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ORIGINAL RESEARCH **Open Access**

COEFFICIENT OF RANGE LABELING OF SOME GRAPHS

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Abstract

The coefficient of range labeling is a new type of labeling since it was introduced in 2022 by Senthamizh Selvan and Jahir Hussain. In this paper, we obtained a coefficient of range labeling for the path graph, cycle graph, sun graph, and double star graph is coefficient of range graph. We can also determine the coefficient of range value of the path graph, cycle graph, sun graph, and double star graph.

Key Words: Labeling, coefficient of Range labeling, double star graph, sun graph.

AMS Classification: 05C78,05C90

1 Introduction

In the present study, the graphs here are only undirected, simple, finite graphs are taken into consideration. Vertex set V(G) and edge set E(G) are present in the graph G (V, E). Graph Labelling was first made available. Rosa's [7] research from 1967 is referenced in a lot of graph labeling studies. Rosa developed the function $\boldsymbol{\alpha}$, which assigns the separate labels | (u)-(v)| to each edge uv in a graph by starting with a set of vertices in the graph G and ending with the set of numbers {0, 1, 2...} Rosa label referred to this as labeling with "β-valuation." In a separate study, Golomb [2] identified the same kind of labeling and dubbed it graceful labeling. [1] has several very intriguing applications of graph labeling. Coefficient of Range labeling is a new type of labeling since it was introduced in 2022 by Senthamizh Selvan and Jahir Hussain [5]. The coefficient of range labeling conditions satisfied graphs; we say the graph is a coefficient of range graph. In this paper coefficient of range labeling applies to some graphs and we can also find the coefficient of range value for those graphs.One of the captivating subfields of graph theory is graph labeling, which has numerous applications in areas like X-ray crystallography, communication networks, and coding theory.

2. PRELIMINARIES

2.1 A **Graph** G is composed of a finite, non-empty set V and a specified set E containing unordered pairs of unique V members. Each component of V is known as a vertex or point of

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G. Any pair (u, v) of vertices in E is a G edge or line. The edge e = (u, v) is said to connect the vertices u and v. If the vertex pairs u and v are close together and e = (u, v) is a G edge.

2.2 The graph called $\mathbf{G^+ = G \Theta K_1}$ is created by linking every vertex of a graph G with exactly one pendant edge.

2.3 The **sun graph** S_n is a series of cycles on n vertices having an edge attached to each cycle vertex that terminates at a vertex of degree 1

2.4 If A and B are the greatest and smallest observations respectively in a distribution, then its **co-efficient of range** is given by

co-efficient of range (or)$C.R = \frac{= X_{max} - X_{min}}{= X_{max} + X_{min}} = \frac{A-B}{A+B}$

2.5 If $G(V,E)$ be a finite graph. If a function $\alpha : V(G) \rightarrow \{1,2,3,\dots,n\}$is called a

co-efficient of range labeling if the edge labels are used

$\beta : E(G) \rightarrow \{1,2,3,\dots,(n-1)\}$ is defined by

$$\beta[E(G)] = Decrease\ Numerator\ of\ \left[\frac{Maximum\ value[\alpha(u),\alpha(v)] - minimum\ value[\alpha(u),\alpha(v)]}{Maximum\ value\ [\alpha(u),\alpha(v)] + minimum\ value[\alpha(u),\alpha(v)]} + 3\ minimum\ value\ [\alpha(u),\alpha(v)]\right]$$

by $Maximum\ value[\alpha(u),\alpha(v)] - minimum\ value[\alpha(u),\alpha(v)]$

(or)

$$\beta[E(G)] = Decrease\ Numerator\ of\ \left[\frac{MAX\ V[\alpha(u),\alpha(v)] - MIN\ V[\alpha(u),\alpha(v)]}{MAX\ V\ [\alpha(u),\alpha(v)] + MIN\ V[\alpha(u),\alpha(v)]} + 3\ MIN\ V[\alpha(u),\alpha(v)]\right]$$

$$by\ MAX\ V[\alpha(u),\alpha(v)] - MIN\ V[\alpha(u),\alpha(v)]$$

If a graph G admits co-efficient of Range labeling, we say G is **co-efficient of Range graph**.

