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# A note on Polynomial Centro-Symmetric Hesitant Fuzzy Matrices

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**Abstract**

Classical and its development towards fuzzy got a wide acceptance due to its divergent real-life applications. Matrix theory is a special tool that is widely used in solving problems of different mathematical situations. Matrix theory can be used in versatile ways in the new concept. Crisis with fuzzy uncertainties are dealt in a good manner with fuzzy matrices. Hesitant Fuzzy Set in an excellent tool for a decision maker to analyze a situation with various possible values. Apart from usual matrices, there exist several useful matrices. Some of them are very helpful in solving complicated practical problems than the existing typical ones! This paper introduces polynomial centro-symmetric hesitant fuzzy matrix with basic properties.

**Key Words:** Centro-Symmetric Matrix, Fuzzy Matrix, Hesitant Fuzzy Matrix, Polynomial Centro-Symmetric Fuzzy Matrix, Polynomial Centro-symmetric hesitant fuzzy matrix.

**AMS Classification: 11C20, 11C20, 52B15, 58J53**

**Notations :**FS - Fuzzy Set, HFS - Hesitant Fuzzy Set, HFM - Hesitant Fuzzy Matrix, HFD - Hesitant Fuzzy Determinant, SM - Symmetric Matrix, CSM - Centro-symmetric Matrix, FM - Fuzzy Matrix, SFM - Square Fuzzy Matrix, FMD - Fuzzy Matrix Determinant, PM - Polynomial Matrix, PCSM - Polynomial Centro-Symmetric Matrix.

## 1 Introduction

Latest extension of set theory is developed in 1965, by American Computer Scientist and father of soft computing, Lotfi Aliasker Zadeh [23]. This set is mostly used in the areas of real life application where the element attains an imprecise degree of membership.

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Hesitant Fuzzy Set was introduced by Swedish mathematician Vincent Torra and Japanian Mathematician, Narukawa [19] and it is capable to accommodate more than one membership value for single object. Its applications can be widely extended to neural network theory, artificial intelligence, machine learning and in different kinds of sophisticated decision making problems.

In 1858, father of matrices Arthur Cayley [6] introduced theoretical definition of matrix. In matrices numbers, symbols and expression are arranged in rows and columns. Applications of matrices can be extended to vast areas like mathematics, physics, engineering and in computer language. For the first time in 1977, Thomson [13] defined Fuzzy Matrix. Kim and Roush [6] developed the theories of fuzzy matrices as a continuation of Boolean matrices. Diophantus of Alexandria [1] is a Greek mathematician, known as father of polynomials. Aryabhatta [8] is the Father of Indian Polynomials. General use of Polynomial matrices [1] is modeling in application areas. Polynomial is word obtained by the words poly and nomial which means many terms.

Symmetric matrix must be a square matrix, then the matrix may be singular or not. Symmetric matrices and linear algebra is strongly connected. Centro-symmetric matrix is a type of matrix which was introduced in 1985 by Weaver, James R [23]. It is widely used in quantum mechanics, optics, graph theory etc. Definitions given by Ann lee [1, 7, 10, 11] in 1962 for centro-symmetric matrices and ideas of cross-symmetric matrices given by Graybill [23] have several similarities.

This paper aims to develop the following concepts as a preliminary work

- Centro-Symmetric Fuzzy Matrix
- Centro-Symmetric Hesitant Fuzzy Matrix
- Polynomial Hesitant Fuzzy Matrix
- Polynomial Centro-Symmetric Hesitant Fuzzy Matrix

As a final stage of the work, major concept polynomial centro-symmetric matrices with hesitant fuzzy matrices, by drawing the benefits of both! For the newly introduced matrix, its determinant and transpose are also introduced along with its elementary properties. A real-life application is also provided.

## 2 Preliminaries

Some basic ideas are discussed below

**Definition 2.1 (FS) [12]:** It is an expansion of crisp set with element membership lies between 0 and 1.

**Definition 2.2 (HFS) [24]:** H.F.S M is a set of ordered pairs $M = \{(x, h_M(x)): x \in X\}$, where $h_M$ is the following function: $h_M$ : X → P([0, 1])

**Definition 2.3 (SM) [3]:** $A = (a_{ij})_{(n\times n)}$ is symmetric if $a_{ij} = a_{ji}$ such that $A = A^T$, where $A^T$, is the transpose of A.

