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Conformable Fractional-Order SEIQRS Epidemic Model and its Solutions

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Abstract

In this paper, we describe the variational iteration method and derive approximate solutions using the conformable fractional differential transform method for the SEIQRS epidemic model. These techniques are based on conformable derivative, which has recently gained a lot of popularity. First, we have redefined the conformable differential transformation method (CDTM) and variational iteration method utilising the fractional order derivative. We then apply these methods to SEIQRS epidemic model to illustrate the effectiveness and accuracy of the suggested strategies.

Key words: Conformable differential transform method, Variational Iteration method, integration byparts.

AMS classification: 26A33, 49k15.

1 Introduction

Infectious illness transmission rates are mostly explained and forecast by epidemic models. The term "epidemic modelling" refers to a variety of methods for analyzing the transmission of communicable infections among populations using mathematical, statistical, and computer techniques (refer [13]). The conformable fractional differential transform method and its application to its solution is discussed in [6]. This method is based on the recently established conformable fractional derivative and its associated mathematical tools. In [2], the brief report on the mathematical model of unequal numbers of different numbers, study of disease transmission from bats to humans, and prove the consistency of the presence and absence of understanding, the stability of the solutions of balance problems, and using the Lyapunov candidate as

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a function of global stability analysis. The study in [3] presents a partial ranking of the SEIQRDP model to examine the multi-field proven COVID-19 pandemic prevention and has demonstrated its ability to predict and understand the pandemic, and therefore epidemiological models will often be useful for epidemic control. The authors in [4] presents a study of the SEIQHR model to predict infectious diseases such as COVID-19; here the small order is considered flexible and reliable for the growth characteristics of the propagation. The results concluded that COVID-19 is still around and requires long-term prevention strategies and interventions. The α-derivative in [7] revise the variance method and variance model and show their results in finding solutions for the SIR transmission model and point their results to other transmission models such as SEIR/SEIRS and SIS applicability. The use of Grunwald-Letnikov derivatives and nabla fractional difference for the fractional generalization of the SIR transmission model for COVID-19 transmission prediction and the use of genetics to estimate model parameters is discussed in [9].

The authors in [10] introduced general conformable fractional derivative (GCFD) as a continuation of classical and conformal derivatives, derived the fractional Taylor power series expansion, proved the product and divergence theorems by integration, and obtained the existence of uniqueness along with the classical solution of the diffusion equation solved by GCFD method of separation of differences. The authors in [11] develop a more efficient solution for the differential equations of the Riemann-Stieltjes environment, examines the points of comparison for the solution problem and explains the results with examples, all using monotonous iterative techniques plus up and down. This solution focuses on the shape-preserving fractional derivative as a modification of the first derivative. Further in [12], authors developed a fractional order SEIR model with vaccination. The approximate solution of the fractional SIR transmission model in [21] was obtained using the conformal fractional differential transformation method expressed as a fast converging series, and the efficiency of the results was proven with examples. Using the Adam-Bashforth-Moulton method, the SEIR model with vaccination is numerically solved. The authors in [14] discussed a nonlinear variation of a cigarette model is analyzed using the conformal fractional differential transformation method with the help of computational Maple software using Taylor power series expansion and fractional stimulation.

A report on the fractional ordinal SEIRD model for the spread of COVID-19, which uses Italian real data for prediction, shows that the fractional ordinal model

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can predict as well as the original model, with the former providing a closer estimate. to real data is given in [15]. In [19], the authors introduces the COVID-19 SEIR transmission model using Caputo fractional derivatives, examines the potential region and the equilibrium equation, examines the stability of the equation, and proves the existence of a unique solution using the fraction Euler method. In addition to stability and feasibility analyses, equilibrium points were indicated (see in [16]). Along with it, the fractional differential system in the Atangana-Baleanu-Caputo derivative analyze a new transmission model for long-term NCDs, including element balance, stability and feasibility, quality and demand analysis through mathematical experiments. The article [17] provides the use of proportional fractional derivatives (PF) in the analysis of epidemiological models and its effects on a variety of epidemics, including significant low birth rates and the SEQIR classification model to identify risk factors and prevention targets for infectious diseases. In [20], the COVID-19 SIES transmission model provides feasibility and stability analysis using fixed points and the Ulam-Hyers method, and supporting the theoretical results, proving the recommendations of the method and the simple method.

Recently, the transmission dynamics of SEIQR model based on difference equation is shown in [1]. To estimate the spread of virus spread, the authors in [5, 18] used the fuzzy fractional laplace transform in Caputo sense.

This article is organised as follows: The preliminaries of conformable fractional order derivative and few properties are covered in Section 2. The mathematical model of SEIQRS and its description are presented in Section 3. The variational iteration method have been developed in Section 4. The conformable fractional differential transform method of SEIQRS model followed with an example is discussed in Section 5. The conclusion is placed in Section 6.

2 Preliminaries of Conformable Fractional Derivative

In this section we recall basic definitions and some of the properties of conformable fractional derivatives, which will be used in the subsequent sections.

