection per period of time among symptomatic infected individuals; I_a and I_s represents the Infection Asymptotically and Symptotically; E_1 and E_2 represents the disease affected individuals by virus 1 and virus 2; P_1 represents the reduction in infectiousness by the I_a class as compared to the I_s class; P_2 represents infectivity reduction factor by the E class relative to the I_s class; P_3 represents the Infectivity reduction factor by the Q class versus the I_s class; P_4 represents the J class's reduced infectivity as compared to I_s class; μ represents the rate of E_1 class persons becoming exposed to disease; δ represents the rate of infection for those in the E_2 class; σ represents the proportion of the infectious Exposed population E_2 that becomes Symptomatic infected $(0 < \sigma < 1)$; η represents the recovery rate of asymptomatic infected persons without medical intervention; η represents the recovery rate of asymptomatic infected persons without medical intervention; ρ represents the incidence of infection-related death for infected individuals with symptoms; ζ_1, ζ_2 and ζ_3 represents the speed at which symptomatic infected people are quarantined, without medical treatment and hospitalization rate; ξ_1 represents the rate of hospitalization for those in isolation; ξ_2 represents the rate of infection recovery after quarantine; ρ_2 represents the rate of hospitalized patients' deaths; Finally υ

represents the rate of hospitalized patients being transferred to the recovered class.

4 Conformable fractional order Derivative model

This section will cover conformable fractional-order SEIQJR epidemic models that are resolved by nonlinear issues. The conformable fractional differential transformation method (CFDTM) and the variational iteration method (VIM) are two such techniques. Since the conformable fractional differential equation (CFDE) that makes up the epidemic models is a nonlinear system with no analytical solution, we can obtain the variational iteration formula by applying integration by parts and Lagrange multipliers.

4.1 Mathematical Model of conformable fractional Operator

Here, we will show how the conformable fraction differential system offers a conformable description of the actual evolution of this epidemic in a population of high N. Now, by replacing the conformable derivative T_α in equation (5), we arrive at the equation system presented in (6).

$$\begin{cases} T_\alpha S(t) = (-\beta S(t)/N)(I_s(t) + P_1 I_a(t) + P_2 E_2(t) + P_3 Q(t) + P_4 J(t)) \\ T_\alpha E_1(t) = (\beta S(t)/N)(I_s(t) + P_1 I_a(t) + P_2 E_2(t) + P_3 Q(t) + P_4 J(t)) - \mu E_1 \\ T_\alpha E_2(t) = \mu E_1(t) - \delta E_2 \\ T_\alpha I_a(t) = (1-\sigma)\delta E_2(t) - \eta I_a(t) \\ T_\alpha I_s(t) = \sigma\delta E_2(t) - (\rho_1 + \zeta_1 + \zeta_2 + \zeta_3) I_s(t) \\ T_\alpha Q(t) = \zeta I_s(t) - (\xi_1 + \xi_2) Q(t) \\ T_\alpha J(t) = \zeta_3 I_s(t) + \xi_1 Q(t) - (\rho_2 + v) J(t) \\ T_\alpha R(t) = \eta I_a(t) + \zeta_2 I_s(t) + \xi_2 Q(t) + v J(t) \end{cases} \tag{6}$$

with the initial conditions, $S(0) = N_S$; $E_1(0) = N_{E_1}$; $E_1(0) = N_{E_1}$; $E_2(0) = N_{E_2}$; $I_a(0) = N_{I_a}$; $I_s(0) = N_{I_s}$; $Q(0) = N_Q$; $J(0) = N_J$; $R(0) = N_R$

5 Variational Iteration Method

If L is a linear, N is a nonlinear, and g is any real function, then $L(u(t)) + N(u(t)) = g(t)$ is known to be a non-homogeneous equation. Given below is the

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relevant correlation function for the non-homogeneous equation.

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda L u_n(s) + N\widetilde{u}_n(s) - g(s)ds \tag{7}$$

where $N\widetilde{u}_n$ is regarded as confined variation, i.e., $\delta N\widetilde{u}_n$=0, and λ is the general Lagrange multiplier, which may be ideally determined using variational theory.

