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Interval-Valued Pythagorean Fuzzy Perfectly Weakly Generalized Continuous Mappings

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Abstract

The purpose of this paper is to introduce and study the concepts of Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mappings and Interval-valued Pythagorean fuzzy perfectly weakly generalized open mappings in Interval-valued Pythagorean fuzzy topological space. Some of their properties are explored.

Key words: Interval-valued Pythagorean fuzzy topology, Interval-valued Pythagorean fuzzy weakly generalized closed set, Interval-valued Pythagorean fuzzy weakly generalized open set, Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mappings, Interval-valued Pythagorean fuzzy perfectly weakly generalized open mappings.

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1. Introduction

Fuzzy set(FS), proposed by Zadeh [18]in 1965, as a framework to encounter uncertainty, vagueness and partial truth , represents a degree of membership for each member of the universe of discourse to a subset of it. Later on, fuzzy topology was introduced by Chang [2]in 1967.By adding the degree of non-membership to Fuzzy Set, Atanassov[1]proposed Interval-valued fuzzy set (IVFS) in 1983 which looks more accurately to uncertainty quantification and provides the opportunity to precisely model the problem based on the existing knowledge and observations. After this, there have been several generalizations of notions of fuzzy sets and fuzzy topology. In the last few years various concepts in fuzzy were extended to Interval-valued fuzzy sets. In 1997, Coker [3]introduced the concept of Interval-valued fuzzy topological space.

In this paper, we introduce the notion of Interval-valued fuzzy perfectly weakly generalized continuous mappings and Interval-valued fuzzy perfectly weakly

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generalized open mappings in Interval-valued fuzzy topological space and study some of their properties. We provide some characterizations of Interval-valued fuzzy perfectly weakly generalized continuous mappings and establish the relationships with other classes of early defined forms of Interval-valued fuzzy mappings.

2.Preliminaries:

Definition 2.1 [15, 16] Let X be non-empty set and I the unit interval [0,1]. A PF set S is an object having the form $P = \{x, \mu_p(x), \nu_p(x) : x \in X\}$ where the function $\mu_p : X \to [0,1]$ and $\nu_p : X \to [0,1]$ denote respectively the degree of membership and degree of non-membership of each element $x \in X$ to the set P, and $0 \leq (\mu_p(x))^2$, $(\nu_p(x))^2 \leq 1$, for each $x \in X$.

Definition 2.2 [1] Let A and B be IFS's of the forms $A = \{[x, \mu_A(x), \nu_A(x)]/x \in X\}$ and $B = \{[x, \mu_B(x), \nu_B(x)]/x \in X\}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$ for all $x \in X$.
(b) A = B if and only if $A \subseteq B$ and $B \subseteq A$.
(c) $A^c = \{[x, \nu_A(x), \mu_A(x)]/x \in X\}$.
(d) $A \cap B = \{[x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)]/x \in X\}$.
(e) $A \cup B = \{[x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)]/x \in X\}$.

Definition 2.3 [4] An Intuitionistic fuzzy topology (IFT in short) on a non empty X is a family τ of IFS in X satisfying the following axioms:

(a) $0, 1 \in \tau$,
(b) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
(c) $\bigcup G_i \in \tau$, for any arbitrary family $\{G_i/i \in J\} \subseteq \tau$.

The pair (X, τ) is called an Interval-valued fuzzy topological space (IFTS in short) and any IFS in τ is known as an Intuitionistic fuzzy open set (IFOS for short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an Intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.4 [3] Let (X, τ) be an IVPFTS and $A = \{x, \mu_1(x), \nu_1(x)/x \in X\}$ be an IVPFs in X. Then the interior and closure of A are defined as
IVPF int(A)=U{G/G is an IVPFOS in X and G⊆A.

IVPF cl(A)=n{K/K is an IVPFCS in X and A⊆K}.
For any IVPFs A in (X,τ) we have,
IVPF $cl(A^c) = (IVPF \quad int(A))^c$ and IVPF $int(A^c) = (IVPF \quad cl(A)^c)$.

Definition 2.5 An IFS $A = \{[x, \mu_A(x), \nu_A(x)]/x \in X\}$ in an IFTS (X,τ) is called an

(a) intuitionistic fuzzy semi closed set (IFSCS) if int(cl (A)) $\subseteq A$[4].
(b) intuitionistic fuzzy α-closed set (IF αCS) if cl(int(cl(A))) $\subseteq A$[4].
(c) intuitionistic fuzzy pre-closed set (IFPCS) if cl(int(A)) $\subseteq A$ [4].
(d) intuitionistic fuzzy regular closed set (IFRCS) if cl(int(A))=A[4].
(e) intuitionistic fuzzy generalized closed set (IFGCS) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS[4].
(f) intuitionistic fuzzy generalized semi closed set (IFGSCS) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS [13].
(g) intuitionistic fuzzy α generalized closed set (IFαGCS) if $\alpha cl(A) \subseteq U$,whenever $A \subseteq U$ and U is an IFOS[12].

An IFS A is called intuitionistic fuzzy semi open set, intuitionistic fuzzy α-open set, intuitionistic fuzzy pre-open set, intuitionistic fuzzy regular open set, intuitionistic fuzzy generalized open set, intuitionistic fuzzy generalized semi open set and intuitionistic fuzzy α generalized open set (IFSOS, IFαOS, IFPOS, IFROS, IFGOS, IFGSOS and IFαGOS) if the complement of A^c is an IFSCS, IFαCS, IFPCS, IFRCS, IFGCS, IFGSCS and IFαGCS respectively.

Definition 2.6 Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be

(a) intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B)\square IFO(X)$ for every $B\square\sigma$[4],
(b) intuitionistic fuzzy α continuous(IFα continuous in short) if $f^{-1}(B)\square IF\alpha O(X)$ for every $B\square\sigma$[6],
(c) intuitionistic fuzzy pre continuous (IFP continuous in short) if $f^{-1}(B)\square IFPO(X)$ for every $B\square\sigma$[6],
(d) intuitionistic fuzzy generalized continuous (IFG continuous in short) if $f^{-1}(B)\square IFGO(X)$ for every $B\square\sigma$[14],
(e) intuitionistic fuzzyα generalized continuous (IFαG continuous in short) if $f^{-1}(B)\square IF\alpha GO(X)$ for every $B\square\sigma$[13],

(f) intuitionistic fuzzy weakly generalized continuous (IFWG continuous in short) if $f^{-1}(B)\square IFWGO(X)$ for every $B\square\sigma$[10],
(g) intuitionistic fuzzy almost continuous (IFA continuous in short) if $f^{-1}(B)\square IFO(X)$ for every IFROS$B\square\sigma$[17],
(h) intuitionistic fuzzy almost weakly generalized continuous (IFAWG continuous in short) if $f^{-1}(B)\square IFWGO(X)$for every IFROS $B\square\sigma$[11],
(i) intuitionistic fuzzy quasi weakly generalized continuous if $f^{-1}(B)\square IFO(X)$ for every IFWGOS $B\square\sigma$[13],
(j) intuitionistic fuzzy weakly generalized irresolute (IFWG irresolute in short) if $f^{-1}(B)\square IFWGO(X)$ for every IFWGOS $B\square\sigma$ [9],
(k) intuitionistic fuzzy totally continuous mapping if the inverse image of every IFCS in Y is an intuitionistic fuzzy clopen subset in X [7],
(l) intuitionistic fuzzy weakly generalized * open mapping if f(A) is an IFWGOS in Y for every IFWGOS A in X [11].

3. Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mappings:

In this section, we introduce Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mappings and study some of their properties.

Definition 3.1 A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous (IVPF perfectly WG continuous in short) mapping if the inverse image of every IVPFWGCS of Y is Interval-valued pythagorean fuzzy clopen in X.

Theorem 3.2 Every Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping is an Interval-valued pythagorean fuzzy continuous mapping but not conversely.

Proof: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping. Let A be an IVPFCS in Y. Since every IVPFCS is an IVPFWGCS, A is an IVPFWGCS in Y. By hypothesis, $f^{-1}(A)$ is Interval-valued Pythagorean fuzzy clopen in X. Thus $f^{-1}(A)$ is an IVPFCS in X. Therefore f is an Interval-valued Pythagorean fuzzy continuous mapping.

Theorem 3.3 Every Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping is an Interval-valued fuzzy αcontinuous mapping but not conversely.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping. Let A be an IVPFCS in Y. Since every IVPFCS is an IVPFWGCS, A is an IVPFWGCS in Y. By hypothesis, $f^{-1}(A)$ is Interval-valued fuzzy clopen in X. Thus $f^{-1}(A)$ is an IVPFCS in X. Since every IVPFCS is an IVPFαCS, $f^{-1}(A)$ is an IVPFαCS in X. Hence f is an Interval-valued Pythagorean fuzzy α continuous mapping.

Theorem 3.4 Every Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping is an Interval-valued Pythagorean fuzzy pre-continuous mapping but not conversely.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping. Let A be an IVPFCS in Y. Since every IVPFCS is an IVPFWGCS, A is an IVPFWGCS in Y. By hypothesis, $f^{-1}(A)$ is Interval-valued Pythagorean fuzzy clopen in X. Thus $f^{-1}(A)$ is an IVPFCS in X. Since every IVPFCS is an IVPFPCS, $f^{-1}(A)$ is an IVPFPCS in X. Hence f is an Interval-valued fuzzy pre-continuous mapping.

Theorem 3.5 Let $f:(X,\tau)\to(Y,\sigma)$ be a mapping from an IVPFTS (X,τ) into an IVPFTS(Y,σ). Then the following statements are equivalent.

(a) f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping,

(b) $f^{-1}(B)$ is Interval-valued Pythagorean fuzzy clopen in X for every IVPFWGOS B in Y.

Proof: (a)⇒ (b): Let B be an IVPFWGOS in Y. Then B^c is an IVPFWGCS in Y. Since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(B^c)=(f^{-1}(B))^c$ is Interval-valued Pythagorean fuzzy clopen in X. This implies $f^{-1}(B)$ is Interval-valued Pythagorean fuzzy clopen in X.

(b) ⇒ (a): Let B be an IVPFWGCS in Y. Then B^c is an IVPFWGOS in Y. By hypothesis, $f^{-1}(B^c)=(f^{-1}(B))^c$ is Interval-valued Pythagorean fuzzy clopen in X, which implies f^{-1}(B) is Interval-valued Pythagorean fuzzy clopen in X. Therefore f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.

Theorem 3.6 If $f:(X,\tau)\to(Y,\sigma)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping from an IVPFTS (X, τ) into an IVPFTS (Y, σ), then $f(cl(A))\square wgcl(f(A))$ for every IVPFS A in X.

Proof: Let A be an IVPFS in X. Then wgcl(f(A)) is an IVPFWGCS in Y. Since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(wgcl(f(A)))$ is Interval-valued Pythagorean fuzzy clopen in X. Thus $f^{-1}(wgcl(f(A)))$ is an IVPFCS in X. Clearly $A\square f^{-1}(wgcl(f(A)))$. Therefore, $cl(A)\square cl(f^{-1}(wgcl(f(A)))) = f^{-1}(wgcl(f(A)))$.
Hence $f(cl(A))\square wgcl(f(A))$ for every IVPFS A in X.