**Definition 2.4 (CSM) [1, 10, 11]:** C = $[C_{ij}]_{n\times n}$ is Centro-symmetric if $c_{ij} = c_{n-i+1,n-j+1}$

**Definition 2.5 (FM) [13]:** $A = (a_{ij})_{(n\times m)}$ is known as fuzzy matrix if its elements are ranging from 0 to 1.

**Definition 2.6 (SFM) [14]:** In definition 2.5, if $m = n$ it is called a square fuzzy matrix.

**Definition 2.7 (FMD) [2]:** The determinant of a Fuzzy Matrix can be calculated by enlarging each column or row of a fuzzy matrix.

**Definition 2.8 (HFM) [22]:** A H.F.M allows multiple membership degree for each element.

**Definition 2.9 (PM) [1]:** If all the entries of $A(\lambda)$ are polynomials then it is called polynomial matrix.

**Definition 2.10 (PCSM) [1]:** A square polynomial matrix $A(\lambda)$ is said to be Polynomial Centro symmetric if $A(\lambda)$=$A(\lambda)^T$.

## 3 Elementary operations of Hesitant Fuzzy Matrices

In this section some new elementary operations of hesitant fuzzy matrices are provided.

**Definition 3.1 (Addition, Multiplication, Determinant of HFM's)**
Consider a hesitant fuzzy matrix of order n

(i) HFM Addition($+$):

$$\{k_1, k_2, k_3 \cdots\} + \{l_1, l_2, l_3, \cdots\} = max\{k_1, k_2, k_3 \cdots, l_1, l_2, l_3, \cdots\}$$

(ii) HFM Multiplication($*$):

$$\{k_1, k_2, k_3 \cdots\} * \{l_1, l, l_3, \cdots\} = min\{k_1, k_2, k_3 \cdots, l_1, l_2, l_3, \cdots\}$$

(iii) HFD: Let $N = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$ be a hesitant fuzzy matrix

$$|N| = \begin{vmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{vmatrix} = k_{11} \begin{vmatrix} k_{22} & k_{23} \\ k_{32} & k_{33} \end{vmatrix} + k_{12} \begin{vmatrix} k_{21} & k_{23} \\ k_{31} & k_{33} \end{vmatrix} + k_{13} \begin{vmatrix} k_{21} & k_{22} \\ k_{31} & k_{32} \end{vmatrix}$$

$$= k_{11} * \max\{min\{k_{22}, k_{33}\}, min\{k_{32}, k_{23}\}\} + k_{12} * \max\{min\{k_{21}, k_{33}\}, min\{k_{31}, k_{23}\}\} + k_{13} * \max\{min\{k_{21}, k_{32}\}, min\{k_{31}, k_{22}\}\}$$

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## 4 Centro-Symmetric Fuzzy Matrix, Centro-Symmetric Hesitant Fuzzy Matrix

In this section some new concepts used as base for the $4^{th}$ section is provided

**Definition 4.1 (Centro-Symmetric Fuzzy Matrix):** A square fuzzy matrix which is symmetric about centre of its array of elements is called centro - symmetric.
Thus C = $[C_{ij}]_{n\times n}$ is Centro symmetric if $a_{ij}$= $a_{n-i+1,\ n-j+1}$, where $a_{ij} \in [0,1]$

**Example 4.2:** Let $N = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$

$k_{11} = k_{3-1+1\quad 3-1+1} = k_{33}$ ; $k_{12} = k_{3-1+1\quad 3-2+1} = k_{32}$ ; $k_{13} = k_{3-1+1\quad 3-3+1} = k_{31}$
$k_{21} = k_{3-2+1\quad 3-1+1} = k_{23}$ ; $\boldsymbol{k_{22}} = k_{3-2+1\quad 3-2+1} = \boldsymbol{k_{22}}$ ; $k_{23} = k_{3-2+1\quad 3-3+1} = k_{21}$
$k_{31} = k_{3-3+1\quad 3-1+1} = k_{13}$ ; $k_{32} = k_{3-3+1\quad 3-2+1} = k_{12}$ ; $k_{33} = k_{3-3+1\quad 3-3+1} = k_{11}$

Corresponding centro-symmetric matrix of N takes entries as follows: $\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & \boldsymbol{k_{22}} & k_{21} \\ k_{13} & k_{12} & k_{11} \end{bmatrix}$

$B = \begin{bmatrix} 0.1 & 0.5 & 0.6 \\ 0.2 & 0.9 & 0.2 \\ 0.6 & 0.5 & 0.1 \end{bmatrix}$ is a centro-symmetric fuzzy matrix.