Definition 2.1 [8] Let $\upsilon \in (0,1)$ and f be $\upsilon-$differentiable at a point $t > 0$, then the conformable fractional derivative on f is defined by

$$T_\upsilon f(t) = t^{1-\upsilon}\frac{d}{dt}f(t). \tag{1}$$

2456-8686, vii(ii), 2023: 015 - 030
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Example 2.2 Consider the function $f(t) = t^{\alpha}$ and $\upsilon = \frac{1}{2}$,since $\upsilon = \frac{1}{2}$ lies in the open interval $(0, 1)$, so f is $\upsilon-$differentiable. The conformable fractional derivative on t^{α} is given by

$$T_{\frac{1}{2}}f(t) = t^{1-\frac{1}{2}}\frac{d}{dt}(t^{\alpha}) = t^{\frac{1}{2}}\frac{d}{dt}(t^{\alpha}) = \frac{1}{2}t^{\frac{1}{2}}\alpha t^{\alpha-1} = \frac{\alpha}{2}t^{\alpha-\frac{1}{2}}. \quad (2)$$

Example 2.3 Consider the function $f(t) = \sin(t)$ and $\upsilon = \frac{2}{3}$,since $\upsilon = \frac{2}{3}$ lies in the open interval $(0, 1)$, so f is $\upsilon-$differentiable. The conformable fractional derivative on $\sin(t)$ is given by

$$T_{\frac{2}{3}}f(t) = t^{1-\frac{2}{3}}\frac{d}{dt}(\sin(t)) = t^{\frac{1}{3}}\frac{d}{dt}(\sin(t)) = \frac{1}{3}t^{\frac{1}{3}}\cos(t). \quad (3)$$

Definition 2.4 [8] The fractional integral of order υ for conformable fractional derivative is defined by

$$I_{\upsilon}(f)(t) = I(t^{\upsilon-1}f) = \int_{0}^{t} s^{\upsilon-1}f(s)ds, \forall\ \upsilon \in (0, 1). \quad (4)$$

Definition 2.5 [22] If f is an infinitely υ-differentiable function, then it can be expanded in a fractional power series expansion around the point $t = 0$ as

$$f(t) = \sum_{k=0}^{\infty} \frac{t^{\upsilon k}}{\upsilon^{k}k!}\left[(T_{\upsilon}f)^{(k)}\right]_{t=0}, 0 < t < R^{\frac{1}{\upsilon}}, R > 0. \quad (5)$$

Here, $\left[(T_{\upsilon}f)^{(k)}\right]_{t=0}$ stands for the fractional derivative's application to the k-times which is defined as

$$F_{\upsilon}(k) = \frac{1}{\upsilon^{k}k!}\left[(T_{\upsilon}f)^{(k)}\right]_{t=0}. \quad (6)$$

Proposition 2.6 [7] If equations (5) and (6) exist, then the following properties holds.

(i) $F_{\upsilon}(k) = U_{\upsilon}(k) \pm V_{\upsilon}(k)$ holds if $f(k) = u(k) \pm v(k)$ exists.

(ii) $F_{\upsilon}(k) = cU_{\upsilon}(k)$ holds if If $f(k) = cu(k), c \in \mathbb{R}$ exists.

(iii) $F_{\upsilon}(k) = \sum_{l=0}^{k} U_{\upsilon}(l)V_{\upsilon}(k-l)$ holds if $f(k) = u(k)v(k)$ exists.

2456-8686, vii(ii), 2023: 015 - 030
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(iv) $F_\upsilon(k) = \upsilon(k+1)U_\upsilon(k+1)$ holds if $f(k) = T_\upsilon u(k)$ exists.

(v) For $\delta(k) = \begin{cases} 1, \ if \ k = 0, \\ 0, \ if \ k \neq 0 \end{cases}$, we have $F_\upsilon(k) = \delta(k - (\frac{p}{\upsilon}))$ if $f(k) = (k - k_0)^p$.

3 SEIQRS: Epidemic Model

The SEIQRS model, an expanded version of SIR simulates the interaction of humans in various conditions: susceptible (S), exposed (E), infected (I), quarantined (Q), and recovered (R) population. The SEIQRS model equations are defined as follows:

$$\begin{cases} \frac{dS}{dt} = \Lambda - \beta S(t)I(t) - dS(t) + \eta R(t) \\ \frac{dE}{dt} = \beta S(t)I(t) - (d+\mu)E(t) \\ \frac{dI}{dt} = \mu E(t) - (d+\alpha+\gamma+\delta)I(t) \\ \frac{dQ}{dt} = \delta I(t) - (d+\alpha+\epsilon)Q(t) \\ \frac{dR}{dt} = \gamma I(t) + \epsilon Q(t) - (d+\eta)R(t) \end{cases} \tag{7}$$

The parameters used in the equations have the following meanings: The disease's transmission rate is β, whereas its recovery rate is γ. The symbol α is the rate at which infected people become seriously ill and need to be hospitalized; δ is the rate at which infected people are found and quarantined; ϵ is the rate at which those who have been quarantined recover or pass away; μ is the rate at which people who have been exposed to the disease become infectious; η is the rate at which persons who have recovered lose their immunity and become susceptible once more. The population's birth rate is λ, and its mortality rate is d. The total population $N(t) = S(t) + E(t) + I(t) + Q(t) + R(t)$.