Theorem 3.7 If $f:(X,\tau)\to(Y,\sigma)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping from an IVPFTS (X, τ) into an IVPFTS (Y, σ), then $cl(f^{-1}(B))\square f^{-1}(wgcl(B))$ for every IVPFS B in Y.

Proof: Let B be an IVPFS in Y. Then wgcl(B) is an IVPFWGCS in Y.
By hypothesis, $f^{-1}(wgcl(B))$ is Interval-valued Pythagorean fuzzy clopen in X. Thus $f^{-1}(wgcl(B))$ is an IVPFCS in X. Clearly $B\square wgcl(B)$ implies $f^{-1}(B)\square f^{-1}(wgcl(B))$.
Therefore $cl(f^{-1}(B))\square cl(f^{-1}(wgcl(B))) = f^{-1}(wgcl(B))$.
Hence $cl(f^{-1}(B))\square f^{-1}(wgcl(B))$ for every IVPFS B in Y.

Theorem 3.8 If $f:(X,\tau)\to(Y,\sigma)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping from an IVPFTS (X, τ) into an IVPFTS (Y,σ), then $f^{-1}(wgint(B))\square int(f^{-1}(B))$ for every IVPFS B in Y.

Proof: Let B be an IVPFS in Y. Then wgint(B) is an IVPFWGOS in Y.
By hypothesis, $f^{-1}(wgint(B))$ is Interval-valued Pythagorean fuzzy clopen in X. Thus $f^{-1}(wgint(B))$ is an IVPFOS in X. Clearly $wgint(B)\square B$ implies $f^{-1}(wgint(B))\square f^{-1}(B)$. Therefore $int(f^{-1}(wgint(B)))$ $\square int(f^{-1}(B))$. Hence, $f^{-1}(wgint(B))\square int(f^{-1}(B))$ for every IVPFS B in Y.

Theorem 3.9 The composition of two Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping in general.

Proof: Let A be an IVPFWGCS in Z. By hypothesis, $g^{-1}(A)$ is Interval-valued Pythagorean fuzzy clopen in Y and hence an IVPFCS in Y. Since every IVPFCS is an IVPFWGCS, $g^{-1}(A)$ is an IVPFWGCS in Y.
Further, since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(g^{-1}(A)) = (gf)^{-1}(A)$ is Interval-valued Pythagorean fuzzy

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clopen in X. Hence gof is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.

Theorem 3.10 Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\delta)$ be any two mappings. Then the following statements hold.
(i) Let $f:(X,\tau)\to(Y,\sigma)$ be an Interval-valued Pythagorean fuzzy continuous mapping and $g:(Y,\sigma)\to(Z,\delta)$ an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping. Then their composition $g\circ f:(X,\tau)\to(Z,\delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.
(ii) Let $f:(X,\tau)\to(Y,\sigma)$ be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping and $g:(Y,\sigma)\to(Z,\delta)$ an Interval-valued Pythgorean fuzzy continuous mapping [respectively Interval-valued Pythagorean fuzzy α continuous mapping, Interval-valued Pythagorean fuzzy pre continuous mapping, Interval-valued Pythagorean fuzzy α generalized continuous mapping and Interval-valued Pythagorean fuzzy generalized continuous mapping]. Then their composition $g\circ f:(X,\tau)\to(Z,\delta)$ is an Interval-valued Pythagorean fuzzy continuous mapping.
(iii) Let $f:(X,\tau)\to(Y,\sigma)$ be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping and $g:(Y,\sigma)\to(Z,\delta)$ an Interval-valued Pythagorean fuzzy weakly generalized continuous mapping. Then their composition $g\circ f:(X,\tau)\to(Z,\delta)$ is an Interval-valued Pythagorean fuzzy continuous mapping.

Proof: (i) Let A be an IVPFWGCS in Z. By hypothesis, $g^{-1}(A)$ is Interval-valued Pythagorean fuzzy clopen in Y and hence an IVPFCS in Y. Since f is an Interval-valued Pythagorean fuzzy continuous mapping, $f^{-1}(g^{-1}(A))=(g\circ f)^{-1}(A)$ is an IVPFCS in X. Hence $g\circ f$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.

(ii) Let A be an IVPFCS in Z. By hypothesis, $g^{-1}(A)$ is an IVPFCS [respectively IVPFαCS, IVPFPCS, IVPF αGCS and IVPFGCS] in Y. Since every IVPFCS [respectively IVPFαCS, IVPFPCS, IVPFαGCS and IVPFGCS] is an IVPFWGCS, $g^{-1}(A)$ is an IVPFWGCS in Y. Then $f^{-1}(g^{-1}(A))=(g\circ f)^{-1}(A)$ is Interval-valued Pythagorean fuzzy clopen in X, by hypothesis. Thus $(g\circ f)^{-1}(A)$ is an IVPFCS in X. Hence $g\circ f$ is an Interval-valued Pythagorean fuzzy continuous mapping.

(iii) Let A be an IVPFCS in Z. By hypothesis, $g^{-1}(A)$ is an IVPFWGCS in Y. Since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(g^{-1}(A))=(g\circ f)^{-1}(A)$ is Interval-valued Pythagorean fuzzy clopen in

X. Thus $(g\circ f)^{-1}(A)$ is an IVPFCS in X. Hence $g\circ f$ is an Interval-valued Pythagorean fuzzy continuous mapping.

Theorem 3.11 If $f:(X,\tau)\to(Y,\sigma)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping and $g:(Y,\sigma)\to(Z,\delta)$ is an Interval-valued Pythagorean fuzzy weakly generalized irresolute mapping, then $g\circ f:(X,\tau)\to(Z,\delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.

Proof:Let A be an IVPFWGCS in Z. By hypothesis, $g^{-1}(A)$ is an IVPFWGCS in Y. Since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping, $f^{-1}(g^{-1}(A))=(g\circ f)^{-1}(A)$ is Interval-valued Pythagorean fuzzy clopen in X. Hence $g\circ f$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.

Theorem 3.12 Let $f:(X,\tau)\to(Y,\sigma)$ be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping and $g:(Y,\sigma)\to(Z,\delta)$ be any mapping. Then $g\circ f:(X,\tau)\to(Z,\delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping if and only if g is an Interval-valued Pythagorean fuzzy weakly generalized irresolute mapping.

Proof: Let $g:(Y,\sigma)\to(Z,\delta)$ be an Interval-valued Pythagorean fuzzy weakly generalized irresolute mapping.
Then the proof follows from the theorem 3.11,
Conversely, let $g\circ f:(X,\tau)\to(Z,\delta)$ be an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping. Let A be an IVPFWGCS in Z. Since $g\circ f:(X,\tau)\to(Z,\delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping, $(g\circ f)^{-1}(A)=f^{-1}(g^{-1}(A))$ is Interval-valued Pythagorean fuzzy clopen in X. Since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping,$g^{-1}(A)$ is an IVPFWGCS in Y. Thus the inverse image of each IVPFWGCS in Z is an IVPFWGCS in Y. Hence g is an Interval-valued Pythagorean fuzzy weakly generalized irresolute mapping.

4. Interval-valued Pythagorean fuzzy perfectly weakly generalized open mappings:

In this section, we introduce Interval-valued Pythagorean fuzzy perfectly weakly generalized open mappings and study some of their properties.

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Definition 4.1 A mapping $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping if the image of every IVPFWGOS in X is Interval-valued Pythagorean fuzzyclopen in Y.

Theorem 4.2 Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a mapping from an IVPFTS (X, τ) into an IVPFTS(Y,σ). Then the following statements are equivalent.
(a) f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping,
(b) f(B) is Interval-valued Pythagorean fuzzy clopen in Y for every IVPFWGCS B in X.
Proof: (a) ⇒ (b): Let B be an IVPFWGCS in X. Then B^c is an IVPFWGOS in X. Since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping, $f(B^c)=(f(B))^c$ is Interval-valued Pythagorean fuzzy clopen in Y. This implies f(B) is Interval-valued Pythagorean fuzzy clopen in Y.

(b) ⇒(a): Let B be an IVPFWGOS in X. Then B^c is an IVPFWGCS in X. By hypothesis, $f(B^c)=(f(B))^c$ is Interval-valued Pythagorean fuzzy clopen in Y, which implies that f(B) is Interval-valued Pythagorean fuzzy clopen in Y. Therefore f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

Theorem 4.3 Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a bijective mapping from an IVPFTS (X, τ) into an IVPFTS(Y,σ), then the following statements are equivalent.
(a) Inverse of f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.
(b) f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

Proof: (a) ⇒(b): Let A be an IVPFWGOS of X. By assumption, $(f^{-1})^{-1}(A)=f(A)$ is Interval-valued Pythagorean fuzzy clopen in Y. Hence f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

(b) ⇒(a): Let B be an IVPFWGOS in X. Then f(B) is Interval-valued Pythagorean fuzzy clopen in Y. That is $(f^{-1})^{-1}(B)=f(B)$ is Interval-valued Pythagorean fuzzy clopen in Y. Therefore f^{-1} is an Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mapping.

Theorem 4.4 The composition of two Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping is again an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

Proof: Suppose $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\delta)$ are any two Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping. Let A be an IVPFWGOS in X. Since f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping, f(A) is Interval-valued Pythagorean fuzzy clopen in Y. Hence it is an IVPFOS in Y. But every IVPFOS is an IVPFWGOS, which implies f(A) is an IVPFWGOS in Y. Since g is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping, $g(f(A))=(g\circ f)(A)$ is Interval-valued Pythagorean fuzzy clopen in Z. Thus the image of each IVPFWGOS in X is Interval-valued Pythagorean fuzzy clopen in Z. Therefore $g\circ f:(X,\tau)\rightarrow(Z,\delta)$ is Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

Theorem 4.5 If $f:(X,\tau)\rightarrow(Y,\sigma)$ is an Interval-valued Pythagorean fuzzy weakly generalized * open mapping and $g:(Y,\sigma)\rightarrow(Z,\delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping, then their composition $g\circ f:(X,\tau)\rightarrow(Z,\delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

Proof: Let A be an IVPFWGOS in X. Since f is an Interval-valued Pythagorean fuzzy weakly generalized * open mapping, f(A) is an IVPFWGOS in Y. Further, since g is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping, $g(f(A))=(g\circ f)(A)$ is Interval-valued Pythagorean fuzzy clopen in Z. Hence $g\circ f:(X,\tau)\rightarrow(Z,\delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

Theorem 4.6 Let $f:(X,\tau)\rightarrow(Y,\sigma)$ and $g:(Y,\sigma)\rightarrow(Z,\delta)$ be two mappings such that $g\circ f:(X,\tau)\rightarrow(Z,\delta)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping. Then the following statements hold.

(a) If f is an Interval-valued Pythagorean fuzzy weakly generalized irresolute mapping and surjective, then g is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

(b) If g is an Interval-valued Pythagorean fuzzy totally continuous mapping and injective, then f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

Proof: ***(a)*** Let A be an IVPFWGOS in Y. Then $f^{-1}(A)$ is an IVPFWGOS in

X, because f is an Interval-valued Pythagorean fuzzy weakly generalized irresolute mapping. Since $(g \circ f)$ is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping, $(g \circ f)(f^{-1}(A)) = g(A)$ is Interval-valued Pythagorean fuzzy clopen in Z. This shows that g is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

(b)Since g is injective, we have, $f(A) = g^{-1}(g \circ f)(A)$ is true for every subset A of X. Let B be an IVPFWGOS in X. Therefore $(g \circ f)(B)$ is Interval-valued Pythagorean fuzzy clopen in Z and hence an IVPFOS in Z. Since g is Interval-valued Pythagorean fuzzy totally continuous, $g^{-1}(g \circ f)(A) = f(A)$ is Interval-valued Pythagorean fuzzy clopen in Y. This shows that f is an Interval-valued Pythagorean fuzzy perfectly weakly generalized open mapping.

