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A Few Separation Axioms on Nα*AS - closed Sets in Nano Topological Space

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Abstract

This work goal is to gain the relation between a number of the existing sets using inducing and investigating the capabilities of Ɲα*AS -T0 space, Nα*AS-T1space, Nα*AS -T2 space.

Key Words: Ɲano-T_0 space, Ɲα*$_{AS}$ -T_0 space, nano-T_1 space, Ɲα*$_{AS}$ -T_1 space, ɲano-T_2 space, Ɲα*$_{AS}$ -T_2 space.

AMS Classification: 54A05

1 Introduction

Lellis Thivagar and Richard introduced the idea of nano topology initially. In ɲano topological spaces, Nasef et al. have found multiple really open sets. Anbarasi Rodrigo and Sahaya Dani developed the concept of Ɲα*AS- closed sets. Sathishmohan et al. introduce the idea of nano neighborhoods in nano topological spaces.This motivates the author to present and examine the properties of Ɲα*AS -T0 space, Ɲα*AS -T1 space, and Ɲα*AS -T2 space in Ɲano topological spaces.

2 Groundworks

Exegesis 2.1: For c, d ∈ U and c d, ∃ a ɲano-open set G such that c ∈ G and d ∉ G, U is referred to as ɲano-T0 (or Ɲ-T0) space.

Exegesis 2.2: For c, d ∈ U and c ≠ d, ∃ a ɲano semi-open [resp. ɲano pre-open] set G such that c ∈ G and d ∉ G, a space U is referred to as a ɲano semi-T0 (or ƝS-T0) [resp. naɲo pre-T0 (or ƝP-T0)] space.

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Exegesis 2.3: U is called ɲano-T1 (or Ɲ-T1) [also called ɲano pre-T1 (or ƝP-T1), ɲano semi-T1 (or ƝS-T1)]. Ɲano-open sets [resp. ɲano semi-open, ɲano pre-open] if there exist G and H such that c ∈ G, d ∉ G and d ∈ H, c ∉ H and c ≠ d.

Exegesis 2.4: A space U is known as ɲano-T2 (or Ɲ-T2) [resp. ɲano semi-T2 (or ƝS-T2), ɲano pre-T2 (or ƝP-T2)] space for c, d ∈ U and c ≠ d, ∃ disjoint ɲano-open sets [resp. ɲano semi-open, ɲano pre-open] G and H such that c ∈ G and d ∈ H.

3 Ɲα^*_{AS} - T_0 space

Exegesis 3.1: A space U is referred to as Ɲα^*_{AS} -T_0 space for c, d ∈ U and c ≠ d, ∃ a Ɲα^*_{AS} – closed set G such that c ∈ G and d ∉ G.

Illustration 3.2: Let X = {1, 2, 3} with τ = {X, ∅, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}}, let σ = {X, ∅, {3}}. Then Ɲα^*_{AS} - closed units are = {X, ∅, {1}, {2}, {1,2}, {1,3}, {2,3}}. permit c = {2} and d = {3} wherein c, d ∈ X and c ≠ d. Let G = {1, 2}, as a consequence c ∈ G however d ∉ G.

Theorem 3.3: Permits (U, $\tau_R(X)$) to be a ɲano topological space, then every Ɲ-T_0 space is Ɲα^*_{AS} -T_0 space but now not conversely.

Illustration 3.4: In illustration 3.2 ɲano closed sets are {X, ∅, {1}, {2}, {3}, {1,2}, {1,3}, {2,3}. let c = {2} and d = {3} then it's miles Ɲα^*_{AS} -T_0 space but now not Ɲ-T_0 space.

Theorem 3.5: Each ƝS-T_0 (resp. ƝP-T_0, Ɲα-T_0) space is Ɲα^*_{AS} -T_0 space but no longer conversely.

Example 3.6: From illustration 3.2, Let c = {2} and d = {3} then it's miles Ɲα^*_{AS} -T_0 space but no longer ƝS-T_0 space.

Illustration 3.7: From illustration 3.2, permit c = {2} and d = {3} then it's miles Ɲα^*_{AS} -T_0 space but no longer ƝP-T_0 space.