Applying the conformable fractional derivative operator T_υ for the time t, the equation (7) becomes

$$\begin{cases} T_vS(t) = \Lambda - \beta S(t)I(t) - dS(t) + \eta R(t) \\ T_vE(t) = \beta S(t)I(t) - (d+\mu)E(t) \\ T_vI(t) = \mu E(t) - (d+\alpha+\gamma+\delta)I(t) \\ T_vQ(t) = \delta I(t) - (d+\alpha+\epsilon)Q(t) \\ T_vR(t) = \gamma I(t) + \epsilon Q(t) - (d+\eta)R(t) \end{cases} \tag{8}$$

with initial constraints, $S(0) = N_S$, $E(0) = N_E$, $I(0) = N_I$, $Q(0) = N_Q$ and $R(0) = N_R$.

To solve the system of equations given in equation (8) we present the following method.

4 Variational Iteration Method

If L is a linear, N is a nonlinear, and g is any real function, then $L(u(t)) + N(u(t)) = g(t)$ is known to be a non-homogeneous equation. Given below is the relevant correlation function for the non-homogeneous equation.

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda L u_n(s) + N\widetilde{u}_n(s) - g(s)ds, \tag{9}$$

where $N\widetilde{u}_n$ is regarded as confined variation, i.e., $\delta N\widetilde{u}_n$=0, and λ is the general Lagrange multiplier, which may be ideally determined using variational theory.

Theorem 4.1 Let (8) is considered as the conformable fractional differential equation. Then, the variational iteration formula is given by

$$\begin{cases} S_{n+1} = S_n(t) - I_v\left\{T_vS_n(t) - \Lambda + \beta S_n(t)I_n(t) + dS_n(t) - \eta R_n(t)\right\} \\ E_{n+1} = E_n(t) - I_v\left\{T_vE_n(t) - \beta S_n(t)I_n(t) + (d+\mu)E_n(t)\right\} \\ I_{n+1} = I_n(t) - I_v\left\{T_vI_n(t) - \mu E_n(t) + (d+\alpha+\gamma+\delta)I_n(t)\right\} \\ Q_{n+1} = Q_n(t) - I_v\left\{T_vQ_n(t) - \delta I_n(t) + (d+\alpha+\epsilon)Q_n(t)\right\} \\ R_{n+1} = R_n(t) - I_v\left\{T_vR_n(t) - \gamma I_n(t) - \epsilon Q(t) + (d+\eta)R_n(t)\right\}. \end{cases} \tag{10}$$

Here the $n^{th}-$approximations are S_n, E_n, I_n, Q_n, and R_n. Then the conformable fractional derivative of order υ is T_υ and the conformable fractional integral of order υ is I_υ for $\upsilon \in (0,1)$. Proof Rewrite the equation (8) in the form

$$\begin{cases} T_vS_n(t) - \Lambda + \beta S_n(t)I_n(t) + dS_n(t) - \eta R_n(t) = 0 \\ T_vE_n(t) - \beta S_n(t)I_n(t) + (d+\mu)E_n(t) = 0 \\ T_vI_n(t) - \mu E_n(t) + (d+\alpha+\gamma+\delta)I_n(t) = 0 \\ T_vQ_n(t) - \delta I_n(t) + (d+\alpha+\epsilon)Q_n(t) = 0 \\ T_vR_n(t) - \gamma I_n(t) - \epsilon Q_n(t) + (d+\eta)R_n(t) = 0. \end{cases}$$

Multiplying the general Lagrange multipliers $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, $\lambda_4(t)$ and $\lambda_5(t)$ in the above equation, we get

$$\begin{cases} \lambda_1(t)\{T_vS_n(t) - \Lambda + \beta S_n(t)I_n(t) + dS_n(t) - \eta R_n(t)\} = 0 \\ \lambda_2(t)\{T_vE_n(t) - \beta S_n(t)I_n(t) + (d+\mu)E_n(t)\} = 0 \\ \lambda_3(t)\{T_vI_n(t) - \mu E_n(t) + (d+\alpha+\gamma+\delta)I_n(t)\} = 0 \\ \lambda_4(t)\{T_vQ_n(t) - \delta I_n(t) + (d+\alpha+\epsilon)Q_n(t)\} = 0 \\ \lambda_5(t)\{T_vR_n(t) - \gamma I_n(t) - \epsilon Q_n(t) + (d+\eta)R_n(t)\} = 0. \end{cases}$$

Now, Inserting the conformable integral operator I_v on both sides of the above equation, we obtain

$$\begin{cases} I_v\Big[\lambda_1(t)\{T_vS_n(t) - \Lambda + \beta S_n(t)I_n(t) + dS_n(t) - \eta R_n(t)\}\Big] = 0 \\ I_v\Big[\lambda_2(t)\{T_vE_n(t) - \beta S_n(t)I_n(t) + (d+\mu)E_n(t)\}\Big] = 0 \\ I_v\Big[\lambda_3(t)\{T_vI_n(t) - \mu E_n(t) + (d+\alpha+\gamma+\delta)I_n(t)\}\Big] = 0 \\ I_v\Big[\lambda_4(t)\{T_vQ_n(t) - \delta I_n(t) + (d+\alpha+\epsilon)Q_n(t)\}\Big] = 0 \\ I_v\Big[\lambda_5(t)\{T_vR_n(t) - \gamma I_n(t) - \epsilon Q_n(t) + (d+\eta)R_n(t)\}\Big] = 0. \end{cases}$$