4. Conclusion

In this paper, We studied the concepts of Interval-valued Pythagorean fuzzy perfectly weakly generalized continuous mappings and Interval-valued Pythagorean fuzzy perfectly weakly generalized open mappings in Interval-valued Pythagorean fuzzy topological space. Some of their properties are explored.

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Pentapartitioned Neutrosophic Almost Resolvable and Irresolvable Spaces

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Abstract

The aim of this paper is to develop many characterizations of Pentapartitioned Neutrosophic (PN) almost resolvable and irresolvable spaces and also the condition under that a PN almost resolvable space becomes a PN baire space. The interrelations between PN almost resolvable spaces and other spaces also are mentioned.

Key Words: Pentapartitioned neutrosophic set, pentapartitioned neutrosophic almost resolvable space, pentapartitioned neutrosophic almost irresolvable spaces.

AMS Classification: 54A40,03E72

1 Introduction

In order to cope with uncertainties, the thought of fuzzy sets and fuzzy set operations was introduced by Zadeh [17]. The speculation of fuzzy topological space was studied and developed by C.L. Chang [3]. The paper of Chang sealed the approach for the following tremendous growth of the various fuzzy topological ideas. Since then a lot of attention has been paid to generalize the fundamental ideas of general topology in fuzzy setting and therefore a contemporary theory of fuzzy topology has been developed. Atanassov and plenty of researchers [1] worked on intuitionistic fuzzy sets within the literature. Florentin Smarandache [14] introduced the idea of Neutrosophic set in 1995 that provides the information of neutral thought by introducing the new issue referred to as uncertainty within the set. thus neutrosophic set was framed and it includes the parts of truth membership function(T), indeterminacy membership function(I), and falsity membership function(F) severally.

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Neutrosophic sets deals with non normal interval of]−0 1+[. Pentapartitioned Neutrosophic set and its properties were introduced by Rama Malik and Surpati Pramanik [13]. In this case, indeterminacy is divided into three components: contradiction, ignorance, and an unknown membership function.

The concept of Pentapartitioned neutrosophic pythagorean sets was initiated by R. Radha and A. Stanis Arul Mary[7]. Many authors have been discussed about the concept of Pythagorean Sets[6-12]. The concept of intuitionistic fuzzy almost resolvable spaces and irresolvable spaces was introduced by Sharmila s [15].R. Radha and A.Stanis Arul Mary introduced Pentapartitioned neutrosophic pythagorean resolvable and irresolvable spaces.

Now we extend the concepts to pentapartitioned neutrosophic sets. In this paper, we discussed about PN almost resolvable and irresolvable spaces in third section, the inter-relations with PN almost resolvable spaces with other spaces have been investigated in fourth section and the levels of Irresolvability can be studied in last section.

2 Preliminaries

Definition 2.1 [14] Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$$

Where $T_A\ , I_A, F_A: U \to [0,1]\ and\ 0 \le T_A(x) + I_A(x) + F_A\ (x) \le 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of inderminancy and $F_A(x)$ is the degree of non-membership.

Definition 2.2 [13] Let P be a non-empty set. A Pentapartitioned neutrosophic set A over P characterizes each element p in P a truth -membership function T_A , a contradiction membership function C_A, an ignorance membership function G_A, unknown membership function U_A and a false membership function F_A , such that for each p in P

$$T_A + C_A + G_A + U_A + F_A \le 5$$

Definition 2.3 [7] The complement of a pentapartitioned neutrosophic set A on R is denoted by A^C or A^* and is defined as

$A^C = \{< x, F_A(x), U_A\ (x), 1 - G_A(x), C_A(x), T_A(x) > : x \in X\}$

Definition 2.4 [7] Let A = $< x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x) >$ and

B = $< x, T_B(x), C_B(x), G_B(x), U_B(x), F\ _B(x) >$ are Pentapartitioned Neutrosophic sets. Then

$A \cup B = <$ x, $max\big(T_A(x), T_B(x)\big), max\big(C_A(x), C_B(x)\big), \min(G_A(x), G\ _B(x)),$

$\min(U_A(x), U\ _B(x)), \min(F_A(x), F_B(x)), >$

$A \cap B = < x, min\big(T_A(x), T_B(x)\big), min\big(C_A(x), C_B(x)\big), \max(G_A(x), G\ _B(x))$

$$, max\big(U_A(x), U_B(x)\big), max(F_A(x), F_B(x))$$

Definition 2.5 [12] A PN topology τ on a nonempty set R is a family of a PN sets in R satisfying the following axioms

1) 0,1$\in \tau$
2) $R_1 \cap R_2 \in \tau$ for any $R_1, R_2 \in \tau$
3) $\cup R_i \in \tau$ for any $R_i : i \in I$

The complement R* of PN open set (PNOS, in short) in PN topological space [PNTS] (R,$\boldsymbol{\tau}$), is called a PN closed set [PNCS].

Definition 2.6 [7] Let (R,$\boldsymbol{\tau}$) be a PNTS and L be a PNTS in R. Then the PN interior and PN Closure of R denoted by

PNCl(L) = $\cap\{\boldsymbol{K}$: K is a PNPCS in R and L$\subseteq$ K}.

PNInt(L) = $\cup${G: G is a PNPOS in R and G$\subseteq$ L}.

Definition 2.7 [11] Let (R,$\boldsymbol{\tau}$) be a PNTS and K be a PN set in (R,$\boldsymbol{\tau}$). Then the PN closure operator satisfy the following properties.

1-PNPCl(K) = PNPInt(1-K)

1-PNPInt(K) = PNPCl(1-K)

Definition 2.8 [11] A PNP A in PNTS (R,$\boldsymbol{\tau}$) is called PN dense if there exists no PNCS L in (R,$\boldsymbol{\tau}$) such that K $\subseteq$ $\boldsymbol{L} \subseteq \mathbf{1}$. That is PNCl(K) = 1.

Definition 2.9 [11] A PN A in PNPTS (R,$\boldsymbol{\tau}$) is called PN nowhere dense if there exists no nonzero PNPOS L in (R,$\boldsymbol{\tau}$) such that $\boldsymbol{L} \subseteq$ PNPCl(K). That is PNPInt (PNPCl(K)) = 0.

Definition 2.10 [11] A PNTS (R,$\boldsymbol{\tau}$) is called PN resolvable if there exists a PN dense set K in (R,$\boldsymbol{\tau}$) such that PNCl (1-K) = 1. Otherwise (R,$\boldsymbol{\tau}$) is called PN irresolvable.

Definition 2.11 [11] A PNTS (R,$\boldsymbol{\tau}$) is called PN submaximal if PNCl(K) = 1 for any non-zero PN set K in (R,$\boldsymbol{\tau}$)

Definition 2.12 [11] A PNTS (R,$\boldsymbol{\tau}$) is called a PN open hereditarily resolvable if PNInt (PNPCl(K)) $\neq \mathbf{0}$ for any PN set K in (R,$\boldsymbol{\tau}$).

Definition 2.13 [11] APNTS (R,$\boldsymbol{\tau}$) is called PN first category if $\cup_{i=1}^{\infty} \boldsymbol{K_i}$, where $\boldsymbol{K_i's}$ are PN nowhere dense sets in (R,$\boldsymbol{\tau}$). A PNTS which is not first category is said to be PN second category.

Definition 2.14 [11] A PNTS (R,$\boldsymbol{\tau}$) is called a PN baire space if PNInt $(\cup_{i=1}^{\infty} \boldsymbol{K_i}) = \mathbf{0}$, where $\boldsymbol{K_i's}$ are PN nowhere dense sets in (R,$\boldsymbol{\tau}$).

3 Pentapartitioned Neutrosophic (PN) Almost Resolvable and Irresolvable Spaces

Definition 3.1 A PNTS is called a PN almost resolvable space if $\bigcup_{i=1}^{\infty} K_i$ =1, where K_i's are PNS s in (R,τ) are such that PNInt (K_i) = 0. Otherwise (R,τ) is called PN almost irresolvable space.

Example 3.2 Let R = {p, q}. Let A3, A4, A5 and A6 be the PN sets defined on R as follows.
A1 = {[p,0.2,0.6,0.5,0.2,0.6], [q,0.4,0.5,0.7,0.4,0.5]}
A2 = {[p,0.5,0.6,0.1,0.2,0.4], [q,0.5,0.6,0.5,0.1,0.1]}.
Then, clearly τ = {0, A1, A2, 1} is a PN topology on R.
Now consider the PN sets defined on R as follows
A3 = {[p,0.3,1,0.6,0,0.5], [q,1,0.5,0,0.2,0.5]},
A4 = {[p,1,0.2,0.7,0.4,0], [q,0.2,0.3,0.5,0.3,0]},
A5 = {[p,0.4,0.3,0.4,0.5,0.2], [q,0.5,1,0.2,0,0.3]},
A6 = {[p,0.2,0.4,0,0.1,0.3], [q,0.2,0.3,0.7,0.4,0.5]}.
Then, PNInt (A3) = 0, PNInt (A4) = 0, PNInt (A5) = 0 and PNInt (A6) = 0 and
{(A3) ∪ (A4) ∪ (A5) ∪ (A6)} = 1.
Hence (R,τ) is a PN almost resolvable space.

Example 3.3 Let R = {p, q}. Let A3, A4, A5 and A6 be the PN sets defined on R as follows.
A1 = {[p,0.2,0.6,0.5,0.2,0.6], [q,0.4,0.5,0.7,0.4,0.5]}
A2 = {[p,0.5,0.6,0.1,0.2,0.4], [q,0.5,0.6,0.5,0.1,0.1]}.
Then, clearly τ = {0, A1, A2, 1} is a PN topology on R.
Now consider the PN sets defined on R as follows
A3 = {[p,0.2,1,0.5,0,0.4], [q,0.4,0.3,0.5,0.2,0.5]},
A4 = {[p,0.4,0.5,0.6,0.1,0], [q,0.2,0.3,0.5,0.3,0.2]},
A5 = {[p,0.2,0.4,0.5,0.2,0.1], [q,0.1,1,0.2,0,0.3]},
A6 = {[p,0.5,0.3,0.2,0.1,0.5], [q,0.2,0.3,0.7,0.4,0.5]}.
Then, PNInt (A3) = 0, PNInt (A4) = 0, PNInt (A5) = 0 and PNInt (A6) = 0 and
{(A3) ∪ (A4) ∪ (A5) ∪ (A6)} ≠1.
Hence (R,τ) is a PN almost irresolvable space.