Illustration 3.8: From illustration 3.2, Let c = {2} and d = {3} then it's far Ɲα^*_{AS} -T_0 space however not Ɲα-T_0 space.

Theorem 3.9: A ɲano topological space U is Ɲcl{c} iff Ɲα^*_{AS} cl{c}≠ Ɲα^*_{AS} cl{d} for c ≠ d in U.

proof: Let c, d ∈ U and c ≠ d with U as Ɲα^*_{AS} -T_0 space. We should show that Ɲα^*_{AS} cl{c} ≠ Ɲα^*_{AS} cl{d}. don't forget the set A = U −{c}, it's far clear that Ɲcl(A) is both A or U. If Ɲcl(A) = A then A is ɲano-closed and consequently Ɲα^*_{AS} -closed. consequently U − A = {c} is a Ɲα^*_{AS} -open set which includes c but no longer d. So, c ∉ Ɲα^*_{AS} cl{d}. however, c ∈ Ɲα^*_{AS} cl{c} and for this reason Ɲα^*_{AS} cl{c}≠ Ɲα^*_{AS} cl{d}. If Ɲcl(A) = U, then A is Ɲα^*_{AS} -open and so U − A = {c} is Ɲα^*_{AS} -closed. therefore Ɲα^*_{AS} cl{c} = {c}. considering that d ∉{c} and d ∈ Ɲα^*_{AS} cl{d}, it follows that Ɲα^*_{AS} cl{c}≠ Ɲα^*_{AS} cl{d}.

Conversely: For c, d ∈ U and c ≠ d. permit Ɲα^*_{AS} cl{c} ≠ Ɲα^*_{AS} cl{d}. consequently ∃ a factor z in U such that z ∈ Ɲα^*_{AS} cl{c} however z ∉ Ɲα^*_{AS} cl{d}. If we suppose that c ∈ Ɲα^*_{AS} cl{d} then Ɲα^*_{AS} cl{c} ⊂ Ɲα^*_{AS} cl{d} and this means z ∈ Ɲα^*_{AS} cl{d} that is a contradiction. consequently, our supposition is incorrect, i.e., c ∉ Ɲα^*_{AS} cl{d} implies c ∈ U − Ɲα^*_{AS} cl{d} and Ɲα^*_{AS} cl{d} is a Ɲα^*_{AS} - closed set containing c but no longer d. this means U is Ɲα^*_{AS} -T_0 space.

4 Ɲα*AS -T1 space

Exegesis 4.1: A space U is called ɲano α-T1 (or Ɲα-T1) space for c, d ∈ U and c ≠ d, ∃ a Ɲα-open sets G and H such that c ∈ G, d ∉ G and d ∈ H, c ∉ H.

Exegesis 4.2: A space U is referred to as Ɲα*AS -T1 space for c, d ∈ U and c ≠ d, ∃ a Ɲα*AS -closed units G and H such that c ∈ G, d ∉ G and d ∈ H, c ∉ H.

Theorem 4.3: Each ɲano-T1 (resp. ƝS-T1, ƝP-T1, Ɲα-T1) space is Ɲα*AS -T1 space however now not conversely.

Illustration 4.4: Let X = {1, 2, 3, 4}, $\tau_R(X)$ = {U, φ, {1}, {1, 2, 4}, {2, 4}} be a ɲano topology on X, we've got Let c = {2} and d = {3} then it's far Ɲα*AS -T1 space but now not Ɲ-T1 space.

Illustration 4.5: From illustration 4.4, let c = {1} and d = {3} then it is $Ɲα^*_{AS}$ -T_1 space however no longer ƝS-T_1 space.

Illustration 4.6: From illustration 4.4, permit c = {2} and d = {3} then it's far $Ɲα^*_{AS}$ -T_1 space but now not Ɲα-T_1 space.

Lemma 4.7: Let C and D be the subsets of U such that C ⊂ D and D is $Ɲα^*_{AS}$ - closed, then C is $Ɲα^*_{AS}$ - closed subset of D if C is $Ɲα^*_{AS}$ - closed subset of U.