**Definition 4.3 (Centro-Symmetric Hesitant Fuzzy Matrix):** C = $[C_{ij}]_{n\times n}$ is centro - symmetric hesitant fuzzy matrix if $k_{ij}$=$k_{n-i+1,\ n-j+1}$, where $k_{ij} \in [0,1]$

**Example 4.4:** $B = \begin{bmatrix} \{0.1, 0.2\} & \{0.5, 0.7, 0.4, 0.3\} & \{0.6, 0.8\} \\ \{0.4, 0.9, 0.1\} & \{0.3, 0.9, 0.5\} & \{0.4, 0.9, 0.1\} \\ \{0.6, 0.8\} & \{0.5, 0.7, 0.4, 0.3\} & \{0.1, 0.2\} \end{bmatrix}$ is a centro-symmetric hesitant fuzzy matrix

## 5 Polynomial Hesitant Fuzzy Matrix, Polynomial Centro- Symmetric Hesitant Fuzzy Matrix

In this section polynomial hesitant fuzzy matrix and polynomial centro-symmetric hesitant fuzzy matrix are introduced with an example

**Definition 5.1 (Polynomial Hesitant Fuzzy Matrix):** A matrix M$(\lambda)$ is a polynomial hesitant fuzzy matrix if all entries are polynomials whose coefficients have hesitant fuzzy degrees.

**Example 5.2:** Let $M(\lambda)$
=

$$\begin{bmatrix} \{0.2,0.5\}\lambda^2 + \{0.1,0.2\}\lambda + \{0.3,0.5,0.6,0.8\} & \{0.9,0.1\}\lambda^2 + \{0.5,0.9,0.7,0.2\}\lambda + \{0.1,0.8\} & \{0.7,0.1,0.2\}\lambda^2 + \{0.2,0.4,0.8,0.9\}\lambda + \{0.5,0.6,0.1,0.2\} \\ \{0.3,0.9,0.1,0.6,0.1\}\lambda^2 + \{0.5,0.8,0.9,0.1\}\lambda + \{0.2,0.3,0.6,0.7\} & \{0.4,0.6,0.1\}\lambda^2 + \{0.5,0.8,0.1,0.2\}\lambda + \{0.9,0.7,0.1,0.3\} & \{0.3,0.4,0.8,0.9\}\lambda^2 + \{0.6,0.1\}\lambda + \{0.5,0.1,0.4\} \\ \{0.8,0.7,0.3\}\lambda^2 + \{0.3,0.2\}\lambda + \{0.4,0.9,0.8,0.1\} & \{0.1,0.5,0.8\}\lambda^2 + \{0.7,0.8\}\lambda + \{0.7,0.8,0.5,0.2\} & \{0.5,0.2\}\lambda^2 + \{0.4,0.9,0.2,0.1\}\lambda + \{0.7,0.2,0.1,0.6,0.9\} \end{bmatrix}$$

$$= M_2\lambda^2 + M_1\lambda + M_0$$

where $M_2 = \begin{bmatrix} \{0.2,0.5\} & \{0.9,0.1\} & \{0.7,0.1,0.2\} \\ \{0.3,0.9,0.1,0.6,0.1\} & \{0.4,0.6,0.1\} & \{0.3,0.4,0.8,0.9\} \\ \{0.8,0.7,0.3\} & \{0.1,0.5,0.8\} & \{0.5,0.2\} \end{bmatrix}$, $M_1 = \begin{bmatrix} \{0.1,0.2\} & \{0.5,0.9,0.7,0.2\} & \{0.2,0.4,0.8,0.9\} \\ \{0.5,0.8,0.9,0.1\} & \{0.5,0.8,0.1,0.2\} & \{0.6, 0.1\} \\ \{0.3,0.2\} & \{0.7,0.8\} & \{0.4,0.9,0.2,0.1\} \end{bmatrix}$,

$$M_0 = \begin{bmatrix} \{0.3,0.5,0.6,0.8\} & \{0.1,0.8\} & \{0.5,0.6,0.1,0.2\} \\ \{0.2,0.3,0.6,0.7\} & \{0.9,0.7,0.1,0.3\} & \{0.5,0.1,0.4\} \\ \{0.4,0.9,0.8,0.1\} & \{0.7,0.8,0.5,0.2\} & \{0.7,0.2,0.1,0.6,0.9\} \end{bmatrix}$$

