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A Study on $J^{(**)}$ - Closed Sets in Topological Spaces

Raviraj R[1] and Mohamed Ali. A[2]

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Abstract

The concept of $J^{(**)}$ - closed sets in initiated here. A few interesting peculiarities of $J^{(**)}$ - closed sets are discussed. Moreover relations of $J^{(**)}$ - closed sets with other existing G-closed sets are analysed. Some important characterizations are also obtained.

Key Words: $J^{(**)}$ - closed set; η*-open set; J*-closed set; J-closed set.

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1 Introduction

For this paper some basic definitions and results in topological spaces are needed which are given in this paper, a topological spces is represented by (Y, ζ).

2 Preliminaries

Definition 2.1. If D⊆Y, then

(i) Cl* (D) is the intersection of all generalized closed sets in Y having D which is said to be generalized closure of the subset D [9].
(ii) int* (D) is the unification of all generalized open sets in Y contained in D which is said to be generalized interior of the subset D [9].

Definition 2.2. A subset D of (Y,ζ) is called

(1) Regular closed set [26] if D=Cl(int (D)).
(2) Semi-closed set [13] if int(Cl (D))⊆D.
(3) α- closed set [21] if Cl (int(Cl (D)))⊆D.
(4) Pre-closed set [16] if Cl (int(D))⊆D.
(5) D⊆Y is said to be semi pre-closed set [1] if int (Cl (int (D)))⊆D
(6) A subset D is known as π-closed set [30] if D is expressed as the finite union of regular closed sets.

[1] Department of Mathematics, Akshara PU & Degree College, Anekal, Karnataka, India.
Email: ravirajr915@gmail.com
[2] Associate professor, Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi – 635752, India. Email: mohamedalihashim@gmail.com

The corresponding components are their corresponding open sets. The corresponding closures are defined as the intersection of corresponding sets having D. A subset D of (Y,ζ) is called cl open if it is both open and closed in (Y,ζ).

Definition 2.3. Let D⊆Y. A subset D in Y is known as J- closed set [18] if Cl(D)⊆M whenever D⊆M, M is η*- open in Y. We represent the collection of all J-closed sets of (Y,ζ) by JC(Y,ζ).

Definition 2.4. Let D⊆Y. A subset D in Y is known as J*- closed set if η*-Cl(D)⊆M whenever D⊆M, M∈ζ. We represent the collection of all J*-closed sets of (Y,ζ) by J* C(Y,ζ).

Definition 2.5. R_1-space is a topological space (Y,ζ) in which unequal points having distinct closure will have non-intersecting neighbourhoods.

Definition 2.6. (Y,ζ) is said to be a A partition space when each openness is closedness.

Remark 2.7

(i) J^*-closed set are gs-closed set.
(ii) J^*-closed set becomes αg-closed set.
(iii) Each J^*-closed is J-closed

3 J^{**}-Closed Sets in Topological Spaces

Definition 3.1. Let D⊆Y. A subset D in Y is known as $J^{(**)}$ -closed set if η*-Cl(D)⊆M whenever D⊆M, M is η*-open in (Y,ζ). We represent the collection of all $J^{(**)}$ -closed sets of (Y,ζ) by $J^{(**)}$ Cl(Y,ζ).

Properties of J^{**}-Closed sets

Theorem 3.2. The arbitrary union of $J^{(**)}$ -closed sets is $J^{(**)}$ -closed.

Proof. Take ∪_(i∈I) D_i⊆M, where M, η*-open set containing D. Then η*-Cl(∪_(i∈I))=∪_(i∈I) η*-Cl(D_i)⊆M (By Theorem 3i(c) from [17]). Hence any unification of $J^{(**)}$ -closed in (Y,ζ).

Remark 3.3. The below example explains that the intersection of any two $J^{(**)}$ - closed sets in Y need not be $J^{(**)}$ - closed.

Example 3.4. Let Y={h,s,n}, ζ={Y,ϕ,{h}}. Consider D and E are subsets of Y. Then the set D={h,s} and E={h,n} are $J^{(**)}$ -closed but the intersection D∩E={h} is not $J^{(**)}$ -closed in (Y,ζ).

Theorem 3.5. If D is a $J^{(**)}$ -closed set and F is a δ-closed set in (Y,ζ), then D∩F becomes $J^{(**)}$ - closed.

Proof. Given D, $J^{(**)}$ -closed and F is δ-closed. Let V=D∩F. Let M be η*-open such that V⊆M. Then D∩F⊆M which implies D⊆M∪F^c. Hence F^c is δ-open. So F^c is η*-open. Hence M∪F^c is η*-open (By Theorem 3i(c) from [17]) and by assumption D⊆M∪F^c which implies η*-Cl(D)⊆M∪F^c which implies η*-Cl(D)∩F⊆M. Now η*-Cl(V)=η*-Cl(D∩F)=η*-Cl(D)∩η*-Cl(F)⊆η*-Cl(D)∩δ-Cl(F) (By Properties 4k.g of [17]) and Theorem 4d of [17] =η*-Cl(D)∪F⊆M. Therefore η*-Cl(V)⊆M. Hence D∪F is $J^{(**)}$ -closed.