Theorem 5.1 Let (6) is considered as the conformable fractional differential equation, then the variational iteration formula is given by

$$\begin{cases} S_{n+1} = S_n(t) - I_\alpha\{T_\alpha S_n(t) + \frac{\beta S(t)}{N}(I_s(t) + P_1 I_a(t) + P_2 E_2(t) + P_3 Q(t) + P_4 J(t))\} \\ E_{1n+1} = E_{1n}(t) - I_\alpha\{T_\alpha E_{1n}(t) - \frac{\beta S(t)}{N}(I_s(t) + P_1 I_a(t) + P_2 E_2(t) + P_3 Q(t) + P_4 J(t)) - \mu E_1\} \\ E_{2n+1} = E_{2n}(t) - I_\alpha\{T_\alpha E_{2n}(t) - \mu E_1(t) + \delta E_2\} \\ I_{an+1} = Q_{an}(t) - I_\alpha\{T_\alpha I_a n(t) - (1-\sigma)\delta E_2(t) + \eta I_a(t)\} \\ I_{sn+1} = Q_{sn}(t) - I_\alpha\{T_\alpha I_s n(t) - \sigma\delta E_2(t) + (\rho_1 + \zeta_1 + \zeta_2 + \zeta_3) I_s(t)\} \\ Q_{n+1} = Q_n(t) - I_\alpha\{T_\alpha Q_n(t) - \zeta I_s(t) - (\xi_1 + \xi_2)Q(t)\} \\ J_{n+1} = J_n(t) - I_\alpha\{T_\alpha J_n(t) - \zeta_3 I_s(t) - \xi_1 Q(t) + (\rho_2 + v)J(t)\} \\ R_{n+1} = R_n(t) - I_\alpha\{T_\alpha R_n(t) - \eta I_a(t) - \zeta_2 I_s(t) - \xi_2 Q(t) + vJ(t)\} \end{cases} \tag{8}$$

Here the n^{th} approximations are S_n, E_n, I_n, Q_n, J_n and R_n. The conformable fractional derivative of order α is T_α and the conformable fractional integral of order α is I_α for $\alpha \in (0,1)$.

Proof: Rewrite the equation (6) in the form

$$\begin{cases} T_\alpha S_n(t) + \frac{\beta S_n(t)}{N}(I_{sn}(t) + P_1 I_{an}(t) + P_2 E_{2n}(t) + P_3 Q_n(t) + P_4 J_n(t)) = 0 \\ T_\alpha E_{1n}(t) - \frac{\beta S_n(t)}{N}(I_{sn}(t) + P_1 I_{an}(t) + P_2 E_{2n}(t) + P_3 Q_n(t) + P_4 J_n(t)) - \mu E_{1n} = 0 \\ T_\alpha E_{2n}(t) - \mu E_{1n}(t) + \delta E_{2n} = 0 \\ T_\alpha I_{an}(t) - (1-\sigma)\delta E_{2n}(t) + \eta I_{an}(t) = 0 \\ T_\alpha I_{sn}(t) - \sigma\delta E_{2n}(t) + (\rho_1 + \zeta_1 + \zeta_2 + \zeta_3) I_{sn}(t) = 0 \\ T_\alpha Q_n(t) - \zeta I_{sn}(t) - (\xi_1 + \xi_2)Q_n(t) = 0 \\ T_\alpha J_n(t) - \zeta_3 I_{sn}(t) - \xi_1 Q_n(t) + (\rho_2 + v)J_n(t) = 0 \\ T_\alpha R_n(t) - \eta I_{an}(t) - \zeta_2 I_{sn}(t) - \xi_2 Q_n(t) + vJ_n(t) = 0 \end{cases} \tag{9}$$

Multiplying the general Lagrange multipliers $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, $\lambda_4(t)$, $\lambda_5(t)$, $\lambda_6(t)$, $\lambda_7(t)$ and $\lambda_8(t)$ in the above equation, we get

$$\begin{cases} \lambda_1(t)\left\{T_\alpha S_n(t) + \frac{\beta S_n(t)}{N}(I_{sn}(t) + P_1 I_{an}(t) + P_2 E_{2n}(t) + P_3 Q_n(t) + P_4 J_n(t))\right\} = 0 \\ \lambda_2(t)\left\{T_\alpha E_{1n}(t) - \frac{\beta S_n(t)}{N}(I_{sn}(t) + P_1 I_{an}(t) + P_2 E_{2n}(t) + P_3 Q_n(t) + P_4 J_n(t)) - \mu E_{1n}\right\} = 0 \\ \lambda_3(t)\{T_\alpha E_{2n}(t) - \mu E_{1n}(t) + \delta E_{2n}\} = 0 \\ \lambda_4(t)\{T_\alpha I_{an}(t) - (1-\sigma)\delta E_{2n}(t) + \eta I_{an}(t)\} = 0 \\ \lambda_5(t)\{T_\alpha I_{sn}(t) - \sigma\delta E_{2n}(t) + (\rho_1 + \zeta_1 + \zeta_2 + \zeta_3) I_{sn}(t)\} = 0 \\ \lambda_6(t)\{T_\alpha Q_n(t) - \zeta I_{sn}(t) - (\xi_1 + \xi_2)Q_n(t)\} = 0 \\ \lambda_7(t)\{T_\alpha J_n(t) - \zeta_3 I_{sn}(t) - \xi_1 Q_n(t) + (\rho_2 + v)J_n(t)\} = 0 \\ \lambda_8(t)\{T_\alpha R_n(t) - \eta I_{an}(t) - \zeta_2 I_{sn}(t) - \xi_2 Q_n(t) + vJ_n(t)\} = 0 \end{cases} \tag{10}$$

