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Solving Fuzzy Assignment Problem Using Nonagonal Fuzzy Numbers In Root Mean Square Method

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Abstract

Assignment Problem is universally used to solve real valued problems. In this paper, the values of the Fuzzy Assignment Problem are considered as nonagonal fuzzy numbers. First the fuzzy numbers are converted into crisp values using Root mean square method. Then the optimum schedule of the Fuzzy Assignment Problem is obtained by usual Hungarian Method. This approach is illustrated by numerical example.

Key words: Nonagonal fuzzy numbers, fuzzy assignment problem, root mean square method, Hungarian method.

AMS classification: 90C70, 90C20

1 Introduction

The assignment problem (AP) is a particular type of transportation problem in which our objective is to assign 'n' number of activities at a maximum cost or maximum profit. First the term Assigment Problem was introduced in Votaw and Orden (1952). All the algorithms developed to find the optimal solution of Transportation problem are applicable to assignment problem. However, due to its highly degenerate nature, a special algorithm known as Hungarian algorithm proposed by Kuhn [4] is used for its solution.

In real life situations, the parameters of AP are imprecise numbers instead of fixed real numbers because time or cost for doing a job by a person or machine might vary due to different reasons. L. A. Zadeh [10] intoduced the concept of fuzzy

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sets in 1965 to deal with imprecision in real life situatins. Chen [1] proposed a fuzzy assignment model that did not consider the differences of individuals, and also proved some theorems. Wang [9] solved a similar model by Graph theory. Dubois and Fortemphs [2] proposed a flexible algorithm which combines with fuzzy theory, multiple criteria decision making and constraint-directed methodology. Based on the labelling method for solving the fuzzy assignment problem, Lin and wen [5] proposed an algorithm.

Feng and Yang [3] investigated a two objective cardinality assignment problem. Liu and Gao [6] proposed an equlibrium optimization problem and extended the assignment problem to the equilibrium multi-job assignment problem. Mukherjee and Basu [7] solved the fuzzy assignment problem by converting the fuzzy numbers into crisp numbers using Yager's ranking technique. E. Melita Vinoliah and K. Ganesan [8] proposed a best candidate method to solve the octogonal fuzzy assignment problem. In this paper the fuzzy assignment problem has been converted into crisp assignment problem using Root Mean Square Method and then we can find the optimal solution using Hungarian Algorithm.

2 Preliminaries

Definition 2.1 ***Fuzzy Set:*** The characteristic function μ_A of crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within the specified range i.e. $\mu_{\tilde{A}} : X \to [0,1]$. The assigned value indicate the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}) : x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 2.2 [10] A Fuzzy set $\tilde{A}$, defined on universal of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\tilde{A}$ is convex i.e, $\mu_{\tilde{A}}\ (\lambda x_1 + (1-\lambda)x_2) \geq$ minimum $(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))\ \forall x_1, x_2 \in R$ and $0 \leq \lambda \leq 1$.

(ii) $\tilde{A}$ is normal, there exists $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$.

(iii) $\mu_{\tilde{A}}$ is piecewise continuous.

Definition 2.3 A fuzzy number $\tilde{A}$ is said to be non negative fuzzy number if and only if $\mu_{\tilde{A}}(x) = 0$ for all $x < 0$.