Thus, the correlation function formula will exists in the form

$$\begin{cases} S_{n+1} = S_n(t) + I_v\Big[\lambda_1(t)\Big\{T_\alpha\tilde{S}_n(t) - \Lambda + \beta\tilde{S}_n(t)\tilde{I}_n(t) + d\tilde{S}_n(t) - \eta\tilde{R}_n(t)\Big\}\Big] \\ E_{n+1} = E_n(t) + I_v\Big[\lambda_2(t)\Big\{T_v\tilde{E}_n(t) - \beta S_n(t)\tilde{I}_n(t) + (d+\mu)\tilde{E}_n(t)\Big\}\Big] \\ I_{n+1} = I_n(t) + I_v\Big[\lambda_3(t)\Big\{T_v\tilde{I}_n(t) - \mu\tilde{E}_n(t) + (d+\alpha+\gamma+\delta)\tilde{I}_n(t)\Big\}\Big] \\ Q_{n+1} = Q_n(t) + I_v\Big[\lambda_4(t)\Big\{T_v\tilde{Q}_n(t) - \delta\tilde{I}_n(t) + (d+\alpha+\epsilon)\tilde{Q}_n(t)\Big\}\Big] \\ R_{n+1} = R_n(t) + I_v\Big[\lambda_5(t)\Big\{T_v\tilde{R}_n(t) - \gamma\tilde{I}_n(t) - \epsilon\tilde{Q}_n(t) + (d+\eta)\tilde{R}_n(t)\Big\}\Big] \end{cases} \tag{11}$$

From Definition 2.4 for order υ, we have

$$\begin{cases} S_{n+1} = S_n(t) + \int_0^t \tau^{\upsilon-1}\Big[\lambda_1(\tau)\Big\{\tau^{\upsilon-1}\frac{d}{d\tau}S_n(\tau) - \Lambda + \beta\tilde{S}_n(\tau)\tilde{I}_n(\tau) + d\tilde{S}_n(\tau) - \eta\tilde{R}_n(\tau)\Big\}\Big]d\tau \\ E_{n+1} = E_n(t) + \int_0^t \tau^{\upsilon-1}\Big[\lambda_2(\tau)\Big\{\tau^{\upsilon-1}\frac{d}{d\tau}E_n(\tau) - \beta\tilde{S}_n(\tau)\tilde{I}_n(\tau) + (d+\mu)\tilde{E}_n(\tau)\Big\}\Big]d\tau \\ I_{n+1} = I_n(t) + \int_0^t \tau^{\upsilon-1}\Big[\lambda_3(\tau)\Big\{\tau^{\upsilon-1}\frac{d}{d\tau}I_n(\tau) - \mu\tilde{E}_n(\tau) + (d+\alpha+\gamma+\delta)\tilde{I}_n(\tau)\Big\}\Big]d\tau \\ Q_{n+1} = Q_n(t) + \int_0^t \tau^{\upsilon-1}\Big[\lambda_4(\tau)\Big\{\tau^{\upsilon-1}\frac{d}{d\tau}Q_n(\tau) - \delta\tilde{I}_n(\tau) + (d+\alpha+\epsilon)\tilde{Q}_n(\tau)\Big\}\Big]d\tau \\ R_{n+1} = R_n(t) + \int_0^t \tau^{\upsilon-1}\Big[\lambda_5(\tau)\Big\{\tau^{\upsilon-1}\frac{d}{d\tau}R_n(\tau) - \gamma\tilde{I}_n(\tau) - \epsilon\tilde{Q}_n(\tau) + (d+\eta)\tilde{R}_n(\tau)\Big\}\Big]d\tau. \end{cases}$$

where $\tilde{S_n}$, $\tilde{E_n}$, $\tilde{I_n}$, $\tilde{Q_n}$ and $\tilde{R_n}$ are the restricted variations with $\delta\tilde{S_n} = 0$, $\delta\tilde{E_n} = 0$, $\delta\tilde{I_n} = 0$, $\delta\tilde{Q_n} = 0$, $\delta\tilde{R_n} = 0$. Therefore the above equation becomes