**Definition 5.3 (Polynomial Centro-Symmetric Hesitant Fuzzy Matrix):** A square polynomial matrix M$(\lambda)$ is Polynomial Centro symmetric hesitant fuzzy matrix if M$(\lambda)$=$M(\lambda)^T$, where $M(\lambda)$ is a polynomial hesitant fuzzy matrix.

**Example 5.4:** Let $H(\lambda)$

$$= \begin{bmatrix} \{0.2,0.5\}\lambda^2 + \{0.1,0.2\}\lambda + \{0.3,0.5,0.6,0.8\} & \{0.9,0.1\}\lambda^2 + \{0.5,0.9,0.7,0.2\}\lambda + \{0.1,0.8\} & \{0.7,0.1,0.2\}\lambda^2 + \{0.2,0.4,0.8,0.9\}\lambda + \{0.5,0.6,0.1,0.2\} \\ \{0.3,0.9,0.1,0.6,0.1\}\lambda^2 + \{0.5,0.8,0.9,0.1\}\lambda + \{0.2,0.3,0.6,0.7\} & \{0.4,0.6,0.1\}\lambda^2 + \{0.5,0.8,0.1,0.2\}\lambda + \{0.9,0.7,0.1,0.3\} & \{0.3,0.9,0.1,0.6,0.1\}\lambda^2 + \{0.5,0.8,0.9,0.1\}\lambda + \{0.2,0.3,0.6,0.7\} \\ \{0.7,0.1,0.2\}\lambda^2 + \{0.2,0.4,0.8,0.9\}\lambda + \{0.5,0.6,0.1,0.2\} & \{0.9,0.1\}\lambda^2 + \{0.5,0.9,0.7,0.2\}\lambda + \{0.1,0.8\} & \{0.2,0.5\}\lambda^2 + \{0.1,0.2\}\lambda + \{0.3,0.5,0.6,0.8\} \end{bmatrix}$$

$$= M_2\lambda^2 + M_1\lambda + M_0$$

where $M_2 = \begin{bmatrix} \{0.2,0.5\} & \{0.9,0.1\} & \{0.7,0.1,0.2\} \\ \{0.3,0.9,0.1,0.6,0.1\} & \{0.4,0.6,0.1\} & \{0.3,0.9,0.1,0.6,0.1\} \\ \{0.7,0.1,0.2\} & \{0.9,0.1\} & \{0.2,0.5\} \end{bmatrix}$, $M_1 = \begin{bmatrix} \{0.1,0.2\} & \{0.5,0.9,0.7,0.2\} & \{0.2,0.4,0.8,0.9\} \\ \{0.5,0.8,0.9,0.1\} & \{0.5,0.8,0.1,0.2\} & \{0.5,0.8,0.9,0.1\} \\ \{0.2,0.4,0.8,0.9\} & \{0.5,0.9,0.7,0.2\} & \{0.1,0.2\} \end{bmatrix}$,

$$M_0 = \begin{bmatrix} \{0.3,0.5,0.6,0.8\} & \{0.1,0.8\} & \{0.5,0.6,0.1,0.2\} \\ \{0.2,0.3,0.6,0.7\} & \{0.9,0.7,0.1,0.3\} & \{0.2,0.3,0.6,0.7\} \\ \{0.5,0.6,0.1,0.2\} & \{0.1,0.8\} & \{0.3,0.5,0.6,0.8\} \end{bmatrix}$$

## 6 Determinant of a Polynomial Centro-Symmetric Hesitant Fuzzy Matrix

In this section determinant of a Centro- symmetric hesitant fuzzy matrix is discussed with example

