2456-8686, vii(ii), 2023: 127-132
https://doi.org/10.26524/cm180

ORIGINAL RESEARCH Open Access

Lucky Labeling for Extended Triplicate Graph of Star

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Received: 31 October 2023/ Accepted: 06 November 2023/ Published online: 16 December 2023

Abstract

In this paper, we investigate the existence of lucky labeling in the context of extended triplicate graph of star.

Key words: Graph labeling, Star graph, Triplicate graph, Lucky labeling.

AMS classification: 05C78

1 Introduction

In 1967 [10], the concept of graph labeling was introduced by Rosa . A graph labeling is an assignment of integers to the edges or vertices or both to the certain conditions. In 2023, the concept of Extended triplicate graph of star ETG($k_{1,n}$)was introduced by S. Bala .et.al., [4]. Lucky labeling of the graphs were studied in recent times by A.Ahari et al[1] and S.Akbari et al [2]. Proper lucky labeling [8] is coloring the vertices such that the coloring is proper as well as lucky. Theleast positive integer k for which a graph G has a lucky labeling from the set $\{1,2,\ldots,k\}$ is calledthe proper lucky number of G. In 2015, Mirka Miller et.al.,[9] introduced the concept of d-Luckylabeling. By interest in this paper we investigate the existence of lucky labeling, proper luckylabeling and d-lucky labeling in the context of extended triplicate graph of star.

2 Preliminaries

In this section, we will discuss the basic notions related to this paper.

Definition 2.1 [4] Let G be a star graph ($K_{1,n}$).The triplicate graph of star graph with vertex set $\delta(G)$ and edge set $\xi(G)$ is given by $\delta(G)$

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$= \{b \cup b' \cup b" \cup c_i \cup c_i' \cup c_i"/1 \leq i \leq n\}and\xi(G)$=$\{bc_i' \cup b'c_i \cup b'c_i" \cup b"c_i'/1 \leq i \leq n\}$.
Clearly ,triplicate graph of star graph $TG(K_{1,n})$ with this vertex set and edge set is disconnected. To make this a connected graph, include a new edgebc_1 to the edge setξG. Thus, we get an extended triplicate graph of star with vertex set$\delta' = \delta$ and edge set$\xi' = \xi \cup bc_1$, denoted by $ETG(K_{1,n})$. clearly,$ETG(K_{1,n})$. has 3(n+1) vertices and 4n+1 edges.

Definition 2.2 [5] let$S : \delta(G) \to N$ be a labeling of the vertices of a graph G by a positive integers. Let $\sigma(b)$ indicate the sum of labels of the neighbors of the vertex b in G we put $\sigma(b) = 0$.A labeling S is lucky if $\sigma(b) \neq \sigma(c)$ for every pair of adjacent vertices b and c.The lucky number of a graph G is,denoted by $\eta(G)$, is the least positive integer k such that G has a lucky labelling with $1, 2, 3, \cdots, k$ as set of labels.

Definition 2.3 [8] A Lucky labeling is proper lucky labeling S is proper as well as lucky ,i.e. if b and c are adjacent in G the $S(b) \neq S(c)$ and $\sigma(b) \neq \sigma(c)$. The proper lucky number of G is denoted by $\eta_p(G)$,is the least positive integer k such that G has a d-lucky labeling with $1, 2, 3, \cdots, k$ as the set of labels.

Definition 2.4 [9] Let $\delta(G) \to 1, 2, 3, \ldots k$ be a labeling of the vertices of a graph G by a positive integers. Define $\mu(c) = \sum_{b \in N(c)} l(b) + d(c)$, where$d(c)$denotes the degree of c. We define a labeling l as d-lucky if $\mu(b) \neq \mu(c)$, for every pair of adjacent vertices b and c in G. The d-lucky number of a graph G, denoted by$\eta_{dl}(G)$,is the least positive k such that G has a d-lucky labeling with $1, 2, 3, \cdots, k$ as the set of labels.