$$\begin{cases} \delta S_{n+1} = \delta S_n(t) + \delta\int_0^t \Big[\lambda_1(\tau)\Big\{\frac{d}{d\tau}S_n(\tau) - \Lambda\tau^{\upsilon-1} + \beta\tau^{\upsilon-1}\tilde{S}_n(\tau)\tilde{I}_n(\tau) + d\tau^{\upsilon-1}\tilde{S}_n(\tau) - \eta\tau^{\upsilon-1}\tilde{R}_n(\tau)\Big\}\Big]d\tau \\ \delta E_{n+1} = \delta E_n(t) + \delta\int_0^t \Big[\lambda_2(\tau)\Big\{\frac{d}{d\tau}E_n(\tau) - \beta\tau^{\upsilon-1}\tilde{S}_n(\tau)\tilde{I}_n(\tau) + (d+\mu)\tau^{\upsilon-1}\tilde{E}_n(\tau)\Big\}\Big]d\tau \\ \delta I_{n+1} = \delta I_n(t) + \delta\int_0^t \Big[\lambda_3(\tau)\Big\{\frac{d}{d\tau}I_n(\tau) - \mu\tau^{\upsilon-1}\tilde{E}_n(\tau) + (d+\upsilon+\gamma+\delta)\tau^{\upsilon-1}\tilde{I}_n(\tau)\Big\}\Big]d\tau \\ \delta Q_{n+1} = \delta Q_n(t) + \delta\int_0^t \Big[\lambda_4(\tau)\Big\{\frac{d}{d\tau}Q_n(\tau) - \delta\tau^{\upsilon-1}\tilde{I}_n(\tau) + (d+\upsilon+\epsilon)\tau^{\upsilon-1}\tilde{Q}_n(\tau)\Big\}\Big]d\tau \\ \delta R_{n+1} = \delta R_n(t) + \delta\int_0^t \Big[\lambda_5(\tau)\Big\{\frac{d}{d\tau}R_n(\tau) - \gamma\tau^{\upsilon-1}\tilde{I}_n(\tau) - \epsilon\tau^{\upsilon-1}\tilde{Q}_n(\tau) + (d+\eta)\tau^{\upsilon-1}\tilde{R}_n(\tau)\Big\}\Big]d\tau. \end{cases} \tag{12}$$

Applying the integration by parts method in the above equation, it becomes

$$\begin{cases} \delta S_{n+1}(t) = (1+\lambda_1(t))\delta S_n(t) - \delta\int_0^t \lambda_1'(\tau)S_n(\tau)d\tau - \delta A\lambda_1(\tau)\frac{\tau^\upsilon}{\upsilon} + \int_0^t \lambda_1'(\tau)\frac{\tau^\upsilon}{\upsilon}d\tau \\ \delta E_{n+1}(t) = (1+\lambda_2(t))\delta E_n(t) - \delta\int_0^t \lambda_2'(\tau)E_n(\tau)d\tau \\ \delta I_{n+1}(t) = (1+\lambda_3(t))\delta I_n(t) - \delta\int_0^t \lambda_3'(\tau)I_n(\tau)d\tau \\ \delta Q_{n+1}(t) = (1+\lambda_4(t))\delta Q_n(t) - \delta\int_0^t \lambda_4'(\tau)Q_n(\tau)d\tau \\ \delta R_{n+1}(t) = (1+\lambda_5(t))\delta R_n(t) - \delta\int_0^t \lambda_5'(\tau)R_n(\tau)d\tau. \end{cases} \tag{13}$$

The Langrange multiplier $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, $\lambda_4(t)$ and $\lambda_5(t)$ can be obtained by $\lambda_j'(\tau) = 0$ for all $j = \{1,2,3,4,5\}$ with boundary constraints $1 + \lambda_j(t) = 0$, for all $j = \{1,2,3,4,5\}$. Solving the last initial value problem for λ_j for all $1 \le j \le 5$, the general Lagrange multiplier λ_j is found to be $\lambda_j = -1$, for all $1 \le j \le 5$.
Hence substituting the values of λ_j into the corresponding correlation function given in (11), we get (10).

2456-8686, vii(ii), 2023: 015 - 030
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5 Conformable Fractional Differential Transform

In this section, we apply Proposition 2.6 on equation (10), and then utilize the initial values to derive the numerical values of the conformable fractional differential transform method SEIQRS epidemic model. An illustrative example is discussed in this section.
Equation (10) can be written as follows:

$$\begin{cases} \upsilon(k+1)S_\upsilon(t) = \Lambda - \beta \sum_{l=0}^{k} S_\upsilon(l)I_\upsilon(k-l) - dS_\upsilon(k) + \eta R_\upsilon(k) \\ \upsilon(k+1)E_\upsilon(t) = \beta \sum_{l=0}^{k} S_\upsilon(l)I_\upsilon(k-l) - (d+\mu)E_\upsilon(k) \\ \upsilon(k+1)I_\upsilon(t) = \mu E_\alpha(k) - (d+\alpha+\gamma+\delta)I_\upsilon(k) \\ \upsilon(k+1)Q_\upsilon(t) = \delta I_\upsilon(k) - (d+\alpha+\epsilon)Q_\upsilon(k) \\ \upsilon(k+1)R_\upsilon(t) = \delta I_\upsilon(k) + \epsilon Q_\upsilon(k) - (d+\eta)R_\upsilon(k). \end{cases} \tag{14}$$

Hence, we obtain the recurrence relation as

$$\begin{cases} S_\upsilon(t) = (1/\upsilon(k+1))\Big[\Lambda - \beta \sum_{l=0}^{k} S_\upsilon(l)I_\upsilon(k-l) - dS_\upsilon(k) + \eta R_\upsilon(k)\Big] \\ E_\upsilon(t) = (1/\upsilon(k+1))\Big[\beta \sum_{l=0}^{k} S_\upsilon(l)I_\upsilon(k-l) - (d+\mu)E_\upsilon(k)\Big] \\ I_\upsilon(t) = (1/\upsilon(k+1))\Big[\mu E_\upsilon(k) - (d+\alpha+\gamma+\delta)I_\upsilon(k)\Big] \\ Q_\upsilon(t) = (1/\upsilon(k+1))\Big[\delta I_\upsilon(k) - (d+\alpha+\epsilon)Q_\upsilon(k)\Big] \\ R_\upsilon(t) = (1/\upsilon(k+1))\Big[\delta I_\upsilon(k) + \epsilon Q_\upsilon(k) - (d+\eta)R_\upsilon(k)\Big]. \end{cases} \tag{15}$$

The assumed initial values for the mathematical model (7) is given in Table:1.