Lemma 4.8: For any subset A of U

(1) $Ɲα^*_{AS}$ int ($Ɲα^*_{AS}$ cl(A)) = $Ɲα^*_{AS}$ cl ($Ɲα^*_{AS}$ int (A)).

(2) Ɲint ($Ɲα^*_{AS}$ cl(A)) = Ɲcl ($Ɲα^*_{AS}$ int (A)).

(3) Ɲcl ($Ɲα^*_{AS}$ int (A)) = Ɲint ($Ɲα^*_{AS}$ cl(A)).

Lemma 4.9: If f: (U, $\tau_R(X)$) → (V, $\tau_R(Y)$) is ɲano-open and ɲano-non-stop then for and subset A of U then

(1) f (Ɲint (A)) ⊂ Ɲint (f(A)).

(2) f (Ɲcl(A)) ⊂ Ɲcl(f(A)).

Theorem 4.10: Let (U, $\tau_R(X)$) be a ɲano topological space, then for each $Ɲα^*_{AS}$ -T_1 (resp. ƝS-T_1, ƝP-T_1) space is $Ɲα^*_{AS}$ -T_0 space.

Illustration 4.11: Let U = {1, 2, 3, 4}, and $\tau_R(X)$ = {U, φ, {1}, {1, 2, 4}, {2, 4}} be a ɲano topology on U, we've got let G = {1,4} and H = {2,3}. Let c = {1} and d = {3}, c, d ∈ U and c ≠ d, then it's miles clean that c ∈ G, d ∉ G and d ∈ H and c H. Then we can say that it's miles $Ɲα^*_{AS}$ -T_0 space.

Theorem 4.12: For a topological space U, each of the subsequent statements is equal

(a) U is $Ɲα^*_{AS}$ -T_1 space.

(b) Every one-factor set is $Ɲα^*_{AS}$ -closed in U.

(c) Every subset of U is the intersection of all $Ɲα^*_{AS}$ -open sets containing it.

(d) The intersection of all $Ɲα^*_{AS}$ -open sets containing the point {c} in U is {c}.

Proof: (a) ⇒ (b): Let c ∈ U, for this reason for and d ∈ V, d ≠ c ∃ a $Ɲα^*_{AS}$ -open set G containing d but not c. for this reason d ∈ G⊂ {c}. Truly {c} = S {G: d ∈ {c} c} so {c} being a union of $Ɲα^*_{AS}$ -open set is $Ɲα^*_{AS}$ -open ⇒ {c} is $Ɲα^*_{AS}$ -closed.

(b) ⇒ (c): Suppose each one-factor set is $Ɲα^*_{AS}$ -closed. let A ⊂U, then for each d ∈ A ∃ a subset {d} such that a ⊂ {d} and each of those units {d} is $Ɲα^*_{AS}$ -open. as a result, A = {{d}: d ∈ A} in order that the intersection of all $Ɲα^*_{AS}$ -open sets containing A is the set A itself.

(c) ⇒ (d): In c) Let A = {c} then d ∉{c} and U−{d} is Ɲα*$_{AS}$ -open set containing c. therefore from (c) {c} = {U −{d} / each U −{d} is Ɲα*$_{AS}$ -open set containing c}

(d) ⇒ (a): assume (d). permit c, d ∈ U and c ≠ d, then using hypothesis for c ∈ U, {c} = {A ∈ Ɲα*$_{AS}$ O(U)/c ∈ A}, it follows that there ought to exist a Ɲα*$_{AS}$ -open set Gc such that c ∈ G and d ∉ G. Similarly, there ought to exist a Ɲα*$_{AS}$ -open set Gd such that d ∈ G and c ∉ G. Subsequently U is Ɲα*$_{AS}$ -T_1.

Lemma 4.13: Let f: (U, $\tau_R(X)$) → (V, $\tau_R(Y)$) be ɲano-open and ɲano-continuous then for each Ɲα*$_{AS}$ -open set A of U, f(A) is Ɲα*$_{AS}$ -open subset of D.

Theorem 4.14: The belongings of being Ɲα*$_{AS}$ -T_1space is a ɲano-topological asset.