2.6 Let $G = (V,E)$ be a co-efficient of Range graph, then the co-efficient of Range value is

$$\boldsymbol{Co-efficient\ of\ Range\ value\ of\ (G)} = \frac{Maximum\ edge\ value\ of\ G - Minimum\ edge\ value\ of\ G}{Maximum\ edge\ value\ of\ G + Minimum\ edge\ value\ of\ G}$$

(or)

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$$\text{COERV(G)} = \frac{MAXEV(G) - MINEV(G)}{MAXEV(G) + MINEV(G)}$$

3.MAIN RESULTS

Theorem 3.1 The Path graph P_n admits co-efficient of Range labeling for every value n.

Proof: Let G be a path Graph.

Where $V(G) = \{ v_i, 1 \leq i \leq n\}$ and

$E(G) = \{v_i v_{i+1}, 1 \leq i \leq n-1\}$

Let $\alpha : V(G) \to \{1,2,3,4, \dots, n\}$

Such that $\alpha(v_i) = i$ for $1 \leq i \leq n$

We have $\beta(e_i)$

$$= Decrease\ Numerator\ of\ \left[\frac{Maximum\ Value\ [\alpha(v_i), \alpha(v_{i+1})] - Minimum\ Value[\alpha(v_i), \alpha(v_{i+1})]}{Maximum\ Value\ [\alpha(v_i), \alpha(v_{i+1})] + Minimum\ Value[\alpha(v_i), \alpha(v_{i+1})]}\right.$$

$$\left. +\ 3\ Minimum\ Value[\alpha(v_i), \alpha(v_{i+1})]\right]$$

$$by\ Maximum\ Value[\alpha(v_i), \alpha(v_{i+1})] - Minimum\ Value[\alpha(v_i), \alpha(v_{i+1})]$$

If $\alpha(v_i)$ is a maximum value and $\alpha(v_{i+1})$ is a minimum value

$$\beta(e_i) = Decrease\ Numerator\ of\ \left[\frac{\alpha(v_i) - \alpha(v_{i+1})}{\alpha(v_i) + \alpha(v_{i+1})} + 3\alpha(v_{i+1})\right]$$

$$by\ [\alpha(v_i) - \alpha(v_{i+1})]$$

$$= \frac{3\ \alpha(v_{i+1})((\alpha(v_i) + \alpha(v_{i+1}))}{(\alpha(v_i) + \alpha(v_{i+1}))} = 3\ \alpha(v_{i+1})$$

Suppose $\alpha(v_{i+1})$ is a maximum value and $\alpha(v_i)$ is a minimum value

$$\beta(ei) = Decrease\ Numerator\ of\ \left[\frac{\alpha(v_{i+1}) - \alpha(v_i)}{\alpha(v_i) + \alpha(v_{i+1})} + 3\alpha(v_i)\right]$$

$$by\ [\alpha(v_{i+1}) - \alpha(v_1)]$$

$$= \frac{3\ \alpha(v_i)[\alpha(v_{i+1}) + \alpha(v_i)]}{\alpha(v_{i+1}) + \alpha(v_i)} = 3\ \alpha(v_i)$$

Hence, every path graph P_n admits co-efficient of Range labeling.