Parameters	Values	Parameters	Values
N_S	$24,518,169$	α	2.4×10^{-5}
N_E	800	μ	3.64×10^{-4}
N_I	150	β	1.0×10^{-3}
N_Q	81	δ	2.0×10^{-5}
N_R	40	γ	0.6×10^{-7}
Λ	1500	ϵ	1.0×10^{-6}
d	1.0×10^{-7}	η	3.0×10^{-4}

Table 1: Initial values of (7)

2456-8686, vii(ii), 2023: 015 - 030
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Now, substituting $k = 0$ in (14), we obtain the first iteration value as

$S_v(1) = (\frac{1}{v})[\Lambda - \beta \sum_{l=0} S_v(l)I_v(-l) - dS_v(0) + \eta R_v(0)]$

$S_v(1) = (\frac{1}{v})[1500 - (1.0 \times 10^{-3} \times 24,518,169 \times 150 - (1.0 \times 10^{-7} \times 24,518,169)) + (3.0 \times 10^{-4} \times 40)]$

$S_v(1) = (-3676227.7898169/v)$

$E_v(1) = (\frac{1}{v})[\beta S_v(0)I_v(0) - (d + \mu)E_v(0)]$

$E_v(1) = (\frac{1}{v})[1.0 \times 10^{-3}(24,518,169)(150) - (1.0 \times 10^{-7} + 3.64 \times 10^{-4})(800)]$

$E_v(1) = (3677725.05872/v)$

$I_v(1) = (\frac{1}{v})[\mu E_v(0) - (d + \alpha + \gamma + \delta)I_v(0)]$

$I_v(1) = (\frac{1}{v})[(3.64 \times 10^{-4})(800) - (1.0 \times 10^{-7} + 2.4 \times 10^{-5} + 0.6 \times 10^{-7} + 2.0 \times 10^{-5})(150)]$

$I_v(1) = (0.284576/v)$

$Q_v(1) = (\frac{1}{v})[\delta I_v(0) - (d + \alpha + \epsilon)Q_v(0)]$

$Q_v(1) = \frac{1}{v}[(2.0 \times 10^{-5})(150) - (1.0 \times 10^{-7} + 2.4 \times 10^{-5} + 1.0 \times 10^{-6})(81)]$

$Q_v(1) = (0.0009669/v)$

$R_v(1) = (\frac{1}{v})[\gamma I_v(0) + \epsilon Q_v(0) - (d + \eta)R_v(0)]$

$R_v(1) = (\frac{1}{v})[(0.6 \times 10^{-7})(150) + (1.0 \times 10^{-6})(81) - (1.0 \times 10^{-7} + 3.0 \times 10^{-4})(140)]$

$R_v(1) = (-0.011914/v)$

Taking $k = 1$ in (14), we obtain

$S_v(2) = (\frac{1}{2v})[\Lambda - \beta \sum_{l=0}^{1} S_v(l)I_v(1-l) - dS_v(1) + \eta R_v(1)]$

$S_v(2) = (\frac{1}{2v})[\Lambda - \beta S_v(0)I_v(1) + S_v(1)I_v(0) - dS_v(1) + \eta R_v(1)]$

$= (\frac{1}{2v})[1500 - (1.0 \times 10^{-3})(24,518,169 \times (0.284576/v) - (3676227.7898169/v)(150)) -(1.0 \times 10^{-7})(-3676227.7898169/v) +(3.0 \times 10^{-4})(-0.011914v)]$

$S_v(2) = (750/v) + (27228.62681519793/v^2)$

$E_v(2) = (\frac{1}{2v}) \left[\beta \sum_{l=0}^{1} S_v(l)I_v(1-l) - (d + \mu)E_v(1)\right]$

$E_v(2) = (\frac{1}{2v}) [\beta(S_v(0)I_v(1) + S_v(1)I_v(0)) - (d + \mu)E_v(1)]$

$E_v(2) = (\frac{1}{2v})[(1.0 \times 10^{-3})(24,518,169 \times (0.284576/v) - (3676227.7898169/v)(150)) -(1.0 \times 10^{-7} + 3.64 \times 10^{-4})(3677725.05872/v)]$

$E_v(2) = (-27897.97285253552/v^2)$

$I_{\upsilon}(2) = (\frac{1}{2\upsilon})[\mu E_{\upsilon}(1) - (d + \alpha + \gamma + \delta)I_{\upsilon}(1)]$
$I_{\upsilon}(2) = (\frac{1}{2\upsilon})[(3.64 \times 10^{-4})(3677725.05872/\upsilon)$
$- (1.0 \times 10^{-7} + 2.4 \times 10^{-5} + 0.6 \times 10^{-7} + 2.0 \times 10^{-5})(0.284576/\upsilon)]$
$I_{\upsilon}(2) = (\frac{1}{2\upsilon})\left[(1338.69192137408/\upsilon) - 0.0000156688\upsilon\right]$
$I_{\upsilon}(2) = (669.3459544036/\upsilon^2)$