3 Main Result

In this section, we investigate the existence of Lucky labeling, Proper lucky labeling and d-lucky labeling for the Extended Triplicate of star.

Theorem 3.1 The Extended triplicate graph of star admits lucky labeling with the lucky number $\eta(ETG(k_{1,n})) = 2$.
Proof: The Extended Triplicate graph of star graph has 3(n+1) vertices and 4n+1 edges.
to prove that $ETG(k_{1,n})$ admits lucky labeling.

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Define a function $S:\delta(G)\to 1,2$ to the vertices such that$S(b)=S(b')=S(b'')$ and $S(c)=S(c')=S(c'')=2$
Then the sum of neighbor vertices are $\sigma(b)=2(n+1),\sigma(b')=4n,\sigma(b'')=2n,\sigma(c_1)=2,\sigma(c_i)=1;2\le i\le n$
for $2\le i\le n$
$\sigma(c_i')=2$and$\sigma(c_i'')=1$
Therefore, the sum of adjacent vertices are distinct.
Hence, The Extended Triplicate graph of star graph admits a lucky labeling with lucky number $\eta(ETG(k_{1,n}))=2$.

Example 3.2 $ETG(K_{1,3})$ and its lucky labeling is shown in figure 1.

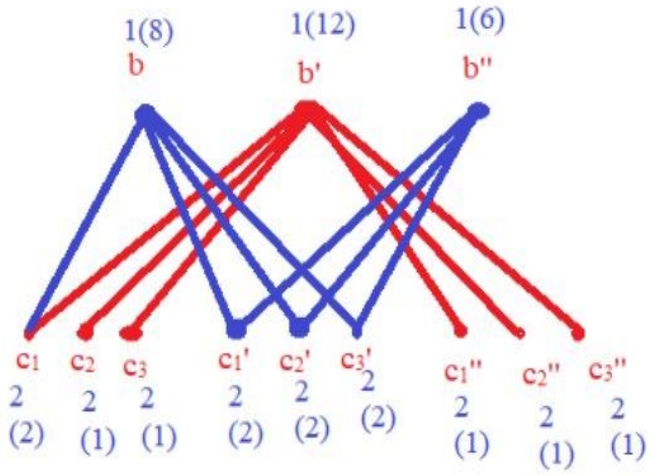

Figure 1:

Theorem 3.3 The Extended triplicate graph of star admits Proper lucky labeling with proper lucky number $\eta(ETG(K_{1,n}))=3$
Proof: The Extended Triplicate graph of star graph has $3(n+1)$ vertices and $4n+1$ edges.
To prove that $ETG(K_{1,n})$ admits proper lucky labeling.

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Define a function $S:\delta(G)\to 1,2,3$ to label the vertices. Such that $S(b)=S(b')=1$ and $S(c_1)=3$, $S(c_i)=2$; $2\le i\le n$ For $1\le i\le n$,
$S(c_i')=S(c_i'')=2$
Then the sum of neighbour vertices are $\sigma(b)=2n+3$, $\sigma(b')=4n+1$, $\sigma(b'')=2n$
$\sigma(c_1)=\sigma(c_i)=1$; $2\le i\le n$ For $1\le i\le n$,
$\sigma(c_i')=2$ and $\sigma(c_i'')=1$
Therfore, the sum of adjacent vertices are distint.
Hence, The Extended Triplicate graph admits a proper lucky labeling with proper lucky number $\eta(ETG(K_{1,n}))=3$.

Example 3.4 $ETG(K_{1,4})$ and its proper lucky labeling is shown in figure 2.