**Definition 6.1 (Determinant of a Centro-symmetric fuzzy matrix):** Determinant is defined as follows:

Let $M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{21} \\ m_{13} & m_{12} & m_{11} \end{bmatrix}$ be a Centro-symmetric fuzzy matrix. Its determinant is defined as follows:

$$|M| = \begin{vmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{21} \\ m_{13} & m_{12} & m_{11} \end{vmatrix} = m_{11}\begin{vmatrix} m_{22} & m_{21} \\ m_{12} & m_{11} \end{vmatrix} + m_{12}\begin{vmatrix} m_{21} & m_{21} \\ m_{13} & m_{11} \end{vmatrix} + m_{13}\begin{vmatrix} m_{21} & m_{22} \\ m_{13} & m_{12} \end{vmatrix}$$

$$= m_{11} * \max\{min\{m_{22}, m_{11}\}, min\{m_{12}, m_{21}\}\} + m_{12} * \max\{min\{m_{21}, m_{11}\}, min\{m_{13}, m_{21}\}\} + m_{13} * \max\{min\{m_{21}, m_{12}\}, min\{m_{13}, m_{22}\}\}$$

**Example 6.2:** Let $M = \begin{bmatrix} 0.1 & 0.5 & 0.6 \\ 0.2 & 0.9 & 0.2 \\ 0.6 & 0.5 & 0.1 \end{bmatrix}$ is a centro-symmetric fuzzy matrix

$$|M| = 0.1\begin{vmatrix} 0.9 & 0.2 \\ 0.5 & 0.1 \end{vmatrix} + 0.5\begin{vmatrix} 0.2 & 0.2 \\ 0.6 & 0.1 \end{vmatrix} + 0.6\begin{vmatrix} 0.2 & 0.9 \\ 0.6 & 0.5 \end{vmatrix} = 0.6$$

**Remark 6.3:** Determinant of a centro-symmetric fuzzy matrix and its transpose are the same.

**Example 6.4:** $M^T = \begin{bmatrix} 0.1 & 0.2 & 0.6 \\ 0.5 & 0.9 & 0.5 \\ 0.6 & 0.2 & 0.1 \end{bmatrix}$

$|M^T| = 0.1 \begin{vmatrix} 0.9 & 0.5 \\ 0.2 & 0.1 \end{vmatrix} + 0.2 \begin{vmatrix} 0.5 & 0.5 \\ 0.6 & 0.1 \end{vmatrix} + 0.6 \begin{vmatrix} 0.5 & 0.9 \\ 0.6 & 0.2 \end{vmatrix} = 0.2 + 0.5 + 0.6 = 0.6$

**Definition 6.5 (Determinant of a Centro-symmetric hesitant fuzzy matrix):**

Determinant of a $(3 \times 3)$ centro-symmetric hesitant fuzzy matrix is defined as follows:

Let $M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{21} \\ m_{13} & m_{12} & m_{11} \end{bmatrix}$ be a centro-symmetric hesitant fuzzy matrix. Its determinant is defined as follows:

$$|M| = \begin{vmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{21} \\ m_{13} & m_{12} & m_{11} \end{vmatrix} = m_{11} \begin{vmatrix} m_{22} & m_{21} \\ m_{12} & m_{11} \end{vmatrix} + m_{12} \begin{vmatrix} m_{21} & m_{21} \\ m_{13} & m_{11} \end{vmatrix} + m_{13} \begin{vmatrix} m_{21} & m_{22} \\ m_{13} & m_{12} \end{vmatrix}$$

$$= m_{11} * \max\{min\{m_{22}, m_{11}\}, min\{m_{12}, m_{21}\}\} + m_{12} * \max\{min\{m_{21}, a_{11}\}, min\{m_{13}, m_{21}\}\} + m_{13} * \max\{min\{m_{21}, m_{12}\}, min\{m_{13}, m_{22}\}\}$$

**Example 6.6:** To find the determinant of example 5.4

$$|M_2| = \begin{vmatrix} \{0.2,0.5\} & \{0.9,0.1\} & \{0.7,0.1,0.2\} \\ \{0.3,0.9,0.1,0.6,0.1\} & \{0.4,0.6,0.1\} & \{0.3,0.9,0.1,0.6,0.1\} \\ \{0.7,0.1,0.2\} & \{0.9,0.1\} & \{0.2,0.5\} \end{vmatrix}$$

$$= \{0.2,0.5\} \begin{vmatrix} \{0.4,0.6,0.1\} & \{0.3,0.9,0.1,0.6,0.1\} \\ \{0.9,0.1\} & \{0.2,0.5,0.7\} \end{vmatrix} +$$