Theorem 3.4 If $\bigcap_{i=1}^{\infty} K_i = 0$,where K_i's are PN dense sets in (R,τ), then (R,τ) is a PN almost resolvable space.
Proof: Suppose that $\bigcap_{i=1}^{\infty} K_i = 0$, where PNCl ($K_i$) = 1 in (R,$\tau$). Then we have 1 - $\bigcap_{i=1}^{\infty} K_i = 1 - 0 = 1$, where 1 - PNCl ($K_i$) = 0. This implies that $\bigcup_{i=1}^{\infty}(1 - K_i) = 1$, where PNInt (1 - K_i) $= 0. Let\ 1 - K_i = L_i$, then we have $\bigcup_{i=1}^{\infty} L_i = 1$, where PNInt ($L_i$)= 0 in (R,$\tau$). Hence (R,$\tau$) is PN almost resolvable space.

Definition 3.5 A PNTS (R,τ) is called a PN hyper- connected space if every PN open set is PN dense in (R,τ). That is PNPCl $(K_i) = 1$ for all $K_i \in \tau$.

Theorem 3.6 If $\bigcap_{i=1}^{\infty} K_i = 0$,where K_i's are PN open set in a PN hyper-connected space (R,τ), then (R,τ) is a PN almost resolvable space.
Proof: Suppose that $\bigcap_{i=1}^{\infty} K_i = 0$, where $K_i \in \tau$ .Since (R,τ) is a PN hyper – connected space, the PN open set K_i is a PN dense set in (R,τ) for each i. Hence we have $\bigcap_{i=1}^{\infty} K_i = 0$, where PNCl ($K_i$) = 1 in (R,$\tau$). Then by theorem 3.2, (R, τ) is a PN almost resolvable space.

Definition 3.7 A PN K in a PNTS (R,τ) is called PNR_1 if K = $\bigcap_{1=1}^{\infty} K_i$ where each $K_i \in \tau$.

Definition 3.8 A PN K in a PNTS (R,τ) is called PNR_2 if K = $\bigcup_{i=1}^{\infty} K_i$ where each $K_i \in \tau$.

Definition 3.9 A PNTS (R,τ) is called PN R-space, if countable intersection of PNOS s in (R,τ) is PN open. That is, every nonzero PNR_1- set in (R,τ) PN open in (R,τ).

Theorem 3.10 If $\bigcap_{1=1}^{\infty} K_i = 0, K_i' s$ are PNR_1- sets in an PN hyper-connected space and PNR-space (R,τ), then (R,τ) is an PN almost resolvable space.
Proof: Let $K_i' s$ be PNR_1- sets in a PNR-space (R,τ). Then $K_i' s$ are PN open sets in (R,τ). Hence, we have $\bigcap_{1=1}^{\infty} K_i = 0,\ where K_i' s$ are PN open sets in an PN hyper-connected space (R,τ). Therefore, by theorem 3.4, (R,τ) is an PN almost resolvable space.

Theorem 3.11 If each PNS K_i is an PNR_2 – set in a PN almost resolvable space (R, τ), then $\bigcap_{1=1}^{\infty}(1 - K_i) = 0, where\ K_i' s$ are PN dense sets in (R,τ).
Proof: Let (R,τ) be a PN almost resolvable space. Then $\bigcup_{i=1}^{\infty} K_i$ = 1, where $K_i's$ are such that PNInt (K_i) = 0. This implies that 1-$\bigcup_{i=1}^{\infty} K_i$ = 0 and 1-PNInt (K_i) = 1. Then $\bigcap_{1=1}^{\infty}(1 - K_i) =$ 0 and PNCl ($1 - K_i$) = 1. Since $K_i' s$ are PNR_2-sets, (1-K_i)$'s$ are PNR_1-sets in (R,τ). Hence we have $\bigcap_{1=1}^{\infty}(1 - K_i) = 0$, where (1-$K_i$)'s are PN dense and PN$R_1$- sets in (R,$\tau$).

Definition 3.12 A PNTS (R,τ) is called Pentapartitioned Neutrosophic nodec space, if every non-zero PN nowhere dense set in (R,τ) is PN closed.
Theorem 3.13
IF the PNTS (R,τ) is a PN first category, then (R,τ) is a PN almost resolvable space.
Proof: Since (R,τ) is of PN first category, we have $\bigcup_{i=1}^{\infty} K_i$ = 1, where $K_i's$ are PN nowhere dense sets in (R,τ). Now K_i is a PN nowhere dense set implies that PNInt(K_i) = 0. Hence $\bigcup_{i=1}^{\infty} K_i$ = 1, where PNInt(K_i) = 0 and therefore (R,τ) is a PN almost resolvable space.

Theorem 3.14 If (R,τ) is a PN first category space and PN nodec space, then (R,τ) is a PN almost resolvable space.
Proof : Let (R,τ) be a first category space. Then we have $\bigcup_{i=1}^{\infty} K_i$ = 1, where $K_i's$ are PN nowhere dense subsets in (R,τ). Since (R,τ) is a PN nodec space, the PN nowhere dense sets are PN closed sets in (R,τ). Hence $K_i's$ are PNCS in (R,τ). That is, PNCl (K_i) = K_i. PNInt (PNCl (K_i)) = PNInt (K_i) = 0. Now PNInt (PNCl (K_i)) = 0 implies that PNInt (K_i) = 0. Hence we have $\bigcup_{i=1}^{\infty} K_i$ = 1, where $K_i's$ in (R,τ) are such that PNInt (K_i) = 0. Hence (R,τ) is a PN almost resolvable space.

Theorem 3.15 If PNCl(PNInt(K_i) = 1, for ach PN dense set K_i in a PN almost resolvable space (R,τ), then (R,τ) is a PN first category space.
Proof: Let (R,τ) be a PN almost resolvable space such that PNCl(PNInt(K_i) = 1, for each PN dense set K_i in (R,τ). Since (R,τ) is PN almost resolvable space, $\bigcup_{i=1}^{\infty} K_i$ = 1, where $K_i' s$ in (R,τ) are such that PNInt(K_i) = 0.Now 1 – PNInt (K_i)= 1, which implies that PNCl(1 - K_i) = 1 , which implies that (1- K_i) is PN dense. Then by hypothesis, PNCl(PNInt(1- K_i)) = 1 for the PN dense set (1- K_i) in (R,τ). This implies that PNInt(PNCl(K_i)) = 0. So 1-PNCl(PNInt(1-K_i)) = 0. Hence $K_i' s$ are PN nowhere dense set in (R,τ). Therefore $\bigcup_{i=1}^{\infty} K_i$ = 1, where $K_i' s$ are PN nowhere dense set in (R,τ), implies that (R,τ) is a PN first category space.

Theorem 3.16 If PNCl(PNInt(K_i) = 1, for ach PN dense set K_i is a PN almost resolvable space (R,τ), then (R,τ) is not a PN Baire space.

Proof: Let (R,τ) be a PN almost resolvable space such that PNCl(PNInt(K_i) = 1, for each PN dense set K_i in (R,τ). Then by theorem 3.13, (R,τ) is a PN first category space. Since $\bigcup_{i=1}^{\infty} K_i$ = 1, where $K_i's$ are PN nowhere dense in (R,τ) This implies that $PNInt(\bigcup_{i=1}^{\infty} K_i)$ = 1 = PNInt(1) = 0 . Hence (R,τ) is not a PN Baire space.

Theorem 3.17 If (R,τ) is a PN second category space, then (R,τ) is a PN almost resolvable space.

Proof: Let (R,τ) be a PN second category space. Then $\bigcup_{i=1}^{\infty} K_i \neq 1$, where $K_i's$ are PN nowhere dense sets in (R,τ). That is $\bigcup_{i=1}^{\infty} K_i \neq 1$, where PNInt(PNCl($K_i$)) = 0. Now PNInt($K_i$) $\subseteq$ PNInt(PNCl(K_i)), implies that PNInt(K_i) = 0. Hence $\bigcup_{i=1}^{\infty} K_i \neq 1$, where PNInt($K_i$) = 0 and therefore (R,$\tau$)is a PN almost resolvable space.

Definition 3.18 A PNTS (R,τ) is called PN Volterra space, if PNCl($\bigcap_{i=1}^{N} K_i$) = 1, where are PN dense and PNR_1 sets in (R,τ).

Definition 3.19 A PNTS (R,τ) is called PN weakly Volterra space, if PNCl ($\bigcap_{i=1}^{N} K_i$) ≠1, where are PN dense and PNR_1 sets in (R,τ).

Theorem 3.20 If PNTS (R,τ) is not a PN weakly Volterra space, then (R,τ) is a PN almost resolvable space.

Proof: Let (R,τ) be a non-weakly volterra space. Then we have PNCl ($\bigcap_{i=1}^{N} K_i$) = 0, where $K_i's$ are PN dense and PNR_1 sets in (R,τ).

Since $K_i's$ are PN dense , 1-PNCl(K_i) = 0. Now PNCl($\bigcap_{i=1}^{N} K_i$) = 0, implies that PNInt($\bigcup_{i=1}^{N}(1-K_i)$) = 1 and PNCl(K_i) = 1, implies that PNInt(1- K_i) = 0. Let $M_j's$ be such that PNInt($M_j's$) = 0 and take the first N $M_j's$ as (1-$M_j's$)'s. Now $\bigcup_{i=1}^{N}(1-K_i) \subseteq \bigcup_{j=1}^{\infty} M_j$, implies that $PNInt(\bigcup_{i=1}^{N}(1-K_i)) \subseteq PNInt(\bigcup_{j=1}^{\infty} M_j) \subseteq \bigcup_{j=1}^{\infty} M_j$. Then we have $1 \subseteq \bigcup_{j=1}^{\infty} M_j$. That is $\bigcup_{j=1}^{\infty} M_j = 1$, where $M_j's$ in (R,τ) are such that PNInt(M_j) = 0. Hence (R,τ) is a PN almost resolvable space.

4. Inter -Relations between PN almost resolvable spaces and Irresolvable spaces with other spaces.

Theorem 4.1 If the PN almost resolvable space (R,τ) is a PN submaximal space, then (R,τ) is a PN first category space.

Proof: Let (R,τ) be a PN almost resolvable space. Then $\bigcup_{i=1}^{\infty} K_i = 1$, where $K_i's$ in (R,τ) are such that PNInt(K_i) = 0. Then we have $\bigcap_{i=1}^{\infty}(1-K_i) = 0 = 0$, where PNCl(1-$K_i$) = 1. Since the space (R,τ) is a PN submaximal space, the PN dense sets (1-K_i)'s are PNOS in (R,τ). Then $K_i's$ are PN closed sets in (R,τ) and hence PNCl(K_i) = K_i. Now PNInt(PNCl(K_i)) = PNInt(K_i) = 0. Then $K_i'sare$ PN nowhere dense sets in (R,τ).Hence $\bigcup_{i=1}^{\infty} K_i = 1$, where $K_i's$ are PN nowhere dense subsets in (R,τ) implies that (R,τ) is a PN first category space.

Theorem 4.2 If the PN almost irresolvable space (R,τ) is a PN submaximal space, then (R,τ) is a PN second category space.

Proof: Let (R,τ) be a PN almost irresolvable space. Then $\bigcup_{i=1}^{\infty} K_i \neq 1$, where $K_i's$in (R,τ) are such that PNInt(K_i) = 0. Now PNInt(K_i) = 0, implies that PNCl(1-K_i) = 1.Then $K_i's$ are PNCS s in (R,τ) and hence PNCl(K_i) = K_i. Now PNInt(K_i) = 0 implies that PNInt (PNCl(K_i)) = PNInt(K_i) = 0. Then $K_i's$ are PN nowhere dense sets in (R,τ). Hence $\bigcup_{i=1}^{\infty} K_i \neq 1$, where $K_i's$ are PN nowhere dense sets in (R,τ). Then (R,τ) is a PN second category space.