Now, taking I_α on both sides of the above equation (10), we get

$$\begin{cases} I_\alpha\left[\lambda_1(t)\left\{T_\alpha S_n(t)+\frac{\beta S_n(t)}{N}(I_{sn}(t)+P_1I_{an}(t)+P_2E_{2n}(t)+P_3Q_n(t)+P_4J_n(t))\right\}\right]=0\\ I_\alpha\left[\lambda_2(t)\{T_\alpha E_{1n}(t)-\frac{\beta S_n(t)}{N}(I_{sn}(t)+P_1I_{an}(t)+P_2E_{2n}(t)+P_3Q_n(t)+P_4J_n(t))-\mu E_{1n}(t)\}\right]=0\\ I_\alpha\left[\lambda_3(t)\{T_\alpha E_{2n}(t)-\mu E_{1n}(t)+\delta E_{2n}\}\right]=0\\ I_\alpha\left[\lambda_4(t)\{T_\alpha I_{an}(t)-(1-\sigma)\delta E_{2n}(t)+\eta I_{an}(t)\}\right]=0\\ I_\alpha\left[\lambda_5(t)\{T_\alpha I_{sn}(t)-\sigma\delta E_{2n}(t)+(\rho_1+\zeta_1+\zeta_2+\zeta_3)I_{sn}(t)\}\right]=0\\ I_\alpha\left[\lambda_6(t)\{T_\alpha Q_n(t)-\zeta I_{sn}(t)-(\xi_1+\xi_2)Q_n(t)\}\right]=0\\ I_\alpha\left[\lambda_7(t)\{T_\alpha J_n(t)-\zeta_3I_{sn}(t)-\xi_1Q_n(t)+(\rho_2+v)J_n(t)\}\right]=0\\ I_\alpha\left[\lambda_8(t)\{T_\alpha R_n(t)-\eta I_{an}(t)-\zeta_2I_{sn}(t)-\xi_2Q_n(t)+vJ_n(t)\}\right]=0 \end{cases} \tag{11}$$

Thus, the correlation function formula of (11) will becomes

$$\begin{cases} S_{n+1}=S_n(t)+I_\alpha\left[\lambda_1(t)\{T_\alpha S_n(t)+\frac{\beta S_n(t)}{N}(I_{sn}(t)+P_1I_{an}(t)+P_2E_{2n}(t)+P_3Q_n(t)+P_4J_n(t))\}\right]\\ E_{1n+1}=E_{1n}(t)+I_\alpha\left[\lambda_2(t)\{T_\alpha E_{1n}(t)-\frac{\beta S_n(t)}{N}(I_{sn}(t)+P_1I_{an}(t)+P_2E_{2n}(t)+P_3Q_n(t)+\right.\\ \qquad\left.P_4J_n(t))-\mu E_{1n}(t)\}\right]\\ E_{2n+1}=E_{2n}(t)+I_\alpha\left[\lambda_3(t)\{T_\alpha E_{2n}(t)-\mu E_{1n}(t)+\delta E_{2n}\}\right]\\ I_{an+1}=I_{an}(t)+I_\alpha\left[\lambda_4(t)\{T_\alpha I_{an}(t)-(1-\sigma)\delta E_{2n}(t)+\eta I_{an}(t)\}\right]\\ I_sn+1=I_{sn}(t)+I_\alpha\left[\lambda_5(t)\{T_\alpha I_{sn}(t)-\sigma\delta E_{2n}(t)+(\rho_1+\zeta_1+\zeta_2+\zeta_3)I_{sn}(t)\}\right]\\ Q_{n+1}=Q_n(t)+I_\alpha\left[\lambda_6(t)\{T_\alpha Q_n(t)-\zeta I_{sn}(t)-(\xi_1+\xi_2)Q_n(t)\}\right]\\ J_{n+1}=J_n(t)+I_\alpha\left[\lambda_7(t)\{T_\alpha J_n(t)-\zeta_3I_{sn}(t)-\xi_1Q_n(t)+(\rho_2+v)J_n(t)\}\right]\\ R_{n+1}=R_n(t)+I_\alpha\left[\lambda_8(t)\{T_\alpha R_n(t)-\eta I_{an}(t)-\zeta_2I_{sn}(t)-\xi_2Q_n(t)+vJ_n(t)\}\right] \end{cases} \tag{12}$$