$$\{0.9,0.1\} \begin{vmatrix} \{0.3,0.9,0.1,0.6,0.1\} & \{0.3,0.9,0.1,0.6,0.1\} \\ \{0.7,0.1,0.2\} & \{0.2,0.5\} \end{vmatrix} +$$

$$\{0.7,0.1,0.2\} \begin{vmatrix} \{0.3,0.9,0.1,0.6,0.1\} & \{0.4,0.6,0.1\} \\ \{0.7,0.1,0.2\} & \{0.9,0.1\} \end{vmatrix}$$

$$= \{0.2,0.5\} * \{0.1\} + \{0.9,0.1\} * \{0.1\} + \{0.7,0.1,0.2\} * \{0.1\} = 0.1$$

$$|M_1| = \begin{vmatrix} \{0.1,0.2\} & \{0.5,0.9,0.7,0.2\} & \{0.2,0.4,0.8,0.9\} \\ \{0.5,0.8,0.9,0.1\} & \{0.5,0.8,0.1,0.2\} & \{0.5,0.8,0.9,0.1\} \\ \{0.2,0.4,0.8,0.9\} & \{0.5,0.9,0.7,0.2\} & \{0.1,0.2\} \end{vmatrix}$$

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$$= \{0.1,0.2\}\begin{vmatrix} \{0.5,0.8,0.1,0.2\} & \{0.5,0.8,0.9,0.1\} \\ \{0.5,0.9,0.7,0.2\} & \{0.1,0.2\} \end{vmatrix} +$$

$$\{0.5,0.9,0.7,0.2\}\begin{vmatrix} \{0.5,0.8,0.9,0.1\} & \{0.5,0.8,0.9,0.1\} \\ \{0.2,0.4,0.8,0.9\} & \{0.1,0.2\} \end{vmatrix} +$$

$$\{0.2,0.4,0.8,0.9\}\begin{vmatrix} \{0.5,0.8,0.9,0.1\} & \{0.5,0.8,0.1,0.2\} \\ \{0.2,0.4,0.8,0.9\} & \{0.5,0.9,0.7,0.2\} \end{vmatrix}$$

$$= \{0.1,0.2\} * \{0.1\} + \{0.5,0.9,0.7,0.2\} * \{0.1\} + \{0.2,0.4,0.8,0.9\} * \{0.1\} = 0.1$$

$$|\mathrm{M}_0| = \begin{vmatrix} \{0.3,0.5,0.6,0.8\} & \{0.1,0.8\} & \{0.5,0.6,0.1,0.2\} \\ \{0.2,0.3,0.6,0.7\} & \{0.9,0.7,0.1,0.3\} & \{0.2,0.3,0.6,0.7\} \\ \{0.5,0.6,0.1,0.2\} & \{0.1,0.8\} & \{0.3,0.5,0.6,0.8\} \end{vmatrix}$$

$$= \{0.3,0.5,0.6,0.8\}\begin{vmatrix} \{0.9,0.7,0.1,0.3\} & \{0.2,0.3,0.6,0.7\} \\ \{0.1,0.8\} & \{0.3,0.5,0.6,0.8\} \end{vmatrix} +$$

$$\{0.1,0.8\}\begin{vmatrix} \{0.2,0.3,0.6,0.7\} & \{0.2,0.3,0.6,0.7\} \\ \{0.2,0.3,0.6,0.7\} & \{0.3,0.5,0.6,0.8\} \end{vmatrix} +$$

$$\{0.5,0.6,0.1,0.2\}\begin{vmatrix} \{0.2,0.3,0.6,0.7\} & \{0.9,0.7,0.1,0.3\} \\ \{0.5,0.6,0.1,0.2\} & \{0.1,0.8\} \end{vmatrix}$$

$$= \{0.3,0.5,0.6,0.8\} * \{0.1\} + \{0.1,0.8\} * \{0.2\} + \{0.5,0.6,0.1,0.2\} * \{0.1\} = 0.1$$

## 7. Real Life Application of Polynomial Centro-Symmetric Hesitant Fuzzy Matrix

In this section one real-life application is provided based on the new concept. Nowadays human-elephant conflict is increasing in a higher level. Three problematic elephants are selected from 3 different states of India, namely Kerala, Tamil-Nadu and Karnataka and an analysis has done on this problem using the new tool introduced in this paper.