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Therefore, any path graph is a co-efficient of range graph

EXAMPLE 3.2

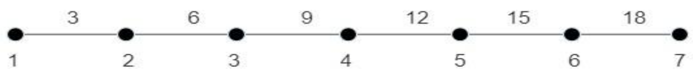

Figure 3.1 co-efficient of range labeling P_7

$$\alpha(e_1) = \text{ Decrease Numerator of } \left[\frac{2-1}{2+1} + 3(1)\right] by\ (2-1)$$

$$= \text{ Decrease Numerator of } \left[\frac{1}{3} + 3\right] \ by\ 1$$

$$= \text{ Decrease Numerator of } \left[\frac{1+3(3)}{3}\right] by\ 1$$

$$= \frac{3(3)}{3}$$

$$\alpha(e_1) = 3$$

$$\alpha(e_6) = \text{ Decrease Numerator of } \left[\frac{7-6}{7+6} + 3(6)\right] by\ (7-6)$$

$$= \text{ Decrease Numerator of } \left[\frac{1}{13} + 3(6)\right] by\ (1)$$

$$= \text{ Decrease Numerator of } \left[\frac{1+3(6)(13)}{13}\right] by\ (1)$$

$$= \frac{3(6)(13)}{13} = 18$$

EXAMPLE 3.3

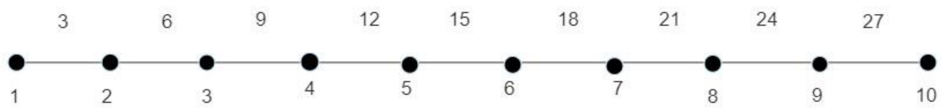

Figure 3.2 Co-efficient of Range labeling

REMARK 3.4

If P_7 and P_{10} is a Co-efficient of Range graph then we can also find

Co-efficient of Range value $P_7$and P_{10}

$$Co - Efficient\ Range\ Value\ of (P_7) = \frac{Maximum\ Edge\ Value\ of\ (P_7) - Minimum\ Edge\ Value\ of (P_7)}{Maximum\ Edge\ Value\ of\ (P_7) + Minimum\ Edge\ Value\ of (P_7)}$$

$$= \frac{18-3}{18+3} = \frac{15}{21}$$

$$CoERV(P_7) = 0.7$$

$$Co - Efficient\ Range\ Value\ of\ (P_{10}) = \frac{Maximum\ Edge\ Value\ of\ (P_{10}) - Minimum\ Edge\ Value\ of (P_{10})}{Maximum\ Edge\ Value\ of\ (P_{10}) + Minimum\ Edge\ Value\ of (P_{10})}$$

$$= \frac{27-3}{27+3} = \frac{24}{30}$$

THEOREM 3.5

The cycle graph C_n admits co-efficient of Range labeling for every value n.

PROOF

Let G be a cycle graph. Where, $V(G) = \{ v_i, 1 \le i \le n\}$ and

$$E(G) = \{ v_i v_{i+1}, 1 \le i \le n-1\} \cup \{v_n v_1\}$$

Let $\alpha : V(G) \to \{1,2,3, \dots, n\}$

Such that $\beta(v_i) = i$ for $1 \le i \le n$

We have $\beta(e_i)$

$$= Decrease\ Numerator\ of\ \left[\frac{Maximum\ Value\ [\alpha\ (v_i), \alpha\ (v_{i+1})] - Minimum\ Value\ [\alpha\ (v_i), \alpha\ (v_{i+1})]}{Maximum\ Value\ [\alpha\ (v_i), \alpha\ (v_{i+1})] + Minimum\ Value\ [\alpha\ (v_i), \alpha\ (v_{i+1})]} + 3\ Minimum\ Value\ [\alpha(v_i), \alpha(v_{i+1})]\right]$$

$$by\ Maximum\ Value\ [\alpha(v_i), \alpha(v_{i+1})] - Minimum\ Value\ [\alpha(v_i), \alpha(v_{i+1})]$$

If $\alpha(v_i)$ is a maximum value and $\alpha(v_{i+1})$ is a minimum value

$$\beta(e_i) = Decrease\ Numerator\ of\ \left[\frac{\alpha(v_i) - \alpha(v_{i+1})}{\alpha(v_i) + \alpha(v_{i+1})} + 3\alpha(v_{i+1})\right]$$

$$by\ [\alpha(v_i) - \alpha(v_{i+1})]$$

$$= \frac{3\ \alpha(v_{i+1})(\alpha(v_i) + \alpha(v_{i+1}))}{\left(\alpha(v_i) + \alpha(v_{i+1})\right)} = 3\ \alpha(v_{i+1})$$