The fractional integral of order α given in Definition 2.2 yields the following:

$$\begin{cases} S_{n+1}=S_n(t)+\int_0^t\tau^{\alpha-1}\left[\lambda_1(\tau)\{\tau^{\alpha-1}\frac{d}{d\tau}S_n(\tau)+\frac{\beta\tilde{S}_n(\tau)}{N}(\tilde{I}_{sn}(\tau)+P_1\tilde{I}_{an}(\tau)+P_2\tilde{E}_{2n}(\tau)+P_3\tilde{Q}_n(\tau)+P_4\tilde{J}_n(\tau))\}\right]d\tau\\ E_{1n+1}=E_1n(t)+\int_0^t\tau^{\alpha-1}\left[\lambda_2(\tau)\{\tau^{\alpha-1}\frac{d}{d\tau}E_{1n}(\tau)-\frac{\beta\tilde{S}_n(\tau)}{N}(\tilde{I}_{sn}(\tau)+P_1\tilde{I}_{an}(\tau)+P_2\tilde{E}_{2n}(\tau)+P_3\tilde{Q}_n(\tau)+P_4\tilde{J}_n(\tau))-\mu\tilde{E}_{1n}(\tau)\}\right]d\tau\\ E_{2n+1}=E_2n(t)+\int_0^t\tau^{\alpha-1}\left[\lambda_3(\tau)\left\{\tau^{\alpha-1}\frac{d}{d\tau}E_{2n}(\tau)-\mu\tilde{E}_{1n}(\tau)+\delta\tilde{E}_{2n}(\tau)\right\}\right]d\tau\\ I_{an+1}=I_an(t)+\int_0^t\tau^{\alpha-1}\left[\lambda_4(\tau)\left\{\tau^{\alpha-1}\frac{d}{d\tau}I_{an}(\tau)-(1-\sigma)\delta\tilde{E}_{2n}(\tau)+\eta\tilde{I}_{an}(\tau)\right\}\right]d\tau\\ I_{sn+1}=I_sn(t)+\int_0^t\tau^{\alpha-1}\left[\lambda_5(\tau)\left\{\tau^{\alpha-1}\frac{d}{d\tau}I_{sn}(\tau)-\sigma\delta\tilde{E}_{2n}(\tau)+(\rho_1+\zeta_1+\zeta_2+\zeta_3)\tilde{I}_{sn}(\tau)\right\}\right]d\tau\\ Q_{n+1}=Q_n(t)+\int_0^t\tau^{\alpha-1}\left[\lambda_6(\tau)\left\{\tau^{\alpha-1}\frac{d}{d\tau}Q_n(\tau)-\zeta\tilde{I}_{sn}(\tau)-(\xi_1+\xi_2)\tilde{Q}_n(\tau)\right\}\right]d\tau\\ J_{n+1}=J_n(t)+\int_0^t\tau^{\alpha-1}\left[\lambda_7(\tau)\left\{\tau^{\alpha-1}\frac{d}{d\tau}J_n(\tau)-\zeta_3\tilde{I}_{sn}(\tau)-\xi_1\tilde{Q}_n(\tau)+(\rho_2+v)\tilde{J}_n(\tau)\right\}\right]d\tau\\ R_{n+1}=R_n(t)+\int_0^t\tau^{\alpha-1}\left[\lambda_8(\tau)\left\{\tau^{\alpha-1}\frac{d}{d\tau}R_n(\tau)-\eta\tilde{I}_{an}(\tau)-\zeta_2\tilde{I}_{sn}(\tau)-\xi_2\tilde{Q}_n(\tau)+v\tilde{J}_n(\tau)\right\}\right]d\tau \end{cases}$$

Where $\tilde{S_n}$, $\tilde{E_{1n}}$, $\tilde{E_{2n}}$, $\tilde{I_{an}}$, $\tilde{I_{sn}}$, $\tilde{Q_n}$, $\tilde{J_n}$ and $\tilde{R_n}$ are the restricted variations with $\delta\tilde{S_n}=0$, $\delta\tilde{E_{1n}}=0$, $\delta\tilde{E_{2n}}=0$, $\delta\tilde{I_{an}}=0$, $\delta\tilde{I_{sn}}=0$, $\delta\tilde{Q_n}=0$, $\delta\tilde{J_n}=0$, $\delta\tilde{R_n}=0$.