Theorem 4.3 If the PN almost irresolvable space (R,τ) is a PN submaximal space, then (R,τ) is not a PN Baire space.
Proof: Let the PN almost irresolvable space (R,τ) be a PN submaximal space. Then, by theorem 3.19 , (R,τ) is a PN first category space and hence $\bigcup_{i=1}^{\infty} K_i = 1$, where $K_i's$ are PN nowhere dense sets in (R,τ). Now PNInt($\bigcup_{i=1}^{\infty} K_i$) = PNInt(1) = 1 ≠0. Hence (R,τ) is not a PN baire space.

Theorem 4.4 If the PN almost irresolvable space (R,τ) is a PN open hereditarily irresolvable space, then (R,τ) is a PN second category space.
Proof: Let (R,τ) be a PN almost irresolvable space. Then $\bigcup_{i=1}^{\infty} K_i \neq 1$, where $K_i{}'s$ in (R,τ) are such that PNInt(K_i) = 0.Since (R,τ) is a PN open hereditarily irresolvable space, PNInt(K_i) = 0, implies that PNInt(PNCl(K_i)) = 0. Then $K_i's$ are PN nowhere dense subsets in (R,τ). Hence $\bigcup_{i=1}^{\infty} K_i \neq 1$, where $K_i's$ are PN nowhere dense subsets in (R,τ) implies that (R,τ) is a PN second category space.

Theorem 4.5 If the PNTS (R,τ) is a PN baire space, then (R,τ) is a PN almost irresolvable space.
Proof: Let (R,τ) be a PN baire space, PNInt($\bigcup_{i=1}^{\infty} K_i$) = 0, where $K_i's$ are PN nowhere dense sets in (R,τ). Now K_i is a PN nowhere dense set implies that PNPInt(PNPCl(K_i)) = 0. Since PNInt(K_i) ⊆ PNInt(PNCl(K_i)), we have PNInt($\bigcup_{i=1}^{\infty} K_i$) = 0, where PNInt($\bigcup_{i=1}^{\infty} K_i$) = 0. Hence PNInt($K_i$) = 0. Suppose that (R,$\tau$) is a PN almost resolvable space. Then $\bigcup_{i=1}^{\infty} K_i$ = 1, where PNInt(K_i) = 0.

Now PNInt($\bigcup_{i=1}^{\infty} K_i$) = PNInt(1) = 1, which implies that 0 = 1, a contradiction. Hence we must have $\bigcup_{i=1}^{\infty} K_i \neq 1$, where PNInt($K_i$) = 0. Therefore (R,$\tau$) is a PN almost resolvable space.

Theorem 4.6 If K is a PN dense and PNOS in a PNTS (R,τ), then 1-K is PN nowhere dense set in (R,τ).
Proof: Since K is PN dense set, PNCl(K) = 1 and let K be a PNOS, then 1-K be PNCS. Hence PNCl(1-K) = 1-K and 1-PNCl(K) =1-1 = 0, which implies that PNCl(1-K) = 1-K and PNInt(1-K) = 0. Therefore PNInt(PNCl(1-K)) = 0, which implies 1-K is a PN nowhere dense set in (R,τ).

Theorem 4.7 If each PN R_1- set is PN fuzzy open and PN dense set in a PNTS (R,τ), then (R,τ) is a PN almost irresolvable space.
Proof: Let K be a PNR_1-set in (R,τ) such that K is a PN dense and PN open set in (R,τ). Then K = $\bigcap_{i=1}^{\infty} K_i$,where K_i's in (R,τ) are PN open set in (R,τ). Now 1-K = 1- $\bigcap_{i=1}^{\infty} K_i$. Then PNCl(1-K) = PNCl(1 –($\bigcap_{i=1}^{\infty} K_i$)) = PNCl($\bigcap_{i=1}^{\infty}(1-K_i)$) and hence PNCl(1-K) = PNCl($\bigcup_{i=1}^{\infty}(1-K_i)$) ⊇ $\bigcup_{i=1}^{\infty} PN$Cl(1- K_i). Since $K_i{}'s$ are PNOS s in (R,τ), (1-K)'s are PNCS in (R,τ) and hence PNCl(1- K_i) = 1- K_i. PNCl(1-K) ⊇ $\bigcup_{i=1}^{\infty}$(1- K_i), which implies that

PNInt(PNCl(1-K)) ⊇ PNInt($\bigcup_{i=1}^{\infty}$(1- K_i)) (1)

Since $\bigcup_{i=1}^{\infty} PN$Int(1- K_i) ⊆ PNInt($\bigcup_{i=1}^{\infty}$(1- K_i)), we have

PNInt(PNPCl(1-K)) ⊆PNInt(1- K_i) (2)

Since K is a PN dense set in (R,τ). By theorem 4.6, the PNS (1-K) is a PN nowhere sense set in (R,τ). Then , we have PNInt(PNCl(1-K)) = 0.

Hence from (2), $0 \supseteq \cup_{i=1}^{\infty}$(PNInt(1- K_i)). *That is* $\cup_{i=1}^{\infty}$(PNInt(1- K_i)) = 0. Then PNInt(1- K_i) $= 0$, for each I = 1 to ∞. Hence PNInt(PNPCl(1- K_i)) $= 0$. [since PNCl(1- K_i) = 1- K_i].This implies that (1-K_i)'s are PN nowhere dense sets in (R,τ). From (1), we have $0 \supseteq$ PNInt($\cup_{i=1}^{\infty}$(1- K_i)).
That is, PNInt((1- K_i)) $= 0$. *Hence* (R, τ) *is a PN baire space*. Therefore , by theorem 4.5 is a PN almost irresolvable space.

Theorem 4.8 If $\cup_{i=1}^{\infty} K_i$ = 1, where K_i's are nonzero PN open sets in PNTS (R, τ), then (R, τ) is a PN almost irresolvable space.

Proof: Suppose that $\cup_{i=1}^{\infty} K_i$ = 1, where the PN sets K_i's are non zero PNOS s in (R, τ). Since $K_i's$ are non zero PNOSs, PNInt(K_i) $= K_i \neq 0$. Hence we have $\cup_{i=1}^{\infty} K_i$ = 1, where PNInt(K_i) $\neq 0$ fo all (i= 1 to ∞). Therefore (R, τ) is a PN almost irresolvable space.

5 Levels of Pentapartitioned Neutrosophic Irresolvability

Theorem 5.1 Let (R, τ) be a PNTS. If (R, τ) is PN open hereditarily irresolvable then PNInt(K) = 0 for any nonzero PN dense sets K in (R, τ) ,implies that PNInt(PNCl(K)) = 0.
Proof: Let K be a PNTS in (R, τ) $\exists PNInt(K)$ = 0. To Prove PNInt(PNCl(K)) = 0. Suppose that PNInt(PNCl(K))≠ 0. Since (R, τ) is PN open hereditarily irresolvable, we have PNInt(K) ≠ 0,which is a contradiction to the assumption. Hence PNInt(PNCl(K)) = 0.

Theorem 5.2 For any PNTS (R, τ), Every PN submaximal space is PN open hereditarily space.
Proof: Let (R, τ) be a PN submaximal space. Then, PNCl(K) = 1 implies that K∈ τ. To Prove (R, τ) is PNO hereditarily irresolvable, suppose that PNInt(K) = 0 for any nonzero PN set K in (R, τ).
Then 1-PNInt(K) = 1-0 = 1 implies that PNCl(1-K) =1. Since (R, τ) is PN submaximal, (1-K) ∈ τ.Then K is PN closed set in (R, τ). Hence A = PNCl(K) = PNInt(K) = PNInt(PNCl(K), Then PNInt(K) = 0 implies that PNInt(PNCl(K)) = 0. By theorem 5.1, (R, τ) is PN open hereditarily space. Hence PN sub-maximality implies PN open hereditarily irresolvable space.

Theorem 5.3 A PN open hereditarily irresolvable space and PN dense set in a PNTS (R, τ) is PN almost irresolvable space.
Proof: Let K be a PN dense set in PNTS (R, τ). Then PNCl(K) = 1 implies that PNInt(1-K) = 0. Since (R, τ) is a PN open hereditarily irresolvable space, PNCl(1-K) = 0 implies that PNInt(PNCl(1-K)) = 0. Now we claim that PNCl(1-K) ≠ 1. Suppose PNCl(1-K) = 1. Then PNInt(PNCl(1-K)) = PNInt(1) = 1 implies that 0=1, a contradiction. Hence we must have PNCl(1-K)≠1. Therefore PNCl(K) = 1 implies that PNCl(1-K)≠ 1, which means that (R, τ) is PN irresolvable space. Now let (R, τ) be PN irresolvable space. Then PNCl(K_i) $= 1$ *implies that* $PNCl(1-K) \neq 1$.Then PNInt(K_i) $\neq 0$.Now PNCl(K_i) $= 1$ implies that PNInt($1-K_i$) $= 0$. We claim that $\cup_{i=1}^{\infty}(1-K_i) \neq 1$.Suppose that $\cup_{i=1}^{\infty}(1-K_i) = 1$.Then 1- $\cap_{i=1}^{\infty}(1-K_i) = 1$ implies that $\cap_{i=1}^{\infty}(K_i) = 0$ Hence there must be atleast two non zero disjoint PN sets K_i and K in (R, τ). Then $K_i + K \subseteq 1$, which implies that $K_i \subseteq 1 - K$.Hence PNCl(K_i) ⊆ PNCl(1-K). Then 1 ⊆ (1-K). That is PNCl($1-K_i$) = 1. Then PNInt(K_i) = 0,a contradiction to PNInt(K_i) $\neq$ 0.Hence $\cup_{i=1}^{\infty}(1-K_i) \neq 1$,where PNInt(1-$K_i$) = 0.Therefore (R, τ) is PN almost irresolvable space.

6. Conclusions

In this paper, we discussed about PN almost resolvable and irresolvable spaces. In future, we extend our ideas to strongly PN resolvable and PN irresolvable spaces.

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Irredundant complete cototal dominating set

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Abstract

The complete cototal domination set is said to be irredundant complete cototal dominating set if for each $u \in S$, $N_G[S - \{u\}] \neq [S]$. The minimum cardinality taken over all an irredundant complete dominating set is called an irredundant complete cototal domination number and is denoted by $\gamma_{ircc}(G)$. Here a new domination parameter called an irredundant complete cototal dominating set was introduced and the study of bounds of $\gamma_{ircc}(G)$ was initiated.

Key Words: Irredundant complete cototal domination set, irredundant complete cototal dominating number.

AMS Classification: 05C62, 05C12.

1 Introduction

Consider a simple graph G with non-isolated vertices. The total number of vertices are denoted by p and the total number of edges by q. The vertex is said to be pendent if its degree is one and the vertex adjacent to the pendent vertex is known as support vertex. Generally, we follow the terminologies of Harary to learn the basic concepts of graph theory. Domination theory was framed by Claude Berge during 1960's. A set S is called total dominating set if the induced subgraph < S > has no isolated vertices. The total dominating number γ_t is minimum cardinality of a total dominating set [3]. A dominating set is said to be a cototal dominating set if the inducedsubgraph < V-S > has no isolated vertices. The cototal domination number is the minimumcardinality of a cototal dominating set [4]. By introducing a parameter in cototal domination, theirredundant complete cototal dominating set is defined. In this paper we have

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defined theirredundant complete cototal dominating set and derived the properties and found the bounds of irredundant complete cototal domination number.