Suppose $\alpha(v_{i+1})$ is a maximum value and $\alpha(v_i)$ is a minimum value

$$\beta(e_i) = Decrease\ Numerator\ of\ \left[\frac{\alpha(v_{i+1}) - \alpha(v_i)}{\alpha(v_i) + \alpha(v_{i+1})} + 3\alpha(v_i)\right]$$

$$by\ [\alpha(v_{i+1}) - \alpha(v_1)]$$

$$= \frac{3\ \alpha(v_i)[\alpha(v_{i+1}) + \alpha(v_i)]}{\alpha(v_{i+1}) + \alpha(v_i)} = 3\ \alpha(v_i)$$

$$\beta(e_i) = Decrease\ Numerator\ of\ \left[\frac{Maximum\ Value\ [\alpha(v_n), \alpha(v_1)] - Minimum\ Value[\alpha(v_n), \alpha(v_1)]}{Maximum\ Value\ \ [\alpha(v_n), \alpha(v_1)] + Minimum\ Value[\alpha(v_n), \alpha(v_1)]} + 3\ Minimum\ Value[\alpha(v_n), \alpha(v_1)]\right]$$

$$by\ Maximum\ Value[\alpha(v_n), \alpha(v_1)] - Minimum\ Value[\alpha(v_n), \alpha(v_1)]$$

If $\alpha(v_n)$ is a maximum value and $\alpha(v_1)$ is a minimum value

$$\beta(e_i) = Decrease\ Numerator\ of\ \left[\frac{\alpha(v_n) - \alpha(v_1)}{\alpha(v_n) + \alpha(v_1)} + 3\alpha(v_1)\right]$$

$$[\alpha(v_n) - \alpha(v_1) = \frac{3\ \alpha(v_1)(\alpha(v_n) + \alpha(v_1))}{(\alpha(v_n) + \alpha(v_1))} = 3\ \alpha(v_1)$$

Suppose $\alpha(v_1)$ is a maximum value and $\alpha(v_n)$ is a minimum value

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$$\beta(e_i) = Decrease\ Numerator\ of\ \left[\frac{\alpha(v_1) - \alpha(v_n)}{\alpha(v_1) + \alpha(v_n)} + 3\alpha(v_n)\right]$$

$$by\ [\alpha(v_1) - \alpha(v_n)]$$

$$= \frac{3\ [(v_n)(\alpha(v_1) + \alpha((v_n)]}{\left(\alpha(v_1) + \alpha(v_n)\right)} = \ 3\ \alpha(v_n)$$

Hence, every cycle graph C_n admits co-efficient of range labeling.

Therefore, any cycle graph is a co-efficient of Range graph.

EXAMPLE 3.6

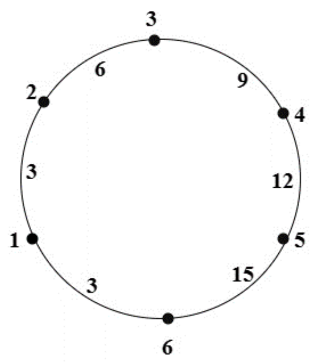

Figure 3.3 co-efficient of range labeling C_6

REMARK 3.7

If C_6 is a co- efficient of range graph then we can also find co-efficient of range value,

$$Co - Efficient\ of\ Range\ Value\ (C_6) = \frac{Maximum\ Edge\ Value\ of\ (C_6) - Minimum\ Edge\ Value\ of\ (C_6)}{Maximum\ Edge\ Value\ of\ (C_6) + Minimum\ Edge\ Value\ of (C_6)}$$

$$= \frac{15-3}{15+3} = \frac{12}{18} = \ 0.7$$

THEOREM 3.8

The sun graph S_n admits co-efficient of range labeling for every value n.