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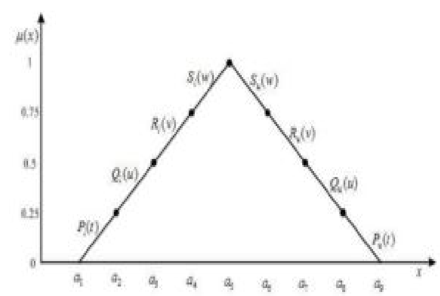
Definition 2.4 A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$ is said to be an nonagonal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{1}{4}\left(\frac{x-a_1}{a_2-a_1}\right) \text{for}\, a_1 \le x \le a_2 \\ \frac{1}{4}+\frac{1}{4}\left(\frac{x-a_2}{a_3-a_2}\right) \text{for}\, a_2 \le x \le a_3 \\ \frac{1}{2}+\frac{1}{4}\left(\frac{x-a_3}{a_4-a_3}\right) \text{for}\, a_3 \le x \le a_4 \\ \frac{3}{4}+\frac{1}{4}\left(\frac{x-a_4}{a_5-a_4}\right) \text{for}\, a_4 \le x \le a_5 \\ 1-\frac{1}{4}\left(\frac{x-a_5}{a_6-a_5}\right) \text{for}\, a_5 \le x \le a_6 \\ \frac{3}{4}-\frac{1}{4}\left(\frac{x-a_6}{a_7-a_6}\right) \text{for}\, a_6 \le x \le a_7 \\ \frac{1}{2}-\frac{1}{4}\left(\frac{x-a_7}{a_8-a_7}\right) \text{for}\, a_7 \le x \le a_8 \\ \frac{1}{4}\left(\frac{a_9-x}{a_9-a_8}\right) \text{for}\, a_8 \le x \le a_9 \\ 0 \quad for \quad otherwise. \end{cases} \tag{1}$$



Definition 2.5 ***Defuzzification:*** Defuzzification is the process of finding singleton value (crisp value) from the output of the aggregated fuzzy set. Here root mean square method is used to defuzzify the nonagonal fuzzy number because of its simplicty and accuracy.

Assignment Problem

The assignment problem can be stated in the form of $n \times n$ cost matrix $[C_{ij}]$ of real numbers as given in the following table:

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Jobs→Personal↓	1	2	3	..j..	n
1	C_{11}	C_{12}	C_{13}	$..C_{1j}..$	C_{1n}
2	C_{21}	C_{22}	C_{23}	$..C_{2j}..$	C_{2n}
-	-	-	-	-	-
i	C_{i1}	C_{i2}	C_{i3}	$..C_{ij}..$	C_{in}
-	-	-	-	-	-
n	C_{n1}	C_{n2}	C_{n3}	$..C_{nj}..$	C_{nn}

Mathematically assignment problem can be stated as

$$Minimize \quad z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij} \tag{2}$$

$$Subject \quad to \sum_{i=1}^{n} x_{ij} = 1 i = i, 2, 3, ..., n \tag{3}$$

$$\sum_{j=1}^{n} x_{ij} = 1 j = 1, 2, 3, ..., n \tag{4}$$

$$Where\ x_{ij} = \begin{cases} 1\ if\ the\ i^{th}\ person\ assigned\ the\ j^{th}\ job \\ 0\ otherwise \end{cases} \tag{5}$$

andC_{ij} represents the cost of assignment of person i to the job j.
When the costs $(\bar{c_{ij}})$ are fuzzy numbers then the total cost becomes a fuzzy number. Then the fuzzy objective function is

$$Minimize \bar{z} = \sum_{i=1}^{n} \sum_{j=1}^{n} \bar{c_{ij}} x_{ij} \tag{6}$$

Where $\bar{C_{ij}} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$, the nonagonal fuzzy numbers. It cannot be minimized directly. We defuzzyfy the fuzzy cost coefficients into crisp ones by our proposed method.