2 Preliminaries

Definition 2.1 [1] A set $S \subseteq V$ is dominating set if each vertex in V is dominated by some vertex in S. Thedomination number (G) is minimum cardinality of a dominating set.

Definition 2.2 [2] A dominating set $S \subseteq V(G)$ is called as connected dominating set if induced subgraph $< S >$ is connected. The connected domination number $\boldsymbol{\gamma_c}(G)$ is the minimum cardinality of a connecteddominating set.

Definition 2.3 [3] A dominating set $S \subseteq V(G)$ is called as total dominating set if induced subgraph $< S >$ has no isolated vertices. The total domination number $\boldsymbol{\gamma_t}(G)$ is the minimum cardinality of a total dominating set.

Definition 2.4 [4] A dominating set $S \subseteq (G)$ is called as cototal dominating set if induced subgraph $< V - S >$ hasno isolated vertices. The cototal domination number $\boldsymbol{\gamma_{cl}}(G)$ is the minimum cardinality of a cototal dominating set.

Definition 2.5 [5] A total dominating set $S \subseteq (G)$ is called as complete cototal dominating set if induced subgraph $< V - S >$ has no isolated vertices. The complete cototal domination number $\boldsymbol{\gamma_{cc}}(G)$ is the minimum cardinality of a complete cototal dominating set.

Definition 2.6 [6] In the mathematical field of graph theory, the friendship graph (or Dutch windmill graph or n-fan) F_n is a planar undirected graph with $2n+1$ vertices and $3n$ edges.

Definition 2.7 [6] A complete bipartite graph is a graph whose vertices can be partitioned into two subsets V_1 and V_2 such that no edge has both endpoints in the same subset, and every possibleedge that could connect vertices in different subsets is part of the graph.

3. Main results

Definition 3.1 A complete cototal dominating set S is called an irredundant complete cototal dominating set iffor each $u \in S$, $N_G[S - \{u\}] \neq N_G[S]$. The irredundant complete cototal dominating number is the cardinality of the smallest irredundant complete cototal dominating set and is denoted by $\boldsymbol{\gamma_{ircc}(G)}$.

Theorem 3.2 If a graph G is a Friendship graph F_n for each $n \geq 2$, then $\boldsymbol{\gamma_{ircc}(G)} = 3$.

Proof: Consider a Friendship graph F_n for each $n \geq 2$ having $2n + 1$ vertices $w_1, w_2, \ldots w_{2n}, v$ and $3n$ edges $w_1w_2, \ldots w_{2n-1}w_{2n}, w_iv$, $1 \leq i \leq 2n$. Middle vertex of F_n for each $n \geq 2$ is v. We take total dominating set as $\{v, u\}$, where u is any w_i such that irredundant complete cototal dominating set is $\{v, u\} \cup \{x\}$, x denotes the isolated vertex. Hence $\gamma_{ircc}(G) = 3$ for a Friendship graph $F_n, n \geq 2$.

Theorem 3.3 If a graph G is complete bipartite graph $k_{m,n}$, then $\boldsymbol{\gamma_{ircc}(G)} = 2$.
Proof: Consider a complete bipartite graph $k_{m,n}$ having $(m + n)$ vertices $x_1, x_2, \ldots, x_m, y_1, \ldots y_n$ and mnedges $x_i y_j$, $i = 1,2,\ldots m$, $j = 1,2, \ldots n$. We take total dominating set as $\{x_1, y_1\}$. Hence irredundant complete cototal dominating set is $\{x_1, y_1\}$. Therefore $\gamma_{ircc}(G) = 2$ for a complete bipartite graph $k_{m,}$.

Bounds for $\gamma_{ircc}(G)$.

Theorem 3.4 Let G be a graph with no isolated vertices, then $2 \leq \boldsymbol{\gamma_{ircc}(G)} \leq p$. The proof of the theorem follows from the definition.

Theorem 3.5 Let G be the graph without isolated vertices , then $\boldsymbol{\gamma_{ircc}(G)} = p$ iff every edge of G is incident to a support vertex.

Proof: If p=2,3 then the theorem holds. Let $p \geq 4$. Consider $S \subseteq V(G)$ as an irredundant complete cototal dominating set. Therefore all pendent vertices and support vertices belong to S. If every edge of G is incident to a support vertex then $S = V(G)$ which implies $\boldsymbol{\gamma_{ircc}(G)} = p$. Suppose every edge of G is incident to a support vertex then there exist an edge uv such that both are not support vertex and $\deg(u) \geq 2$, $\deg(v) \geq 2$. If $S = V - \{u, v\}$, then S is dominating set and $< V-S >$ is connected. If $< S >$ has no isolated vertices then S is an irredundant complete cototal dominating set with cardinality $p-2$, which is a contradiction. Hence $\boldsymbol{\gamma_{ircc}(G)} = p$.

Theorem 3.6 If G is a nontrivial tree, then $1+\Delta(G) \leq \boldsymbol{\gamma_{ircc}(G)}$.
Proof Consider a nontrivial tree G with an irredundant complete cototal dominating set S. Then S has all pendent vertices and support vertices of G. Hence $\gamma_{ircc}(G) \geq 1 + \Delta(G)$
Theorem 3.7 For any nontrivial connected graph G, $\boldsymbol{\gamma_{ircc}(G)} \leq 2q - p + 2$
Proof: Consider a nontrivial connected graph G. Then $q \geq p - 1$. Also we know that $\gamma_{ircc}(G) \leq p$. Hence it can be rewritten as $\gamma_{ircc}(G) \leq 2(p - 1) - p + 2$, which implies $\gamma_{ircc}(G) \leq 2q - p + 2$.

Theorem 3.5. Let G be a tree then every edge of a tree is incident with a support vertex iff $\boldsymbol{\gamma_{ircc}(G)} = 2q - p + 2$.
Proof: Let G be tree and every edge of the tree is incident with a support vertex. We have to prove $\gamma_{ircc}(G) = 2q - p + 2$. Since every edge of G is incident to a support vertex, $\gamma_{ircc}(G) = p$. Also we know that $q = p - 1$ for any tree. Hence $\gamma_{ircc}(G) = p = 2(p - 1) - p + 2$ becomes $\gamma_{ircc}(G) = p = 2q - p + 2$.

Theorem 3.9 If G be a graph with $\Delta(G) = p - 1$, then $\gamma_{ircc}(G) \geq \gamma_c(G) + 1$.

Proof: Let the maximum degree of a graph G be $p - 1$. Then $\gamma_c = 1$. Since $\gamma_{ircc}(G) \leq 2$ we get $\gamma_{ircc}(G) \geq \gamma_c(G) + 1$.

Theorem 3.10 Let G be any graph with non-isolated vertices, then $\frac{2p}{\Delta(G)+1} \leq \gamma_{ircc}(G)$.

Proof: Let G be a graph without isolates and let the irredundant complete cototal dominating set of G be S. The number of edges in G is denoted by t, having only one vertex in S and all other vertices in $V-S$. Since $\deg(v) \leq \Delta(G)\ \forall\, v \in S$. Each vertex in S is adjacent to atleast one member of S, then $t \leq (\Delta(G)-1)|S| = (\Delta(G)-1)\gamma_{ircc}(G)$. And each vertex in $V-S$ is adjacent to atleast one vertex of $V-S$, then $t \leq 2|V-S| = 2[p-\gamma_{ircc}(G)]$. Hence $\Delta(G)\gamma_{ircc}(G) - \gamma_{ircc}(G) \geq 2(p-\gamma_{ircc}(G)]$. Therefore $\frac{2p}{\Delta(G)+1} \leq \gamma_{ircc}(G)$.

Theorem 3.11 For any connected graph G, $\gamma_{ircc}(G) = \left\lceil \frac{n}{\Delta(G)} \right\rceil$.

Proof :Let the irredundant complete cototal dominating set of G be S. Each vertex of S is a neighbor of at most $\Delta(G)-1$ vertices of $V-S$ and neighbor of atleast one vertex in S. Therefore $|S|(\Delta(G)-1) + |S| > n$. Hence $\gamma_{ircc}(G) = \left\lceil \frac{n}{\Delta(G)} \right\rceil$.

3 Conclusion

In the field of domination theory a new domination parameter has been defined and its properties are discussed.

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A Study On The Cross Product Of Fuzzy Numbers

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Abstract

In fuzzy arithmetic, the multiplication operation based on Zadeh's extension principle owns several unnatural properties both from theoretical and practical points of view. To overcome some of these shortcomings, a new operation called cross product has been introduced recently. We show the main properties of the cross product. We also present a comparative study of the traditional multiplication and the cross product in geological applications, especially for estimating of resources of solid mineral deposits.

Key words: Fuzzy number, cross product, solid mineral deposit estimation.

AMS classification: 03E72, 16Y80, 03B52, 20N25.

1 Introduction

In this section we study the theoretical properties of the cross product of fuzzy numbers. Let $R_F^* = \{u \in R_F : u \text{ is positive or negative}\}$. Firstly we begin with a theorem which was obtained by using the stacking theorem.

Theorem 1.1 If u and v are positive fuzzy numbers then $w = u \odot v$ defined by $[w]^r = [\underline{w}^r, \overline{w}^r]$, where $\underline{w}^r = \underline{u}^r \underline{v}^1 + \underline{u}^1 \underline{v}^r - \underline{u}^1 \underline{v}^1$ and $\overline{w}^r = \overline{u}^r \overline{u}^1 + \overline{u}^1 \overline{v}^r - \overline{u}^1 \overline{v}^1$, for every $r \in [0,1]$, is a positive fuzzy number.

Corollary 1.2 Let u and v be two fuzzy numbers.

(i) If u is positive and v is negative then $u \odot v = -(u \odot (-v))$ is a negative fuzzy number;

(ii) If u is negative and v is positive then $u \odot v = -((-u) \odot v)$ is a negative fuzzy number;

(iii) If u and v are negative then $u \odot v = (-u) \odot (-v)$ is a positive fuzzy Number.

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Definition 1.3 The binary operation $\odot$ on R_F^* introduced by Theorem 1.1 and Corollary 1.2 is called cross product of fuzzy numbers.

Remark 1.4 1) The cross product is defined for any fuzzy numbers in $R_F^\wedge = \{u \in R_F^*;$ there exists an unique $x_0 \in \mathbf{R}$ such that $u(x_0) = 1\}$, so implicitly for any triangular fuzzy number. In fact, the cross product is defined for any fuzzy number.
2) The below formulas of calculus can be easily proved $(r \in [0,1])$:

$$\underline{(u \odot v)}^r = \overline{u}^r \underline{v}^1 + \overline{u}^1 \underline{v}^r - \overline{u}^1 \underline{v}^1,$$

$$\overline{(u \odot v)}^r = \underline{u}^r \overline{v}^1 + \underline{u}^1 \overline{v}^r - \underline{u}^1 \overline{v}^1$$

If u is positive and v is negative, $\underline{(u \odot v)}^r = \underline{u}^r \overline{v}^1 + \underline{u}^1 \overline{v}^r - \underline{u}^1 \overline{v}^1$, $\overline{(u \odot v)}^r = \overline{u}^r \underline{v}^1 + \overline{u}^1 \underline{v}^r - \overline{u}^1 \underline{v}^1$
If u is negative and v is positive. In the last possibility, if u and v are negative then

$$\underline{(u \odot v)}^r = \overline{u}^r \overline{v}^1 + \overline{u}^1 \overline{v}^r - \overline{u}^1 \overline{v}^1, \ \overline{(u \odot v)}^r = \underline{u}^r \underline{v}^1 + \underline{u}^1 \underline{v}^r - \underline{u}^1 \underline{v}^1$$

3) The cross product extends the scalar multiplication of fuzzy numbers. Indeed, if one of operands is the real number k identified with its characteristic function then $\underline{k}^r = \overline{k}^r = k, \forall r \in [0,1]$ and following the above formulas of calculus we get the result.