PROOF

Let $G = (V, E)$ be a graph. Let $v_1, v_2, \dots, v_n$ is a vertex of cycle S_n and let $u_1, u_2, \dots, u_n$ is a end vertices of every fixed to $v_1, v_2, \dots, v_n$

Let $\alpha : V(G) \rightarrow \{1,2,3,4, \dots, n\}$

$\alpha_1 : U(G) \rightarrow \{n+1, n+2, \dots, 2n\}$

Such that $\alpha(v_i) = i \; for \; 1 \le i \le n$

$\alpha_1(u_i) = i \; for \; n+1 \le i \le 2n$

we have $\beta(e_i)$

$$= Decrease\ Numerator\ of\ \left[\frac{Maximum\ Value\ [\alpha(v_i),\alpha(v_{i+1})] - Minimum\ Value[\alpha(v_i),\alpha(v_{i+1})]}{Maximum\ Value\ [\alpha(v_i),\alpha(v_{i+1})] + Minimum\ Value[\alpha(v_i),\alpha(v_{i+1})]} + 3\ Minimum\ Value[\alpha(v_i),\alpha(v_{i+1})]\right]$$

$$by\ Maximum\ Value[\alpha(v_i),\alpha(v_{i+1})] - Minimum\ Value[\alpha(v_i),\alpha(v_{i+1})]$$

If $\alpha(v_i)$ is a maximum value and $\alpha(v_{i+1})$ is a minimum value

$$\beta(e_i) = Decrease\ Numerator\ of\ \left[\frac{\alpha(v_i) - \alpha(v_{i+1})}{\alpha(v_i) + \alpha(v_{i+1})} + 3\alpha(v_{i+1})\right]$$

$$by\ [\alpha(v_i) - \alpha(v_{i+1})]$$

$$= \frac{3\,\alpha(v_{i+1})((\alpha(v_i)+\alpha(v_{i+1}))}{(\alpha(v_i)+\alpha(v_{i+1}))} = 3\,\alpha(v_{i+1})$$

Suppose $\alpha(v_{i+1})$ is a maximum value and $\alpha(v_i)$ is a minimum value

$$\beta(e_i) = Decrease\ Numerator\ of\ \left[\frac{\alpha(v_{i+1}) - \alpha(v_i)}{\alpha(v_i) + \alpha(v_{i+1})} + 3\alpha(v_i)\right]$$

$$by\ [\alpha(v_{i+1}) - \alpha(v_i)]$$

$$= \frac{3\,\alpha(v_i)[\alpha(v_{i+1})+\alpha(v_i)]}{\alpha(v_{i+1})+\alpha(v_i)} = 3\,\alpha(v_i)$$

$$\beta(e_{1i})$$

$$= Decrease\ Numerator\ of\ \left[\frac{Maximum\ value[\alpha(v_i),\alpha_1(u_i)] - Minimum\ value[\alpha(v_i),\alpha_1(u_i)]}{Maximum\ value[\alpha(v_i),\alpha_1(u_i)] + Minimum\ value[\alpha(v_i),\alpha_1(u_i)]} + 3\ Minimum\ value[\alpha(v_i),\alpha_1(u_i)]\right]$$

By $[Maximum\ value[\alpha(v_i),\alpha_1(u_i)] - Minimum\ value[\alpha(v_i),\alpha_1(u_i)]]$

If $\alpha(v_i)$ is a maximum value and $\alpha_1(u_i)$ is a minimum value

$$\beta(e_{1i}) = Decrease\ Numerator\ of\ \left[\frac{[\alpha(v_i) - \alpha_1(u_i)]}{[\alpha(v_i) + \alpha_1(u_i)]} + 3\alpha_1(u_i)\right]$$

If $\alpha(v_i)$ is a maximum value and $\alpha(v_{i+1})$ is a minimum value

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$$\beta(e_i) = Decrease\ Numerator\ of\ \left[\frac{\alpha(v_i)-\alpha(v_{i+1})}{\alpha(v_i)+\alpha(v_{i+1})} + 3\alpha(v_{i+1})\right]$$

$$by\ [\alpha(v_i)-\alpha(v_{i+1})]$$

$$= \frac{3\ \alpha(v_{i+1})((\alpha(v_i)+\alpha(v_{i+1}))}{(\alpha(v_i)+\alpha(v_{i+1}))} = 3\ \alpha(v_{i+1})$$