Theorem 4.15: Each open subspace of a Ɲα*$_{AS}$ -T_1 space is Ɲα*$_{AS}$ -T_1space.

Theorem 4.16: Let U be Ɲ-T_1 space and f: (U, $\tau_R(X)$) → (V, $\tau_{R*(}Y)$) be Ɲα*$_{AS}$ -closed

surjection then D is Ɲα*$_{AS}$ -T_1 space.

5 Ɲα*$_{AS}$ -T_2 space

Exegesis 5.1: A space U is known as ɲano α-T_2 (or Ɲα-T_2) space for c, d ∈ U and c ≠ d, ∃ disjoint Ɲα-open units G and H such that c ∈ G and d ∈ H.

Exegesis 5.2: A space U is known as Ɲα*$_{AS}$ -T_2 space for c, d ∈ U and c ≠ d, ∃ disjoint Ɲα*$_{AS}$ - open units G and H such that c ∈ G and d ∈ H.

Lemma 5.3: If A is ɲano open in U and V is Ɲα*$_{AS}$ -open in U then A∩V is Ɲα*$_{AS}$ -open in U.

Lemma 5.4: If f: (U, $\tau_R(C)$) → (V, $\tau_R(D)$) is Ɲα-open and Ɲα*$_{AS}$ -continuous, then inverse image of Ɲα*$_{AS}$ -open set is Ɲα*$_{AS}$ -open.

Theorem 5.5: Every ɲano-T_2 space is Ɲα*$_{AS}$ -T_2 space however, not conversely.

Illustration 5.6: From the illustration 4.4, permit c = {2} and d = {3} then it is Ɲα*$_{AS}$ -T_2 space however now not Ɲ-T_2 space.

Theorem 5.7: Every ƝS-T_2 space is Ɲα*$_{AS}$ -T_2 space however no longer conversely.

Proof: same as Theorem 5.5

Illustration 5.8: From illustration 4.4, permit c = {1} and d = {3} then it's far Ɲα*$_{AS}$ -T_2 space but no longer ƝS-T_2 space.

Theorem 5.9: Every Ɲα-T_2 space is Ɲα*$_{AS}$ -T_2 (resp. ƝS-T_2, ƝP-T_2) space however no longer conversely.

Proof: identical to Theorem 5.5

Illustration 5.10: From illustration 4.4, Let c = {b} and d = {c} then it's miles Ɲα*$_{AS}$ -T_2 space however no longer Ɲα-T_2 space.

Theorem 5.11: For the ɲano topological space U the subsequent are equal (a) U is Ɲα*$_{AS}$ -T_2 space. (b) If c ∈ U, then for every d ≠ c ∃ a Ɲα*$_{AS}$ -neighborhood G of c such that d ∉ Ɲα*$_{AS}$ cl (G). (c) For each c ∈ U, {Ɲα*$_{AS}$ cl(G): G is a Ɲα*$_{AS}$ -the neighborhood of c} = {c}.

Theorem 5.12: The assets of being Ɲα*$_{AS}$ -T_2 space is ɲano topological a belonging.

Proof: Identical to Theorem 4.14

Theorem 5.13: Every ɲano-open subspace of Ɲα*$_{AS}$ -T_2 space is Ɲα*$_{AS}$ -T_2 space.

Theorem 5.14: If f: (U, τ_R(C)) → (V, τ_R(D)) is injective, Ɲα-open, Ɲα*$_{AS}$ -non-stop and V is Ɲα*$_{AS}$ -T_2 space then U is Ɲα*$_{AS}$ -T_2 space.

Theorem5.15: If f: (U, τ_R(C)) → (V, τ_R(D)) is injective, Ɲα*$_{AS}$ -continuous and V is ɲano-T_2 space then U is Ɲα*$_{AS}$ -T_2 space.

6 Conclusion

Using a few deductions to prove, we have explored and analyzed the resources of the aforementioned arguments on Ɲα*$_{AS}$ -T_0, Ɲα*$_{AS}$ -T_1, and Ɲα*$_{AS}$ -T_2 spaces with some of the prevailing sets.

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