The Langrange multiplier $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, $\lambda_4(t)$, $\lambda_5(t)$, $\lambda_6(t)$, $\lambda_7(t)$ and $\lambda_8(t)$ can be obtained by $\lambda_j'(\tau)=0$ for all j=$\{1,2,3,4,5,6,7,8\}$ with boundary condition $1+\lambda_j(t)=0$,for all j=$\{1,2,3,4,5,6,7,8\}$. Solving the last initial value problem for λ_j for all $1\le j\le 8$, for general lagrange multiplier λ_j is found to be $\lambda_j=-1$, for all $1\le j\le 8$. Hence, substituting the values of λ_j into the corresponding correction functional, we get (9).

2456-8686, vii(ii), 2023: 001-014
https://doi.org/10.26524/cm171

6 Numerical Illustration

6.1 Scheme of conformable fractional differential transform

With the help of the Properties 1-5, the equation (6) will be constructed in the form of

$$\begin{cases} \alpha(k+1)S_{\alpha}(t) = (-\beta S(t)/N)(I_s(t) + P_1 I_a(t) + P_2 E_2(t) + P_3 Q(t) + P_4 J(t)) \\ \alpha(k+1)E_{1\alpha}(t) = (\beta S(t)/N)(I_s(t) + P_1 I_a(t) + P_2 E_2(t) + P_3 Q(t) + P_4 J(t)) - \mu E_1 \\ \alpha(k+1)E_{2\alpha}(t) = \mu E_1(t) - \delta E_2 \\ \alpha(k+1)I_{a\alpha}(t) = (1-\sigma)\delta E_2(t) - \eta I_a(t) \\ \alpha(k+1)I_{s\alpha}(t) = \sigma\delta E_2(t) - (\rho_1 + \zeta_1 + \zeta_2 + \zeta_3)I_s(t) \\ \alpha(k+1)Q_{\alpha}(t) = \zeta I_s(t) - (\xi_1 + \xi_2)Q(t) \\ \alpha(k+1)J_{\alpha}(t) = \zeta_3 I_s(t) + \xi_1 Q(t) - (\rho_2 + v)J(t) \\ \alpha(k+1)R_{\alpha}(t) = \eta I_a(t) + \zeta_2 I_s(t) + \xi_2 Q(t) + vJ(t) \end{cases} \tag{13}$$

Therefore, we obtain the recurrence relation as

$$\begin{cases} S_{\alpha}(t) = (1/\alpha(k+1))\Big[(-\beta S(t)/N)(I_s(t) + P_1 I_a(t) + P_2 E_2(t) + P_3 Q(t) + P_4 J(t))\Big] \\ E_{1\alpha}(t) = (1/\alpha(k+1))\Big[(\beta S(t)/N)(I_s(t) + P_1 I_a(t) + P_2 E_2(t) + P_3 Q(t) + P_4 J(t)) - \mu E_1\Big] \\ E_{2\alpha}(t) = (1/\alpha(k+1))\Big[\mu E_1(t) - \delta E_2\Big] \\ I_{a\alpha}(t) = (1/\alpha(k+1))\Big[(1-\sigma)\delta E_2(t) - \eta I_a(t)\Big] \\ I_{s\alpha}(t) = (1/\alpha(k+1))\Big[\sigma\delta E_2(t) - (\rho_1 + \zeta_1 + \zeta_2 + \zeta_3)I_s(t)\Big] \\ Q_{\alpha}(t) = (1/\alpha(k+1))\Big[\zeta I_s(t) - (\xi_1 + \xi_2)Q(t)\Big] \\ J_{\alpha}(t) = (1/\alpha(k+1))\Big[\zeta_3 I_s(t) + \xi_1 Q(t) - (\rho_2 + v)J(t)\Big] \\ R_{\alpha}(t) = (1/\alpha(k+1))\Big[\eta I_a(t) + \zeta_2 I_s(t) + \xi_2 Q(t) + vJ(t)\Big] \end{cases} \tag{14}$$

Let us take the assumed initial conditions of equation (9) per day as $N = 24518970$, $S(0) = 24218169$, $E_1(0) = 300$, $E_2(0) = 500$, $I_a(0) = 60$, $I_s(0) = 90$, $Q(0) = 31$, $J(0) = 50$ and $R(0) = 40$. The other parameter values are mentioned in Table 1.