Suppose $\alpha(v_{i+1})$ is a maximum value and $\alpha(v_i)$ is a minimum value

$$\beta(e_i) = Decrease\ Numerator\ of\ \left[\frac{\alpha(v_{i+1})-\alpha(v_i)}{\alpha(v_i)+\alpha(v_{i+1})} + 3\alpha(v_i)\right]$$

$$by\ [\alpha(v_{i+1})-\alpha(v_i)]$$

$$= \frac{3\alpha(v_i)[\alpha_1(u_i)+\alpha(v_i)]}{[\alpha_1(u_i)+\alpha(v_i)]} = 3\alpha(v_i)$$

Hence, every sun graph admits co-efficient of range labeling.

Therefore, Any Sun graph is a co-efficient of range labeling.

EXAMPLE 3.9

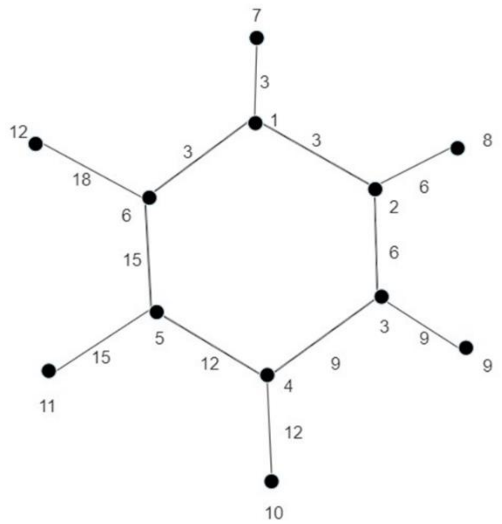

Figure 3.4 Co-efficient of range labeling of S_6

REMARK 3.10

If S_6 is co-efficient of range graph then we can also find co-efficient of range value.

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$$Co - Efficient\ Range\ Value\ (S_6) = \frac{Maximum\ edge\ value\ of\ (S_6) - Minimum\ edge\ value\ of\ (S_6)}{Maximum\ edge\ value\ of\ (S_6) + Minimum\ edge\ value\ of (S_6)}$$

$$= \frac{18-3}{18+3} = \frac{15}{21} = 0.7$$

THEOREM 3.11

The double star graph $S_{m,n}$ admits co-efficient of range labeling for every value n.

PROOF

Let G be a double star graph.

It is fixed by $S_{m,n}$ and v_1, v_2these two vertices in $S_{m,n}$ Type equation here.which are not pendant. Consider u_i's are m pendant vertices to v_1 and w_j's are n pendant vertices to v_2.

Let $\alpha : V(G) \rightarrow \{1,2,3, \ldots, n\}$

Such that $\alpha(v_1) = 1$

$$\alpha(v_2) = 2$$

$$\alpha(u_i) = i \ if \ 3 \leq i \leq m$$

$$\alpha_1(w_j) = i \ if \ m+1 \leq i \leq n$$

We have

$$\beta(e_i) = Decrease\ Numerator\ of\ \left[\frac{Maximum\ value[\alpha(v_1), \alpha(v_2)] - Minimum\ value[\alpha(v_1), \alpha(v_2)]}{Maximum\ value[\alpha(v_1), \alpha(v_2)] + Minimum\ value[\alpha(v_1), \alpha(v_2)]} + 3Minimum\ value[\alpha(v_1), \alpha(v_2)]\right]$$

$$by\ Maximum\ value[\alpha(v_1), \alpha(v_2)] - Minimum\ value[\alpha(v_1), \alpha(v_2)]$$