Remark 3.6. The below example explains that the difference of any two $J^{(**)}$ -closed sets in Y need not be $J^{(**)}$ - closed.

Example 3.7. Let Y={h,s,n},ζ={Y,φ,{h}}. Consider D and E are subsets of Y.Then Analysing we get D={h,s} and E={s,n} are $J^{(**)}$ - closed but the difference D–E={h} is not $J^{(**)}$ - closed in (Y,ζ).

Theorem 3.8. Let D be a $J^{(**)}$ - closed set of (Y,ζ). Then η*-Cl(D)–D does not contain a non-empty η*-closed set.

Theorem 3.9. Let D be a $J^{(**)}$ -closed set of (Y,ζ). Then η*-Cl(D)–D does not contain a non-empty η*-closed set.

Proof. Suppose that D is $J^{(**)}$ -closed, let M be a η*-closed set contained in η*-Cl(D)–D. Now M^c is a η*-open set in Y such that D⊆M^c. Since D is $J^{(**)}$- closed set of Y, η*-Cl(D)⊆M^c. Thus M⊆$(η^*\text{-Cl(D)})^c$. Also M⊆η*-Cl(D)–D. Therefore ⊆$(η^*\text{-Cl(D)})^c$ ∪η*-Cl(D)=φ . Hence M=φ.

Proposition 3.10. If D is a η*-open set and a $J^{(**)}$ -closed set of (Y,ζ), then D is a η*-closed set of Y.

Proof. Since D is η*-open and $J^{(**)}$ - closed, η*-Cl(D)⊆D. Obviously, D⊆η*-Cl(D). Hence D is η*-closed in (Y,ζ).

Theorem 3.11. If D is $J^{(**)}$ - closed and η*-open and F is η*- closed in (Y,ζ), then D∪F is η*-closed.

Proof. Since D $J^{(**)}$ - closed and η*-open, D is η*-closed by Proposition 3.10.,Since F is η*-closed in Y, D∪F is η*-closed in Y (by Theorem 3i (c) of [17]).

Proposition 3.12. If D is a $J^{(**)}$ - closed set in a space (Y,ζ) and D⊆B⊆η*-Cl(D) then B becomes a $J^{(**)}$ - closed set.

Proof. Let M be η*-open set of Y such that B⊆M. Then D⊆M. Since D is $J^{(**)}$-closed set, η*-Cl(D)⊆M. Since B⊆η*-Cl(D), η*-Cl(B)⊆η*-Cl(η*-Cl(D)) = η*- Cl(D) (Remark 2.11.(i)). Hence η*-Cl(B)⊆M. Therefore B is also a $J^{(**)}$ - closed set .

Theorem 3.13. Let D be a $J^{(**)}$ -closed set of (Y,ζ).Then D is η*-closed iff η*-Cl(D)–D is η*-closed.

Proof. (Necessity) Let D be a η*-closed subset of (Y,ζ).Then η*-Cl(D)=D and therefore η*-Cl(D)–D=φ which is η*-closed.

(Sufficiency) Let η*-C1(D)-D be η*-closed set. Since D is $J^{(**)}$ -closed, by Theorem 3.46., η*-Cl(D)-D does not contain a non- empty η*- closed set which implies η*-Cl(D)-D=φ . That is η*-Cl(D)=D. Hence D is η*- closed.

Definition 3.14. Let B⊆A⊆Y. Then B is $J^{(**)}$ - closed relative to A if η*-Cl_A (B)⊆M, whenever B⊆M, M is η*-open in A.

Theorem 3.15. B⊆A⊆Y and suppose that B is $J^{(**)}$-closed in Y, then B is $J^{(**)}$ -closed relative to A. The converse is true if A is η*- closed in Y.

Proof. Suppose that B is $J^{(**)}$ - closed in Y. Let B⊆M, M is η*-open in A. Since M is η*-open in A, M=V∩A where V is η*-open in Y. Hence ⊆M⊆V . Since B is $J^{(**)}$ - closed in Y, η*-Cl(B)⊆V. Hence η*-Cl(B)∩A⊆V∩A which in turn implies that η*-Cl_A (B)∩A⊆V∩A=M. Therefore B is $J^{(**)}$ - closed relative to A.

Now to prove that the converse, assume that B⊆A⊆Y where A is η*- closed in Y and B is $J^{(**)}$ - closed relative to A. Let B⊆M, M is η*-open in Y. Then A∩M is η*- open in A by the definition of subspace topology. Since B⊆A and B⊆M, B⊆A∩M. Since B is $J^{(**)}$ - closed relative to A, η*-Cl_A (B)⊆A∩M. Since B⊆A, η*-Cl(B)⊆ η*-Cl_A (By Properties 4k.d of [17]). Hence η*-Cl(B)⊆A. Therefore η*-Cl(B)∩A=η*-Cl(B) which implies η*-Cl_A (B)=η*-Cl(B) . Hence η*-Cl(B)⊆A∩M⊆M. Thus B is $J^{(**)}$ - closed in Y.

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