Table 1: Initial values of transmission rates

Parameters	Description	Prameters	Description
β	1.12	P_1, P_2, P_3, P_4	0.195
δ	0.75	v	0.025
η	0.5	μ	0.29
σ	0.5	ρ_1	0.075
ζ_1	0.035	ζ_2	0.05
ζ_3	0.05	ξ_1	0.07
ξ_2	0.075	ρ_2	0.035

2456-8686, vii(ii), 2023: 001-014
https://doi.org/10.26524/cm171

Now, substituting $k = 0$ in (13), we obtain the first iteration value as

$$S_\alpha(1) = \frac{1}{\alpha(1)}\Big[(-\beta S(0)/N)(I_s(0) + P_1 I_a(0) + P_2 E_2(0) + P_3 Q(0) + P_4 J(0))\Big]$$

$$= \frac{1}{\alpha(1)} \frac{(-1.12)24518169}{24518970}\Big[90 + 0.195(60) + 0.195(500) + 0.195(31) + 0.195(50)\Big]$$

$$S_\alpha(1) = \frac{-240.7729}{\alpha}$$

$$E_{1\alpha}(1) = \frac{1}{\alpha(1)}\Big[\frac{-\beta S(0)}{N}(I_s(0) + P_1 I_a(0) + P_2 E_2(0) + P_3 Q(0) + P_4 J(0)) - \mu E_{1\alpha}(0)\Big]$$

$$= \frac{1}{\alpha(1)} \frac{(1.12)24518169}{24518970}\Big[90 + 0.195(60) + 0.195(500) + 0.195(31) + 0.195(50)\Big]$$

$$- 0.29(300)$$

$$E_{1\alpha}(1) = \frac{143.3416}{\alpha}$$

$$E_{2\alpha}(1) = \frac{1}{\alpha(1)}\Big[\mu E_1(0) - \delta E_{2\alpha}(0)\Big] = \frac{1}{\alpha(1)}\Big[0.29(300) - 0.75(500)\Big]$$

$$E_{1\alpha}(1) = \frac{-288}{\alpha}$$

$$I_{a\alpha}(1) = \frac{1}{\alpha(1)}\Big[(1-\sigma)\delta E_2(0) - \eta I_a(0)\Big] = \frac{1}{\alpha(1)}\Big[(1-0.5)(0.75)(500) - 0.5(60)\Big]$$

$$I_{a\alpha}(1) = \frac{157.5}{\alpha}$$

$$I_{s\alpha}(1) = \frac{1}{\alpha(1)}\Big[\sigma\delta E_2(0) - (\rho_1 + \zeta_1 + \zeta_2 + \zeta_3) I_s(0)\Big]$$

$$= \frac{1}{\alpha(1)}\Big[(0.5)(0.75)(500) - (0.075 + 0.035 + 0.05 + 0.07)(90)\Big]$$

$$I_{s\alpha}(1) = \frac{166.8}{\alpha}$$

$$Q_\alpha(1) = \frac{1}{\alpha(1)}\Big[\zeta I_s(0) - (\xi_1 + \xi_2) Q(0)\Big]$$

$$= \frac{1}{\alpha(1)}\Big[(0.035)(0.75)(500) - (0.075 + 0.035 + 0.05 + 0.07)(90)\Big]$$

$$Q_\alpha(1) = \frac{-7.575}{\alpha}$$

$$J_\alpha(1) = \frac{1}{\alpha(1)}\Big[\zeta_3 I_s(0) + \xi_1 Q(0) - (\rho_2 + v) J(0)\Big]$$

$$= \frac{1}{\alpha(1)}\Big[0.07(90) - 0.07(31) - (0.035 + 0.025)(50)\Big]$$

$$J_\alpha(1) = \frac{1.13}{\alpha}$$

$$R_\alpha(1) = \frac{1}{\alpha(1)}\Big[\eta I_a(0) + \zeta_2 I_s(0) + \xi_2 Q(t) + v J(0)\Big]$$

$$= \frac{1}{\alpha(1)}\Big[0.5(60) - 0.05(90) - 0.075(31) + 0.025(50)\Big]$$

$$R_\alpha(1) = (24.425/\alpha)$$

Taking $k = 1$ in equation (13), we obtain

$S_\alpha(2) = \frac{1}{\alpha(2)}\Big[(-\beta S(1)/N)(I_s(1) + P_1 I_a(1) + P_2 E_2(1) + P_3 Q(1) + P_4 J(1))\Big]$
$= \frac{1}{\alpha(2)}\frac{(-1.12)(-240.7729)}{24518970\alpha}((1.668 + 30.7125 - 56.16 - 1.4771 + 0.22035)/\alpha)$

$S_\alpha(2) = \frac{-0.000137674}{\alpha^3}$

$E_{1\alpha}(2) = \frac{1}{\alpha(2)}\Big[\frac{-\beta S(1)}{N}(I_s(1) + P_1 I_a(1) + P_2 E_2(1) + P_3 Q(1) + P_4 J(1)) - \mu E_{1\alpha}(1)\Big]$
$= \frac{1}{(2)\alpha}(0.00001099824/\alpha^2)\Big[-25.03625 - 41.56906\Big]$