If $g(v_1)$ is a maximum value and $\alpha(v_2)$ is a minimum value

$$\beta(e_i) = Decrease\ Numerator\ of\ \left[\frac{\alpha(v_1) - \alpha(v_2)}{\alpha(v_1) + \alpha(v_2)} + 3\alpha(v_2)\right]$$

by $\alpha(v_1) - \alpha(v_2)$

$$= \frac{3\,\alpha(v_2)[\alpha(v_1) + \alpha(v_2)]}{\alpha(v_1) + \alpha(v_2)} = 3\alpha(v_2)$$

Suppose $\alpha(v_2)$is a maximum value and $\alpha(v_1)$is a minimum value

$$\beta(e_i) = Decrease\ Numerator\ of\ \left[\frac{\alpha(v_2)-\alpha(v_1)}{\alpha(v_2)+\alpha(v_1)} + 3\alpha(v1)\right]$$

by [α (v_2)- α (v_1)]

$$= \frac{3\ \alpha(v_1)[(\alpha(v_1)+\alpha(v_2)]}{\alpha(v_1)+\alpha(v_2)} = 3\ \alpha(v_1)$$

$$\beta(e_{1i}) = Decrease\ Numerator\ of\ \left[\frac{Maximum\ value[\alpha(v_1),\alpha(u_i)] - Minimum\ value[\alpha(v_1),\alpha(u_i)]}{Maximum\ value[\alpha(v_1),\alpha(u_i)] + Minimum\ value[\alpha(v_1),\alpha(u_i)]} + 3Minimum\ value[\alpha(v_1),\alpha(u_i)]\right]$$

By $Maximum\ value[\alpha(v_1),\alpha(u_i)] - Minimum\ value[\alpha(v_1),\alpha(u_i)]$

If $\alpha(v_1)$ is a maximum value and $\alpha(u_i)$ is a minimum value

$$\beta(e_{1i}) = Decrease\ Numerator\ of\ \left[\frac{[\alpha(v_1)-\alpha(u_i)]}{[\alpha(v_1)+\alpha(u_i)]} + 3\alpha(u_i)\right]$$

By $[\alpha(v_1)-\alpha(u_i)]$

$$= \frac{3\alpha(u_i)[\alpha(v_1)+\alpha(u_i)]}{\alpha(v_1)+\alpha(u_i)} = 3\alpha(u_i)$$

Suppose $\alpha(u_i)$ is a maximum value and α (v_1)is a minimum value.

$$\beta(e_{1i}) = Decrease\ Numerator\ of\ \left[\frac{\alpha(u_i)-\alpha(v_1)}{\alpha(u_i)+\alpha(v_1)} + 3\alpha(v_1)\right]$$

By $[\alpha(u_i)-\alpha(v_1)]$

$$= \frac{3\alpha(v_1)[\alpha(v_1)+\alpha(u_i)]}{\alpha(v_1)+\alpha(u_i)} = 3\ \alpha(v_1)$$

$$\beta(e_{2i}) = Decrease\ Numerator\ of\ \left[\frac{Maximum\ value[\alpha(v_2),\alpha_1(w_j)] - Minimum\ value[\alpha(v_2),\alpha_1(w_j)]}{Maximum\ value[\alpha(v_2),\alpha_1(w_j)] + Minimum\ value[\alpha(v_2),\alpha_1(w_j)]} + 3\ Minimum\ value[\alpha(v_2),\alpha_1(w_j)]\right]$$

by $Maximum\ value[\alpha(v_2),\alpha_1(w_j)] - Minimum\ value[\alpha(v_2),\alpha_1(w_j)]$

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If $\alpha(v_2)$ is a maximum value and $\alpha(w_j)$ is a minimum value

$$\beta(e_{2i}) = Decrease\ Numerator\ of\ \left[\frac{\alpha(v_2) - \alpha_1(w_j)}{\alpha(v_2) + \alpha_1(w_j)} + 3\alpha_1(w_j)\right]$$

by$[\alpha(v_2) - \alpha_1(w_j)]$

$$= \frac{3\alpha_1(w_j)[\alpha(v_2)+\alpha_1(w_j)]}{\alpha(v_2)+\alpha_1(w_j)} = 3\alpha_1(w_j)$$