Let$E_1 = Arikomban$, $E_2 = Karuppan$ and $E_3 = Belur\ Megna$ are the three elephants in news recently because of some issues happened.

Let $R_1 = Habitat\ Loss$,

$R_2 = Anti - Social\ activities\ inside\ forest$,

$R_3 = Encroachment(for\ construction\ of\ roads, railwaylines\ in\ elephant\ corridors\ \&\ farming\ inside\ forest\ areas.)$

**Step 1:** Consider a **elephant-reason** polynomial Centro-symmetric hesitant fuzzy matrix of order $(3 \times 3)$

$$\begin{matrix} & R_1 & R_2 & R_3 \end{matrix}$$

$$K = \begin{matrix} E_1 \\ E_2 \\ E_3 \end{matrix}\begin{bmatrix} \{0.1,0.2,0.3\}\lambda^2 + \{0.3,0.4\}\lambda + \{0.7,0.8\} & \{0.6,0.8,0.9\}\lambda^2 + \{0.2,0.6\}\lambda + \{0.4,0.5\} & \{0.3,0.4,0.5\}\lambda^2 + \{0.2,0.5\}\lambda + \{0.5,0.6\} \\ \{0.2,0,7\}\lambda^2 + \{0.6,0.7,0.9\}\lambda + \{0.2,0.5,0.7\} & \{0.3,0,4\}\lambda^2 + \{0.5,0.8,0.9\}\lambda + \{0.7,0.8,1\} & \{0.2,0,7\}\lambda^2 + \{0.6,0.7,0.9\}\lambda + \{0.2,0.5,0.7\} \\ \{0.3,0.4,0.5\}\lambda^2 + \{0.2,0.5\}\lambda + \{0.5,0.6\} & \{0.6,0.8,0.9\}\lambda^2 + \{0.2,0.6\}\lambda + \{0.4,0.5\} & \{0.1,0.2,0.3\}\lambda^2 + \{0.3,0.4\}\lambda + \{0.7,0.8\} \end{bmatrix}$$

**Step 2:** Consider a **state - elephant** polynomial $K^c$ Centro-symmetric hesitant fuzzy matrix of order $(3 \times 3)$

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$$\begin{array}{cc} & \begin{matrix} E_1 & E_2 & E_3 \end{matrix} \\ L = \begin{matrix} S_1 \\ S_2 \\ S_3 \end{matrix} & \begin{bmatrix} \{0.2,0.5\}\lambda^2 + \{0.6,0.9,0.3\}\lambda + \{0.1,0.4\} & \{0.7,0.5,0.8\}\lambda^2 + \{0.3,0.2\}\lambda + \{0.9,0.4,0.7\} & \{0.8,0.3,0.5\}\lambda^2 + \{0.2,0.5,0.9\}\lambda + \{0.4,0.7\} \\ \{0.9,0.5\}\lambda^2 + \{0.7,0.1,0.3\}\lambda + \{0.6,0.8\} & \{0.1,0.4\}\lambda^2 + \{0.6,0.9\}\lambda + \{0.2,0.5,0.9\} & \{0.9,0.5\}\lambda^2 + \{0.7,0.1,0.3\}\lambda + \{0.6,0.8\} \\ \{0.8,0.3,0.5\}\lambda^2 + \{0.2,0.5,0.9\}\lambda + \{0.4,0.7\} & \{0.7,0.5,0.8\}\lambda^2 + \{0.3,0.2\}\lambda + \{0.9,0.4,0.7\} & \{0.2,0.5\}\lambda^2 + \{0.6,0.9,0.3\}\lambda + \{0.1,0.4\} \end{bmatrix} \end{array}$$

**Step 3:** Compute T= $Ko^iL$

$$= \begin{bmatrix} 0.3\lambda^2 + 0.3\lambda + 0.4 & 0.3\lambda^2 + 0.2\lambda + 0.4 & 0.2\lambda^2 + 0.2\lambda + 0.4 \\ 0.3\lambda^2 + 0.6\lambda + 0.6 & 0.5\lambda^2 + 0.5\lambda + 0.4 & 0.3\lambda^2 + 0.6\lambda + 0.6 \\ 0.2\lambda^2 + 0.2\lambda + 0.4 & 0.3\lambda^2 + 0.2\lambda + 0.4 & 0.3\lambda^2 + 0.3\lambda + 0.4 \end{bmatrix}$$