$E_{1\alpha}(2) = \frac{-0.0003662625}{\alpha^3}$

$E_{2\alpha}(2) = \frac{1}{\alpha(2)}\Big[\mu E_1(1) - \delta E_{2\alpha}(1)\Big] = (\frac{1}{(2)\alpha})(41.5690 + 216/\alpha)$

$E_{2\alpha}(2) = \frac{128.7845}{\alpha^2}$

$I_{a\alpha}(2) = \frac{1}{\alpha(2)}\Big[(1-\sigma)\delta E_2(1) - \eta I_a(1)\Big] = (1/2\alpha)(-108 - 78.75/\alpha)$

$I_{a\alpha}(2) = \frac{-93.375}{\alpha^2}$

$= \frac{1}{\alpha(2)}\Big[\sigma\delta E_2(1) - (\rho_1 + \zeta_1 + \zeta_2 + \zeta_3)I_s(1)\Big] = (\frac{1}{(2)\alpha})(-108 - 38.364/\alpha)$

$I_{s\alpha}(2) = \frac{-73.182}{\alpha^2}$

$Q_\alpha(2) = \frac{1}{\alpha(2)}\Big[\zeta I_s(1) - (\xi_1 + \xi_2)Q(1)\Big] = (\frac{1}{(2)\alpha})(-7.56 - 38364/\alpha)$

$Q_\alpha(2) = \frac{-22.962}{\alpha^2}$

$J_\alpha(2) = \frac{1}{\alpha(2)}\Big[\zeta_3 I_s(1) + \xi_1 Q(1) - (\rho_2 + v)J(1)\Big]$
$= \frac{1}{(2)\alpha}(11.676 - 0.53025 - 0.0678/\alpha)$

$J_\alpha(2) = \frac{5.53895}{\alpha^2}$

$R_\alpha(2) = \frac{1}{(2)\alpha}\Big[\eta I_a(1) + \zeta_2 I_s(1) + \xi_2 Q(t) + vJ(1)\Big]$
$= \frac{1}{(2)\alpha}(78.75 - 8.34 + 0.56812 + 0.02825/\alpha)$

$R_\alpha(2) = \frac{35.503185}{\alpha^2}$

Similarly, inserting $k = 2$ in equation (13), we obtain the third iteration value as

$S_\alpha(3) = \frac{1}{\alpha(3)}\Big[(-\beta S(2)/N)(I_s(2) + P_1 I_a(2) + P_2 E_2(2) + P_3 Q(2) + P_4 J(2))\Big]$
$= \frac{1}{\alpha(3)}\frac{(-1.12)0.000137674}{24518970\alpha^2}(-73.182 - 18.208125 + 25.112977 - 4.47759 + 1.080095/\alpha^2)$

$S_\alpha(3) = (1.460566141 * 10^{-10}/\alpha^5)$

$E_{1\alpha}(3) = \frac{1}{\alpha(3)}\Big[\frac{-\beta S(2)}{N}(I_s(2) + P_1 I_a(2) + P_2 E_2(2) + P_3 Q(2) + P_4 J(2)) - \mu E_{1\alpha}(2)\Big]$

$$= \frac{1}{\alpha(3)} \frac{(6.288799244)(10^{-12})}{\alpha^2} \left[\frac{-25.03625}{\alpha^2} + \frac{(7.14211875)(10^{-5})}{\alpha^3} \right]$$

$$E_{1\alpha}(3) = \frac{(-5.248265)(10^{-11})}{\alpha^3} + \frac{(1.497178367)(10^{-16})}{\alpha^4}$$

$$E_{2\alpha}(3) = \frac{1}{\alpha(3)} \left[\mu E_1(2) - \delta E_{2\alpha}(2) \right] = \frac{1}{\alpha(3)} \left[\frac{(-1.06216125)(10^{-4})}{\alpha^3} - \frac{96.588375}{\alpha^2} \right]$$

$$E_{2\alpha}(3) = \frac{(-3.5405375)(10^{-5})}{\alpha^4} - \frac{32.196125}{\alpha^3}$$

$$I_{a\alpha}(3) = \frac{1}{\alpha(3)} \left[(1-\sigma)\delta E_2(2) - \eta I_a(2) \right] = \frac{1}{\alpha(3)} \left[\frac{45.2941875 + 46.6875}{\alpha^2} \right]$$