$Q_{\upsilon}(2) = (\frac{1}{2\upsilon})[\delta I_{\upsilon}(1) - (d + \alpha + \epsilon)Q_{\upsilon}(1)]$
$Q_{\upsilon}(2) = (\frac{1}{2\upsilon})[(2.0 \times 10^{-5}(0.284576/\upsilon) - (1.0 \times 10^{-7}$
$+ 2.4 \times 10^{-5} + 1.0 \times 10^{-6})(0.0009669/\upsilon)]$
$Q_{\upsilon}(2) = (0.00000283363/\upsilon^2)$

$R_{\upsilon}(2) = (\frac{1}{2\upsilon})\left[\gamma I_{\upsilon}(1) + \epsilon Q_{\upsilon}(1) - (d + \eta)R_{\alpha}(1)\right]$
$R_{\upsilon}(2) = (\frac{1}{2\upsilon})[(0.6 \times 10^{-7})(0.284576/\upsilon) + (1.0 \times 10^{-6})(0.0009669/\upsilon)$
$-(1.0 \times 10^{-7} + 3.0 \times 10^{-4})(-0.011914/\upsilon)]$
$R_{\upsilon}(2) = (0.00000180082/\upsilon^2)$

Applying $k = 2$ in equation (14), we arrive

$S_{\upsilon}(3) = (\frac{1}{3\upsilon})\left[A - \beta \sum_{l=0}^{2} S_{\upsilon}(l)I_{\upsilon}(2-l) - dS_{\upsilon}(2) + \eta R_{\upsilon}(2)\right]$
$S_{\upsilon}(3) = (500/\upsilon) - (3750025/\upsilon^2) - (5483641.776711221/\upsilon^3)$

$E_{\upsilon}(3) = (\frac{1}{3\upsilon})\left[\beta \sum_{l=0}^{2} S_{\upsilon}(l)I_{\upsilon}(2-l) - (d + \mu)E_{\upsilon}(2)\right]$
$E_{\upsilon}(3) = (37.5/\upsilon^2) + (5483645.17166948/\upsilon^3)$

$I_{\upsilon}(3) = (\frac{1}{3\upsilon})\left[\mu E_{\upsilon}(2) - (d + \alpha + \gamma + \delta)I_{\upsilon}(2)\right]$
$I_{\upsilon}(3) = (-3.39480681188/\upsilon^3)$

$Q_{\upsilon}(3) = (\frac{1}{3\upsilon})\left[\delta I_{\upsilon}(2) - (d + \alpha + \epsilon)Q_{\upsilon}(2)\right]$
$Q_{\upsilon}(3) = (0.00446230634/\upsilon^3)$

$R_{\upsilon}(3) = (\frac{1}{3\upsilon})\left[\gamma I_{\upsilon}(2) + \epsilon Q_{\upsilon}(2) - (d + \eta)R_{\alpha}(2)\right]$
$R_{\upsilon}(3) = (0.00001338674/\upsilon^3)$

Here, the susceptible series will be

$S(t) = S_{\upsilon}(0) + S_{\upsilon}(1)t^{\upsilon} + S_{\upsilon}(2)t^{2\upsilon} + S_{\upsilon}(3)t^{3\upsilon} + \cdots$
$S(t) = 24518169 - \frac{3676227.7898169}{\alpha}t^{\upsilon} + \frac{750}{\upsilon}t^{2\upsilon} + \frac{27228.62681519793}{\upsilon^2}t^{2\upsilon} + \frac{500}{\upsilon}t^{3\upsilon}$

$$-\frac{3750025}{\upsilon^2}t^{3\upsilon}-\frac{5483641.776711221}{\upsilon^3}t^{3\upsilon}+\cdots$$

Similarly, the other series are

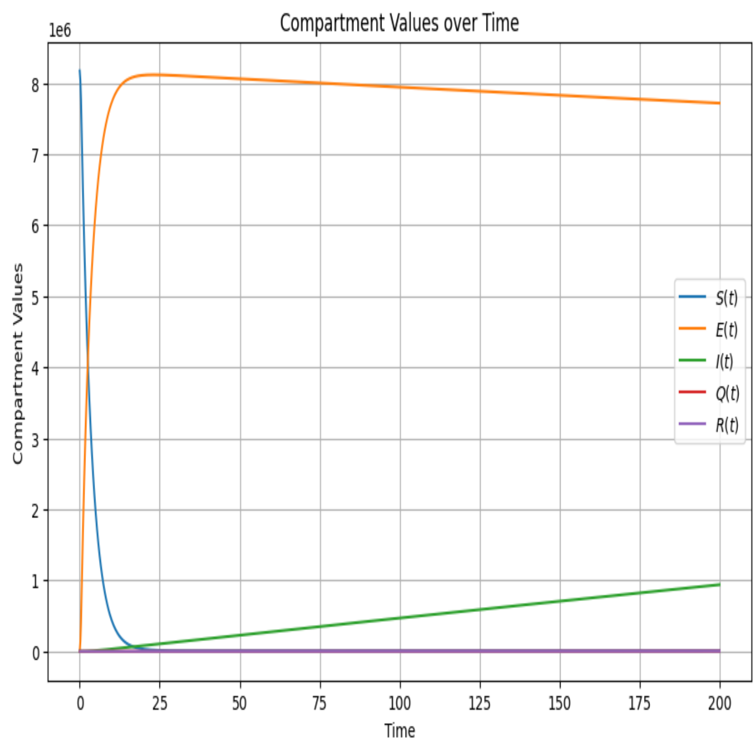
$$E(t)=800+\frac{3677725.05872}{\alpha}t^{\upsilon}-\frac{27897.97285253552}{\upsilon^2}t^{2\upsilon}+\frac{37.5}{\upsilon^2}t^{3\upsilon}$$
$$+\frac{5483645.17166948}{\upsilon^3}t^{3\upsilon}+\cdots$$