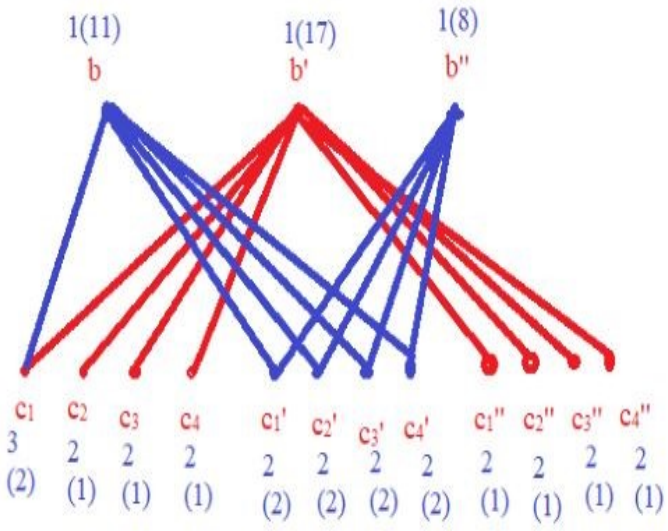

Figure 2:

Theorem 3.5 The Extended triplicate graph of star admits d- lucky labeling with lucky number $\eta_{dl}(ETG(k_{1,n}))=3$.
Proof: The Extended Triplicate graph of star graph has 3(n + 1) vertices and 4n+1 edges.
To prove that $ETG(k_{1,n})$ admits d- lucky labeling.
Define a function $S:\delta(G)\longrightarrow 1,2$ to label the vertices.
Such that, $S(b)=S(b')=S(b'')=1$ and $S(c_i)=S(c_i')=S(c_i'')=2; 1\le i\le n$,

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The degree of the vertices are
$d(b) = n + 1, d(b') = 2n, d(b'') = n$
$d(c_1) = 2$, $d(c_i) = 1$; $2 \leq i \leq n$
for $1 \leq i \leq n$,
$d(c_i') = 2$ and $d(c_i'') = 1$
Then by $\mu(b) = d(b) + \sum_{\delta \in N(b)} l(c)$ we obtain a labeling
$\mu(b) = 3(n+1), \mu(b') = 6n, \mu(b'') = 3n$ $\mu(c_1) = 4, \mu(c_i) = 2; 2 \leq i \leq n$
for $1 \leq i \leq n$,
$\mu(c_i') = 4$ and $\mu(c_i'') = 2$
For every pair of adjacent vertices of b and c in $ETG(k_{1,n})$ are distinct.
Therefore, $\mu(b) \neq \mu(c)$
Hence, The Extended Triplicate graph of star graph admits a d - lucky labeling with lucky number $dl(ETG(K_{1,n}) = 2$.

Example 3.6 $ETG(K_{1,4})$ and its d- lucky labeling is shown in figure 3.

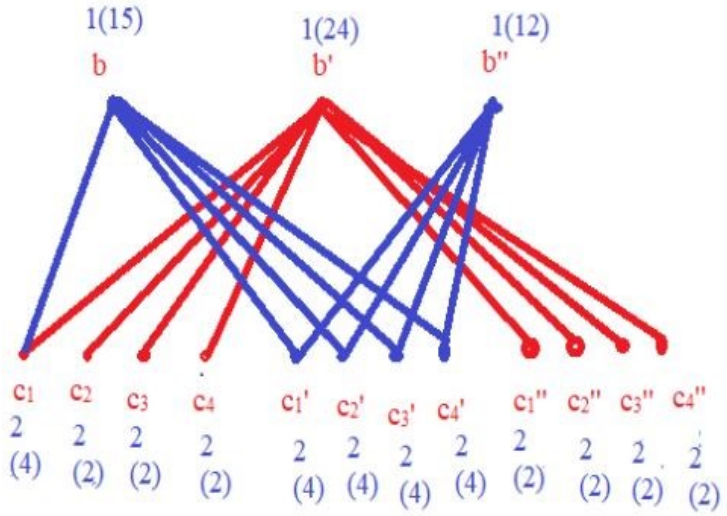

Figure 3:

3 Conclusion

In this paper, we have proved that the extended triplicate graph of star admits lucky labeling, proper lucky labeling and d-lucky labeling.

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