$$I_{a\alpha}(3) = \frac{30.6605625}{\alpha^3}$$

$$I_{s\alpha}(3) = \frac{1}{\alpha(3)} (\sigma\delta E_2(2) - (\rho_1 + \zeta_1 + \zeta_2 + \zeta_3) I_s(2)) = \frac{1}{\alpha(3)} (15.09806825 + 5.61062/\alpha^2)$$

$$I_{s\alpha}(3) = (20.7086825/\alpha^3)$$

$$Q_\alpha(3) = \frac{1}{\alpha(3)} \left[\zeta I_s(2) - (\xi_1 + \xi_2) Q(2) \right] = \frac{1}{\alpha(3)} \left[\frac{1.60980625 + 5.61962}{\alpha^2} \right]$$

$$Q_\alpha(3) = \frac{7.22042625}{\alpha^3}$$

$$J_\alpha(3) = \frac{1}{\alpha(3)} \left[\zeta_3 I_s(2) + \xi_1 Q(2) - (\rho_2 + v) J(2) \right]$$

$$= \frac{1}{\alpha(3)} \left[(-5.12274 + 1.60734 - 0.332337/\alpha^2) \right]$$

$$J_\alpha(3) = \frac{-1.282579}{\alpha^3}$$

$$R_\alpha(3) = \frac{1}{\alpha(3)} \left[\eta I_a(2) + \zeta_2 I_s(2) + \xi_2 Q(t) + v J(2) \right]$$

$$= \frac{1}{\alpha(3)} \left[(-15.5625 + 1.2197 + 0.57405 + 0.138474/\alpha^2) \right]$$

$$R_\alpha(3) = \frac{-13.630276}{\alpha^3}$$

Proceeding like this for $k = 3, 4, 5, \ldots$, we get the other terms. Hence the general form for all the transmission series will be

$$S(t) = \sum_{l=0}^{k} S(k) t^{k\alpha} = 2518269 - 240.7729(t^\alpha/\alpha) - 0.000137674(t^{2\alpha}/\alpha^2) + 1.460566141 \times 10^{-10}(t^{3\alpha}/\alpha^3) + \cdots$$

$$E_1(t) = \sum_{l=0}^{k} E_1(k) t^{k\alpha} = 300 + 143.3416(t^\alpha/\alpha) - 0.0003662625(t^{2\alpha}/\alpha^2) - \left[(5.248265 * 10^{-11}/\alpha^3) + (1.49717836 * 10^{-16}/\alpha^4) \right] t^{3\alpha} + \cdots$$

$$E_2(t) = \sum_{l=0}^{k} E_2(k) t^{k\alpha} = 500 - 288(t^\alpha/\alpha) + 129.7845(t^{2\alpha}/\alpha^2) - \left[(3.5405375 * 10^{-5}/\alpha^4) - (32.196125/\alpha^3) \right] t^{3\alpha} + \cdots$$

$$I_a(t) = \sum_{l=0}^{k} I_a(k) t^{k\alpha} = 60 + 157.5(t^\alpha/\alpha) - 93.375(t^{2\alpha}/\alpha^2) + 30.6605625(t^{3\alpha}/\alpha^3) + \cdots$$

$$I_s(t) = \sum_{l=0}^{k} I_s(k)t^{k\alpha} = 90 + 166.8(t^{\alpha}/\alpha) - 73.182(t^{2\alpha}/\alpha^2) + 20.7086825(t^{3\alpha}/\alpha^3) + \cdots$$

$$Q(t) = \sum_{l=0}^{k} Q(k)t^{k\alpha} = 31 - 7.575(t^{\alpha}/\alpha) - 22.962(t^{2\alpha}/\alpha^2) + 7.22042625(t^{3\alpha}/\alpha^3) + \cdots$$

$$J(t) = \sum_{l=0}^{k} J(k)t^{k\alpha} = 50 + 1.13(t^{\alpha}/\alpha) + 5.53895(t^{2\alpha}/\alpha^2) - 1.282579(t^{3\alpha}/\alpha^3) \cdots$$

$$R(t) = \sum_{l=0}^{k} R(k)t^{k\alpha} = 40 + 24.425(t^{\alpha}/\alpha) + 35.503185(t^{2\alpha}/\alpha^2) - 13.630276(t^{3\alpha}/\alpha^3) + \cdots$$

If we put the α value, one can easily analyze the transmission of SEIQJR model.

7 Conclusion

In our research work, the conformable fractional-order SEIQJR model are solved by conformable fractional differential transform method and variational iteration method. We analyse the SEIQJR model using the conformable fractional differentiation technique with an initial condition and numerical values and using the conformable fractional differential transformation approach and variational iteration to predict the Disease transmission.

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