4 Algorithms

4.1 Hungarian Assignment Algorithm

Step 1. Check whether the number of rows are equal to number of columns or not in the assignment problem.If not add dummy row or dummy column with cost value 0 and make it as a balanced one.
Step 2. In each row subtract the smallest cost element from each element so that there will be at least one zero in each row. In the same way proceed in the column also.
Step 3. Examine the rows successively until a row with exactly one zero is found. Circle the zero as an assigned cell and cross out all other zero in this column. Proceed in this manner until all the rows have been examined. If there are more than one zero in any row do not consider that row and pass on to the next row.
Step 4. Repeat the procedure for the columns of the cost matrix. If there is no single zero in any row or column then arbitrarily choose a new row or column having the minimum number of zeros. Repeat the step 3 and 4 until all the zeros are either assigned or crossed out.
Step 5. If the number of assigned cells equals the number of rows then there is an optimal assignment schedule.If not go to next step.
Step 6. Draw the minimum number of horizontal or vertical lines through all the zeros as follows:
(i) Mark ($\sqrt{}$) to those rows where no assignment has been made.
(ii) Mark ($\sqrt{}$) to those columns which have zeros in the marked rows.
(iii) Mark ($\sqrt{}$) rows (not already marked) which have assignment in marked columns.
(iv) Draw straight lines through unmarked rows and marked columns.
Step 7. If the number of lines is equal to the number of rows or columns. The optimum solution is attained by arbitrary allocation in the position of the zeros not crossed in step:3. If not go to the next Step.
Step 8. Choose the smallest element from the uncrossed elements and subtract this element from them and add the same at the point of intersection of two lines. Other elements crossed by the lines remain unchanged.
Step 9. Go to Step 4 and repeat the procedure till an optimum solution is attained.

4.2 Algorithm to solve fuzzy assignment problem

Step 1. First convert the cost values of the fuzzy Assignment Problem which are all in nonagonal fuzzy numbers into crisp value by using the following formula.

(i.e.) If $cij = (a_1, a_2, ..., a_n)$ then the crisp value of cij' which is denoted by cij' is defined by

$$cij' = Sq.root(a_1^2 + a_2^2 + ...a_n^2)$$

Step 2. Check whether the assignment problem is balanced or not. If not add dummy row or dummy column with cost value 0 and make it as a balanced one.
Step 3. Using Hungarian method, find the optimum assignment schedule.

5 Numerical Example

To illustrate a fuzzy assignment problem whose elements are nanogonal fuzzy numbers by using the proposed method. Let us consider a fuzzy assignment problem with rows representing five persons **P1, P2, P3, P4, P5** and columns representing the five jobs **J1, J2, J3, J4, J5** and the cost matrix $(\bar{c_{ij}})$ is given whose elements are nanogonal fuzzy numbers. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum.

	J1	J2	J3	J4	J5
P1	(-3,-1,0,1,2,3,4,7,8)	(-3,-2,0,2,3,4,5,8,10)	(-2,-1,0,1,4,5,7,9,11)	(3,4,5,6,7,8,9,10,11)	(-4,-2,-1,0,1,2,3,5,6)
P2	(1,2,3,5,7,9,11,12,13)	(8,9,10,11,12,13,14,15,16)	(2,3,4,5,6,9,10,12,13)	(5,6,7,8,9,10,11,12,13)	(-6,-5,0,5,6,18,24,30,36)
P3	(-3,-2,-1,0,1,2,4,5,6)	(2,3,4,5,6,7,8,9,10)	(3,5,6,7,8,9,10,11,12)	(1,2,3,5,6,7,8,10,11)	(0,1,2,3,4,5,6,7,8)
P4	(5,6,8,9,10,12,13,14,15)	(-1,0,1,3,5,7,8,9,10)	(8,9,10,11,12,13,14,15,16)	(-3,-2,-1,1,2,3,4,7,10)	(-3,-2,-1,1,3,5,7,9,11)
P5	(4,5,6,7,10,12,14,15,17)	(-2,-1,0,1,2,3,4,5,6)	(-1,0,1,2,3,4,5,6,7)	(2,4,5,6,7,8,9,11,13)	(2,3,4,5,6,7,10,11,12)

solution.

The nonagonal fuzzy number $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ is changed to a crisp one by applying