$$I(t)=150+\frac{0.284576}{\upsilon}t^{\upsilon}+\frac{669.3459544036}{\upsilon^2}t^{2\upsilon}-\frac{-3.39480681188}{\upsilon^3}t^{3\upsilon}+\cdots$$

$$Q(t)=81+\frac{0.0009669}{\upsilon}t^{\upsilon}+\frac{0.00000283363}{\upsilon^2}t^{2\upsilon}+\frac{0.00446230634}{\upsilon^3}t^{3\upsilon}+\cdots$$

$$R(t)=40-\frac{0.011914}{\upsilon}t^{\upsilon}+\frac{0.00000180082}{\upsilon^2}t^{2\upsilon}+\frac{0.00001338674}{\upsilon^3}t^{3\upsilon}+\cdots$$

Hence, we found the values of S(t), E(t), I(t), Q(t) and R(t). Now, we will represent the nature of the compartments as 2D and 3D respectively.



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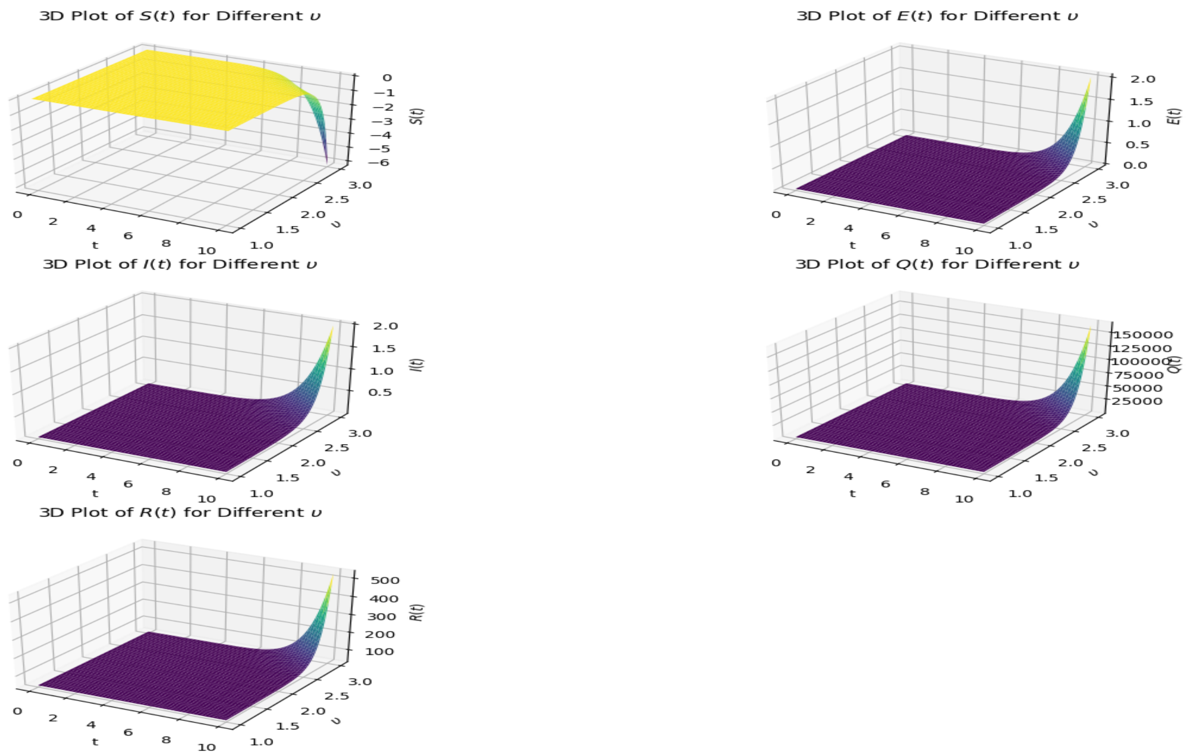
https://doi.org/10.26524/cm172



6 Conclusion

Our research focuses on the most common methods of the SEIQRS outbreak model in the broadest sense. To solve this problem, we use two different methods: conformable fractional differential transform method and variational iteration method. To estimate the transmission rate, we have applied the initial values for all the parameters and solved the compartment model. Our results demonstrate the validity and reliability of SEIQRS infectious disease models. The accuracy of the solutions obtained by this method supports its applicability in real-world epidemic models. These results have important implications for researchers and practitioners in epidemic modeling. From our illustrative example, solutions with respect to time have been discussed. Finally, we have shown the transmission of virus by plotting the diagram.

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