Suppose $\alpha_1(w_j)$is a maximum value and $\alpha(v_2)$ is a minimum value,

$$\beta(e_{2i}) = Decrease\ Numerator\ of\ \left[\frac{\alpha_1(w_j) - \alpha(v_2)}{\alpha_1(w_j) + \alpha(v_2)} + 3\alpha(v_2)\right]$$

by$[\alpha_1(w_j) - \alpha(v_2)]$

$$= \frac{3\alpha(v_2)[\alpha_1(w_j)+\alpha(v_2)]}{\alpha_1(w_j)-\alpha(v_2)} = 3\alpha(v_2)$$

Hence, Every Double star graph admits co-efficient of range labeling.

Therefore, Any Double star graph is a co-efficient of range graph.

EXAMPLE 3.12

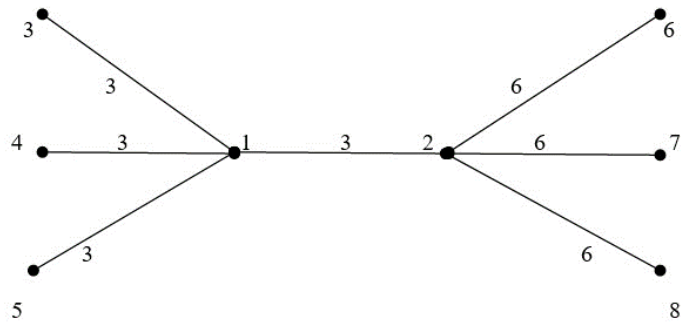

Figure 3.5 Co-efficient of range labeling of $S_{3,3}$

REMARK 3.13

If $S_{3,3}$ is a co-efficient of range graph, then we can also find co-efficient of range value.

$$Co - Efficient\ of\ Range\ Value\ (S_{3,3}) = \frac{Maximum\ edge\ value\ of\ (S_{3,3}) - Minimum\ edge\ value\ of\ (S_{3,3})}{Maximum\ edge\ value\ of\ (S_{3,3}) + Minimum\ edge\ value\ of\ (S_{3,3})}$$

$$= \frac{6-3}{6+3} = \frac{3}{9} = 0.3$$

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4 Conclusion

This research has significantly advanced coefficient of range labeling, a novel form of labeling. In this work, the ideas behind coefficient of range labeling of graphs have been covered. We concentrated on demonstrating the coefficient of range graph status of the path graph, cycle graph, sun graph, double star graphs. moreover, we may discover the coefficient of range value of those graphs. There are still a few additional families that may be investigated further.

References

[1] Gallian JA, Dynamic Survey of Graph Labeling, Electronic Journal of combinatorics 18, DS6(2015).

[2] Golomb SW, How to number a graph, in Graph Theory and Computing, R.C. Read, ed., Academic Press, New York 23-37(1972).

[3] Gupta SC and Kapoor VK, Fundamentals of mathematical statistics. Sultan Chand & Sons (2020).

[4] Jahir Hussain R and Senthamizh Selvan J, prime edge magic labeling for some graphs. Advances and Applications in Mathematical Sciences, Vol-20(6),1093-1100(2021).

[5] Jahir Hussain R and Senthamizh Selvan J, Co-Efficient of Range Labeling for Some Trees. *Journal of Algebraic Statistics*, *13*(3), 3792-3805(2022).

[6] Jahir Hussain R and Senthamizh Selvan J, “Range labeling for Some graphs” International Journal of mathematics and computer research, vol-11, issue-5,3362-3365(2023).

[7] Rosa A, On certain valuations of the vertices of a graph. Theory of Graphs (Internet. Symposium, Rome, July 1966), Gordon Breach, NY and Dunod paris,349-355(1967).

[8] Senthamizh Selvan J and Jahir Hussian R, “some results on range labeling” Ratio Mathematica, Vol-46(2023).