The main algebraic properties of the cross product are the following.

Theorem 1.5 If $u, v, w \in R_F^*$ then

(i) $(-u) \odot v = u \odot (-v) = -(u \odot v)$;

(ii) $u \odot v = v \odot u$;

(iii) $(u \odot v) \odot w = u \odot (v \odot w)$;

(iv) If u and v have the same sign then $(u \oplus v) \odot w = (u \odot w) \oplus (v \odot w)$;

(v) $(u \odot v)^{\odot n} = u^{\odot n} \odot v^{\odot n}, \forall n \in N^*$, where $a^{\odot n} = \underbrace{a \odot \ldots \odot a}_{n \text{ times}}$ for any $a \in R_F^*$.

Remark 1.6 1) If u is positive and v negative (or u is negative and v positive) then the property of distributivity in (iv) is not verified even if u and v are real numbers.

2) The above properties (i)-(iii) hold for the usual product "$\cdot$" based on the extension

principle. The property (iv) holds in a weaker form: If u and v are on the same side of 0 then for any $w, w \prec 0$ or $0 \prec w$ we have $(u \oplus v) \cdot w = (u \cdot w) \oplus (v \cdot w)$

The so-called $L-R$ fuzzy numbers are considered important in fuzzy arithmetic. These and their particular cases triangular and trapezoidal fuzzy numbers are used almost exclusively in applications.

Definition 1.7 Let $L, R : [0, +\infty) \to [0, 1]$ be two continuous, decreasing functions fulfilling $L(0) = R(0) = 1, L(1) = R(1) = 0$, invertible on $[0, 1]$. Moreover, let a^1 be any real number and suppose $\underline{a}, \bar{a}$ be positive numbers. The fuzzy set $u : \mathbf{R} \to [0, 1]$ is an $L-R$ fuzzy number if $u(t) = \begin{cases} L\left(\frac{a^1-t}{\underline{a}}\right), & \text{for } t \leq a^1 \\ R\left(\frac{t-a^1}{\bar{a}}\right), & \text{for } t > a^1 \end{cases}$.

Symbolically, we write $u = \left(a^1, \underline{a}, \bar{a}\right)_{L,R}$, where a^1 is called the mean value of $u, \underline{a}, \bar{a}$ are called the left and the right spread. If u is an $L-R$ fuzzy number then

$$[u]^r = \left[a^1 - L^{-1}(r)\underline{a}, a^1 + R^{-1}(r)\bar{a}\right]$$

Theorem 1.8 If u and v are strict positive $L-R$ fuzzy numbers then $u \odot v$ is a strict positive $L-R$ fuzzy number.

Since we are interested mainly in the applications of the cross product we may restrict our attention to positive fuzzy numbers, however in other cases some similar properties can be obtained.
The cross product verifies the following metric property.

Theorem 1.9 If u, v have the same sign and $w \in R_F^*$ then

$$D(w \odot u, w \odot v) \leq K_w D(u, v), \text{ where } K_w = \max\left\{\left|\overline{w}^1\right|, \left|\underline{w}^1\right|\right\} + \overline{w}^0 - \underline{w}^0.$$

Definition 1.10 Let u be a fuzzy number. The crisp number $\Delta_L^r(u) = \underline{u}^1 - \underline{u}^r$ is called $r-$ error to left of u and the crisp number $\Delta_R^r(u) = \bar{u}^r - u^{-1}$ is called r -error to right of u, where $r \in [0, 1]$. The sum $\Delta^r(u) = \Delta_L^r(u) + \Delta_R^r(u)$ is called r -error of u.

If u expresses the fuzzy concept A then $\Delta_L^r(u)$ and $\Delta_R^r(u)$ can be interpreted as the values of tolerance of level r from the concept A to left and to right, respectively.

For example, if the triangular fuzzy number $u=(5,7,9)$ expresses "early morning" then $\Delta_L^{\frac{1}{2}}(u)=1$ (one hour) is the tolerance of level $\frac{1}{2}$ of u towards night from the concept of "early morning" and $\Delta_R^{\frac{1}{4}}(u)=0.5$ (30 minutes) is the tolerance of level $\frac{1}{4}$ of u towards moon from the concept of "early morning".

A new argument in the use of addition of fuzzy numbers as extension (by Zadeh's principle) of real addition is the validity of the formula $\Delta^r(u\oplus v)=\Delta^r(u)+\Delta^r(v)$ which is consistent to the classical error theory. It is an immediate consequence of the obvious formulas $\Delta_L^r(u\oplus v)=\Delta_L^r(u)+\Delta_L^r(v)$ and $\Delta_R^r(u\oplus v)=\Delta_R^r(u)+\Delta_R^r(v)$

Now, let us study the relative error of the cross product.

Definition 1.11 Let u be a fuzzy number such that $\underline{u}^1\neq 0$ and $\bar{u}^1\neq 0$. The crisp numbers $\delta_L^r(u)=\frac{\Delta_L^r(u)}{|\underline{u}^1|}$ and $\delta_R^r(u)=\frac{\Delta_R^r(u)}{|\overline{u}^1|}$ are called relative r -errors of u to left and to right. The quantity $\delta^r(u)=\delta_L^r(u)+\delta_R^r(u)$ is called relative $r-$ error of u.

Theorem 1.12 If u and v are strict positive or strict negative fuzzy numbers then

$$\delta^r(u\odot v)=\delta^r(u)+\delta^r(v)$$

2 Applications of the Cross Product in Geology

Recently, fuzzy arithmetic has found several applications in geology. In the above cited work the usual (Zadeh's extension principle based) product is used for estimation of resources of solid mineral deposits. In this section we propose an alternative study of the same problem, by using the cross product. The reasons of the possible usefulness of the cross product are the following.

Firstly, in this case the shape of the result of the product is conserved, i.e. the product of triangular numbers is triangular and the product of trapezoidal numbers is trapezoidal. Secondly, the 1-level sets are better taken into account by the use of cross product. Also, the consistency of the cross product with the classical error theory motivates this study.

As we perform resource estimation on several bauxite deposits in Hungary. In the same way as with the traditional methods, the tonnage of the resources is obtained by the product of the deposit area, the average thickness and the average bulk-density of the studied ore or mineral commodity. Large deposits can be split into blocks, preferably along natural boundaries, such as tectonic lines. We present the results obtained by the usual multiplication and the results obtained by using the cross product.

Furthermore, if we defuzzify the two results obtained by the two different producttype operations we conclude that the results are different. Also, we observe that after defuzzification (by centroid method) the result of the cross product in the study of the Óbarok deposit is smaller than that of the usual product i.e. the cross product leads to a more pessimistic result than the usual multiplication in this case. So, the risks of an investment at this site can be more realistically evaluated.

3 Conclusion

In this paper, we discussed about the concept of Cross product of fuzzy numbers and the applications of cross product in Geology.

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A Study on Analytical Solutions For Stochastic Differential Equations Via Martingale Processes

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Abstract

In this paper, we propose some analytical solutions of stochastic differential equations related to Martingale processes. In the first resolution, the answers of some stochastic differential equations are connected to other stochastic equations just with diffusion part (or drift free). The second suitable method is to convert stochastic differential equations into ordinary ones that it is tried to omit diffusion part of stochastic equation by applying Martingale processes. Finally, solution focuses on change of variable method that can be utilized about stochastic differential equations which are as function of Martingale processes like Wiener process, exponential Martingale process and differentiable processes.

Key words: Martingale process, Itô formula, Change of variable, Differentiable process, Analytical solution

AMS classification: 60G10, 60H10, 60H30

1 Introduction

The purpose of this article is to put forward some analytical and numerical solutions to solve the Itô stochastic differential equation (SDE):

$$\begin{cases} dX(t) = \mathcal{A}(X(t),t)dt + \mathcal{B}(X(t),t)dW_t, \\ X(0) = X_0, \end{cases} \tag{1}$$

where $W(t)$ is a Wiener process and triple $(\Omega, \mathcal{F}, \mathbb{P})$ is a probability space under some conditions and special relations between drift and volatility.

Both the drift vector $\mathcal{A} : \mathbb{R} \times [0,T] \longrightarrow \mathbb{R}$ and the diffusion matrix $a := \mathcal{B}\mathcal{B}^T : \mathbb{R} \times [0,T] \longrightarrow \mathbb{R}$ are considered Borel measurable and locally bounded functions. It is

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assumed that X_0 is a non-random vector. As usual, $\mathcal{A}$ and $\mathcal{B}$ are globally Lipschitz in $\mathbb{R}$ that is:

$$|\mathcal{A}(X,t)-\mathcal{A}(Y,t)|+|\mathcal{B}(X,t)-\mathcal{B}(Y,t)|\leq D|X-Y|,\quad X,Y\in\mathbb{R}\quad \text{and}\quad t\in[0,T],$$

and result in the linear growth condition:

$$|\mathcal{A}(X,t)|+|\mathcal{B}(X,t)|\leq C(1+|X|).$$

These conditions guarantee the Eq. (1) has a unique t-continuous solution adapted to the filtration $\mathcal{F}_t t\geq 0$ generated by $W(t)$ and

$$E\left[\int_0^T |X(s)|^2\,\mathrm{d}s\right]<\infty. \tag{2}$$

It is generally accepted that, analytical solutions of partial and ordinary differential equations are so important particularly in physics and engineering, whereas most of them do not have an exact solution and even a limited number of these equations, (e.g., in classical form), have implicit solutions. Analytical methods and solutions, especially in stochastic differential equations, could be excessive fundamental in some cases therefore we draw to take a comparison and analyze computation error between them and different numerical methods. Numerous numerical methods can be applied to solve stochastic differential equations like Monte Carlo simulation method, finite elements and finite differences.

2 Change of measure and Martingale process

In this section under some conditions, we intend to make a Martingale process from a random one in $\mathbb{L}^2(\mathbb{R}\times[0,T])$, where T is called maturity time. The exponential Martingale process associated with $\lambda(t)$ is defined as follows:

$$\mathrm{d}Z^{\lambda}=\exp\left(\int_0^t \lambda(s)dW_s-\frac{1}{2}\int_0^t \lambda^2(s)\,ds\right). \tag{3}$$

It can be indicated by Itô formula that Z_t^{λ} is a Martingale due to the drift-free property:

$$\mathrm{d}Z_t^{\lambda}=\lambda Z_t^{\lambda}\mathrm{d}W_t,\quad Z_t^{\lambda}(0)=1. \tag{4}$$

Theorem 2.1 Suppose that stochastic processes X_t verify in differential equation:

$$dX_t = \mu(X_t, t)dt + \sigma(X_t, t)dW_t, \tag{5}$$

and let $\lambda(t) := -\mu(X_t, t)/\sigma(X_t, t)$. Therefore, $X Z_t^\lambda$ is a Martingale process.