$$c_{ij'} = Sq.root(a_1 2^2 + a_2^2 + ... + a_8^2)$$

$C'_{11} = (-3, -1, 0, 1, 2, 3, 4, 7, 8)' = 12.37$, $C'_{12} = (-3, -2, 0, 2, 3, 4, 5, 8, 10)' = 15.20$
$C'_{13} = (-2, -1, 0, 1, 4, 5, 7, 9, 11)' = 17.26$, $C'_{14} = (3, 4, 5, 6, 7, 8, 9, 10, 11)' = 22.38$
$C'_{15} = (-4, -2, -1, 0, 1, 2, 3, 5, 6)' = 9.80$, $C'_{21} = (1, 2, 3, 5, 7, 9, 11, 12, 13)' = 24.56$
$C'_{22} = (8, 9, 10, 11, 12, 13, 14, 15, 16)' = 36.82$, $C'_{23} = (2, 3, 4, 5, 6, 9, 10, 12, 13)' = 24.17$
$C'_{24} = (5, 6, 7, 8, 9, 10, 11, 12, 13)' = 28.09$, $C'_{25} = (-6, -5, 0, 5, 6, 18, 24, 30, 36)' = 56.73$
$C'_{31} = (-3, -2, -1, 0, 1, 2, 4, 5, 6)' = 9.80$, $C'_{32} = (2, 3, 4, 5, 6, 7, 8, 9, 10)' = 19.60$
$C'_{33} = (3, 5, 6, 7, 8, 9, 10, 11, 12)' = 25.08$, $C'_{34} = (1, 2, 3, 5, 6, 7, 8, 10, 11)' = 20.22$
$C'_{35} = (10, 1, 2, 3, 4, 5, 6, 7, 8)' = 17.44$, $C'_{41} = (5, 6, 8, 9, 10, 12, 13, 14, 15)' = 32.25$
$C'_{42} = (-1, 0, 1, 3, 5, 7, 8, 9, 10)' = 18.17$, $C'_{43} = (8, 9, 10, 11, 12, 13, 14, 15, 16)' = 36.82$
$C'_{44} = (-3, -2, -1, 1, 2, 3, 4, 7, 10)' = 13.89$, $C'_{45} = (-3, -2, -1, 1, 3, 5, 7, 9, 11)' = 17.32$

$C'_{51} = (4,5,6,7,10,12,14,15,17)' = 32.86$, $C'_{52} = (-2,-1,0,1,2,3,4,5,6)' = 9.80$
$C'_{53} = (-1,0,1,2,3,4,5,6,7)' = 11.87$, $C'_{54} = (2,4,5,6,7,8,9,11,13)' = 23.77$
$C'_{55} = (2,3,4,5,6,7,10,11,12)' = 22.45$
The new cost table is

-	J1	J2	J3	J4	J5
P1	12.37	15.20	17.26	22.38	9.80
P2	24.56	36.82	24.17	28.09	56.73
P3	9.80	19.60	25.08	20.22	17.44
P4	32.25	18.17	36.82	13.89	17.32
P5	32.86	9.80	11.87	23.77	22.45

Using Hungarian Algorithm, the optimal allocations are

-	J1	J2	J3	J4	J5
P1	2.57	5.4	7.46	12.58	**0**
p2	0.39	12.65	**0**	3.92	32.56
p3	**0**	9.8	15.28	10.42	7.64
p4	18.36	4.28	22.93	**0**	3.43
p5	23.06	**0**	2.07	13.97	12.65

The optimal schedule is $P1 \rightarrow J5, P2 \rightarrow J3, P3 \rightarrow J1, P4 \rightarrow J4, P5 \rightarrow J2$
The optimum assignment cost is 9.80 + 24.17 + 9.80 + 13.89 + 9.80 = 67.46
(-4,-2,-1,0,1,2,3,5,6) + (2,3,4,5,6,9,10,12,13) + (-3,-2,-1,0,1,2,4,5,6)
+ (-3,-2,-1,1,2,3,4,7,10) + (-2,-1,0,1,2,3,4,5,6) = (-10,-4,0,7,12,19,25,34,41).
Comparing the assignment cost which has been found in the above example with assignment cost calculated by existing method is minimum.

6 Conclusion

In this paper the fuzzy costs of nonagonal fuzzy assignment problem have been

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defuzzified into crisp value by using root mean square formula and then solved by Hungarian method. We hope that this approach will be effective in fuzzy assignment problem involving imprecise data. Not only the nonagonal fuzzy numbers, we can use this method for type of fuzzy numbers like pentagon, hexagon etc.

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