Proof: With attention to real function $\lambda(t)$, we have:

$$\begin{cases} dX = \mu(X,t)dt + \sigma(X,t)dW_t = -\lambda(t)\sigma(X,t)dt + \sigma(X,t)dW_t, \\ dZ_t^\lambda = Z_t^\lambda \lambda dW_t. \end{cases}$$

By utilizing Itô product formula, we get:

$$\begin{aligned} d(XZ_t^\lambda) &= Xd(Z_t^\lambda) + Z_t^\lambda dX + dXd(Z_t^\lambda) \\ &= \lambda X Z_t^\lambda dW_t + \mu(X,t)Z_t^\lambda dt + \sigma(X,t)Z_t^\lambda dW_t + \lambda\sigma(X,t)Z_t^\lambda dt. \end{aligned}$$

According to theorem assumption, we obtain:

$$d(XZ_t^\lambda) = Z_t^\lambda (X\lambda + \sigma(X,t))dW_t. \tag{6}$$

It emphasizes that XZ_t^λ is a P-Martingale.
Therefore, $\lambda(t) = \frac{-\mu(X,t)}{\sigma(X,t)}$ is the sufficient condition for following SDEs equivalence:

$$dX = \mu(X,t)dt + \sigma(X,t)dW_t \Leftrightarrow d(XZ_t^\lambda) = Z_t^\lambda (X\lambda(t) + \sigma(X,t))dW_t. \tag{7}$$

Consequently, by solving the obtained equation in Eq. (6), we obtain the following result when $Z_0^\lambda = 1$:

$$XZ_t^\lambda = \int_0^t Z_t^\lambda (X\lambda(s) + \sigma(X,t))\, dW_t + X_0. \tag{8}$$

By taking mathematical expectation from both sides of Eq. (8):

$$E^P[XZ_t^\lambda] = X_0 \Rightarrow E^P[X] = X_0 (Z_t^\lambda)^{-1}. \tag{9}$$

In addition, to compute the variance of this stochastic process:

$$\begin{aligned} E^P[(X Z_t^\lambda)^2] &= X_0^2 + E\left[\int_0^t (Z_s^\lambda)^2 (X\lambda(s)+\sigma(X,t))^2 \,\mathrm{d}s\right] \quad (\text{ by Itǒ isometry }) \\ &= X_0^2 + \int_0^t (Z_s^\lambda)^2 E\left(\left[(X\lambda(s)+\sigma(X,t))^2\right]\right) \mathrm{d}s. \ \text{var}\ (X Z_t^\lambda) = (Z_t^\lambda)^2 \ \text{var}\ (X) \\ &= \int_0^t (Z_s^\lambda)^2 E\left(\left[(X\lambda(s)+\sigma(X,t))^2\right]\right) \mathrm{d}s. \end{aligned} \tag{10}$$

Applying (6) and using numerical approximation by EM method, we have:

$$\begin{aligned} \Delta X_i Z_{t_i}^\lambda &= Z_{t_i}^\lambda (X_i \lambda(t_i) + \sigma_i)\Delta W_i. \\ X_{t_{i+1}} Z_{t_{i+1}}^\lambda &= X_{t_i} Z_{t_i}^\lambda + Z_{t_i}^\lambda (X_{t_i}\lambda(t_i)+\sigma_i)\Delta W_i. \\ X_{t_{i+1}} &= (Z_{t_{i+1}}^\lambda)^{-1} Z_{t_i}^\lambda (X_{t_i} + (X_{t_i}\lambda(t_i)+\sigma_i)\Delta W_i). \end{aligned}$$

Direct calculations would lead to the conclusion that:

$$R_{t_i} = (Z_{t_{i+1}}^\lambda)^{-1} Z_{t_i}^\lambda = \exp\left(-\int_{t_i}^{t_{i+1}} \lambda(s)\,\mathrm{d}W_s + \frac{1}{2}\int_{t_i}^{t_{i+1}} |\lambda^2(s)|\,\mathrm{d}s\right).$$

So the following Milstein recursive method is inferred as a good numerical method to find $X(t_{i+1})$:

$$X_{t_{i+1}} = R_{t_i}(X_{t_i} + (X_{t_i}\lambda(t_i)+\sigma_i)\Delta W_i) + \frac{1}{2} R_{t_i}^2 \lambda(t_i)\,(X_{t_i}\lambda(t_i)+\sigma_i)\,(\Delta^2 W_i - \Delta t_i). \tag{11}$$

In example, we compare this method with usual Milstein method in the case that a stochastic differential equation contains drift and volatility both parts and indicate that this method could be better in some cases.

3 Change of variable method

This section intends to analyze the change of variable method like, to get explicitly the solution of arbitrary SDE:

$$\mathrm{d}X = \mathcal{A}(X,t)\mathrm{d}t + \mathcal{B}(X,t)\mathrm{d}W_t, \quad X(0) = x.$$

By finding appropriate variables $u(Y) = X$ and their conditions so that Y is the answer of a well-known SDEs related to Martingale processes.

$$dY = f(X,t)dt + g(X,t)dW_t, \quad y(0) = y.$$

For more explanation and different conditions under which they are possible. Now we consider following various cases.

Case 1 Consider the following SDE:

$$dY = a(t)dt + b(t)dW_t. \tag{12}$$

Applying Itô formula for $u(Y) = X$, to (12), we get:

$$\begin{cases} u'(a(t)) + \frac{1}{2}u''b^2(t) = \mathcal{A}(u(Y),t), \\ u'b(t) = \mathcal{B}(u(Y),t). \end{cases} \tag{13}$$

Thus, it concludes that:

$$\frac{a(t)}{b(t)}\mathcal{B} + \frac{1}{2}\mathcal{B}\mathcal{B}' = \mathcal{A} \Rightarrow \frac{\mathcal{A}}{\mathcal{B}} - \frac{1}{2}\mathcal{B}' = \frac{a(t)}{b(t)}. \tag{14}$$

Finally, the equation $\frac{\partial}{\partial Y}\left(\frac{\mathcal{A}}{\mathcal{B}} - \frac{1}{2}\mathcal{B}'\right) = 0$ is necessary condition to solve an equation via change of variable in (12) $\left(\mathcal{B}' = \frac{\partial \mathcal{B}}{\partial X}\right)$.

Case 2 Consider the exponential Martingale process SDE (3):

$$\begin{cases} dY = \lambda(t)YdW_t, \\ Y(0) = Y_0. \end{cases} \tag{15}$$

Applying Itô formula for $u(Y) = X$, to (15), we acquire:

$$\begin{cases} u'\lambda Y = \mathcal{B}(u,t) = \lambda(t)Y\hat{\mathcal{B}}(u) \quad \text{or} \quad u' = \hat{\mathcal{B}}(u), \\ \frac{1}{2}u''\lambda^2Y^2 = \mathcal{A}(u,t). \end{cases} \tag{16}$$

So from the last equality, we have $\frac{\mathcal{B}'}{\lambda(t)} - \frac{2\mathcal{A}}{\mathcal{B}} = \lambda(t)$. Therefore, $\frac{\partial}{\partial u}\left(\mathcal{B}'_u - \frac{2\lambda(t)\mathcal{A}}{\mathcal{B}}\right) = 0$ is necessary condition to solve SDE, with this change of variable.

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Case 3 Consider the well-known equation:

$$\begin{cases} \mathrm{d}Y = a(t)Y\mathrm{d}t + b(t)Y\mathrm{d}W_t, \\ Y(0) = Y_0. \end{cases} \tag{17}$$

Which is Black-Scholes equation with exact solution

$$Y_0 = \exp\left(\int_0^t b(s)\mathrm{d}W_s + \int_0^t \left(a(s) - \frac{1}{2}b^2(s)\right)\mathrm{d}s\right).$$

Applying Itô formula for $u(Y) = X$, to (17), we get:

$$\begin{cases} u'a(t)Y + \frac{1}{2}u''b^2(t)Y^2 = \mathcal{A}(u,t), \\ u'Yb(t) = \mathcal{B}(u,t) = b(t)Y\hat{\mathcal{B}}(u). \end{cases} \tag{18}$$

For this reason, $u' = \hat{\mathcal{B}}(u)$ and we have:

$$\frac{a(t)}{b(t)} = \frac{\mathcal{A}}{\mathcal{B}} - \frac{1}{2}(\mathcal{B}'_u - b(t)) = \gamma(u,t). \tag{19}$$

It means that $\frac{\partial}{\partial u}\gamma(u,t) = 0$, is a necessary condition to solve the initial stochastic differential equation by this change of variable.

Case 4 Another appropriate and prominent case is as follows:

$$\begin{cases} \mathrm{d}Y_t = f(Y_t,t)\mathrm{d}t + c(t)Y_t\mathrm{d}W_t, \\ Y(0) = Y_0. \end{cases} \tag{20}$$

This kind of equations, applying Itô formula on $X_t = Y_t Z_t^c(t)^{-1}$, is converted to a ordinary differential equations.

Theorem 3.1 The stochastic differential equations in (20) given by continuous functions $f : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ and $C : \mathbb{R} \to \mathbb{R}$ can be written as:

$$\mathrm{d}(Y_t(Z_t^c(t))^{-1}) = (Z_t^c(t))^{-1} f(Y_t,t)\mathrm{d}t, \tag{21}$$

where $Z_t^c(t)$ is an exponential Martingale process.

To be more precise, using change of variable $V = X(Z_t^{c(t)})^{-1}$, it is enough to solve

$$\begin{cases} X_t' = (Z_t^{c(t)})^{-1} f(X_t Z_t^{c(t)}), \\ X(0) = X_0 \end{cases} \tag{22}$$

Applying Itô formula for $u(Y) = M_t$, in (20) we get:

$$dM_t = M_t' dY + \frac{1}{2} M_t''(dY)^2.$$
$$\begin{cases} f(Y,t)M_t' + \frac{1}{2}M_t'' c^2(t) Y^2 = \mathcal{A}(M_t, t), & (1) \\ c(t) Y M_t' = \mathcal{B}(M_t, t), \ u(Y_0) = M_0. & (2) \end{cases} \tag{23}$$

According to (23), we have $\mathcal{B}(M_t, t) = c(t)\hat{\mathcal{B}}(M_t)$. Besides, if the new stochastic differential equation is related to a Martingale process, we have $\mathcal{A}(M_t, t) = 0$ and:

$$f(Y,t) = -\frac{c^2(t)Y}{2}(\hat{\mathcal{B}}(M_t)' - 1). \tag{24}$$

Again, applying Itô formula for $\phi(M_t) = V_t$ to Martingale equation contributes to

$$dM_t = \mathcal{B}(M_t, t) dW_t = c(t)\hat{\mathcal{B}}(M_t) dW_t,$$

we can achieve to a novel group of stochastic differential equation that its solution is as a function of a Martingale process.

4 Conclusions

In this paper, a couple of analytical solutions of some determined set of stochastic differential equations was indicated via making the Martingale process from a stochastic process. Converting stochastic differential equations to ordinary ones as another suitable method was posed. Indeed, it is tried to omit diffusion part of stochastic equation by applying Martingale processes. In addition, change of variable method on SDEs related to Martingale processes was discussed.

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