**Step 4:** Complement matrix of $K$ and $L$

$$K^c = \begin{bmatrix} \{0.9,0.8,0.7\}\lambda^2 + \{0.7,0.6\}\lambda + \{0.3,0.2\} & \{0.4,0.2,0.1\}\lambda^2 + \{0.8,0.4\}\lambda + \{0.6,0.2\} & \{0.7,0.6,0.5\}\lambda^2 + \{0.8,0.5\}\lambda + \{0.5,0.4\} \\ \{0.8,0.3\}\lambda^2 + \{0.4,0.3,0.1\}\lambda + \{0.8,0.5,0.3\} & \{0.7,0.6\}\lambda^2 + \{0.5,0.2,0.1\}\lambda + \{0.3,0.2,0\} & \{0.8,0.3\}\lambda^2 + \{0.4,0.3,0.1\}\lambda + \{0.8,0.5,0.3\} \\ \{0.7,0.6,0.5\}\lambda^2 + \{0.8,0.5\}\lambda + \{0.5,0.4\} & \{0.4,0.2,0.1\}\lambda^2 + \{0.8,0.4\}\lambda + \{0.6,0.2\} & \{0.9,0.8,0.7\}\lambda^2 + \{0.7,0.6\}\lambda + \{0.3,0.2\} \end{bmatrix}$$

$$L^c = \begin{bmatrix} \{0.8,0.5\}\lambda^2 + \{0.4,0.1,0.7\}\lambda + \{0.9,0.6\} & \{0.4,0.5,0.2\}\lambda^2 + \{0.7,0.8\}\lambda + \{0.1,0.6,0.3\} & \{0.2,0.7,0.5\}\lambda^2 + \{0.8,0.5,0.1\}\lambda + \{0.6,0.3\} \\ \{0.1,0.5\}\lambda^2 + \{0.3,0.9,0.7\}\lambda + \{0.4,0.2\} & \{0.9,0.6\}\lambda^2 + \{0.4,0.1\}\lambda + \{0.8,0.5,0.1\} & \{0.1,0.5\}\lambda^2 + \{0.3,0.9,0.7\}\lambda + \{0.4,0.2\} \\ \{0.2,0.7,0.5\}\lambda^2 + \{0.8,0.5,0.1\}\lambda + \{0.6,0.3\} & \{0.4,0.5,0.2\}\lambda^2 + \{0.7,0.8\}\lambda + \{0.1,0.6,0.3\} & \{0.8,0.5\}\lambda^2 + \{0.4,0.1,0.7\}\lambda + \{0.9,0.6\} \end{bmatrix}$$

**Step 5:** Compute composition matrix $T = K^C \overset{i}{o} L^C$.

**Step 6:** Compute $V = C(-)T$, which represents relation between the reason for conflicts of elephants in each state

**Step 7:** Compute relativity values and comparison matrix

$$f\left(\frac{S_i}{R_i}\right) = \frac{\mu^h_{R_i}(S_i) - \mu^h_{S_i}(R_i)}{max\left\{\mu^h_{R_i}(S_i),\ \mu^h_{S_i}(R_i)\right\}}$$

$$f\left(\frac{S_1}{R_1}\right) = 0;\ f\left(\frac{S_1}{R_2}\right) = \frac{-0.1\lambda^2 - 0.2\lambda}{0.3\lambda^2 + 0.3\lambda + 0.2};\ f\left(\frac{S_1}{R_3}\right) = 0$$

$$f\left(\frac{S_2}{R_1}\right) = \frac{0.1\lambda^2 + 0.2\lambda}{0.3\lambda^2 + 0.3\lambda + 0.2};\ f\left(\frac{S_2}{R_2}\right) = 0;\ f\left(\frac{S_2}{R_3}\right) = \frac{0.1\lambda^2 + 0.2\lambda}{0.3\lambda^2 + 0.3\lambda + 0.2}$$

$$f\left(\frac{S_3}{R_1}\right) = 0;\ f\left(\frac{S_3}{R_2}\right) = \frac{-0.1\lambda^2 - 0.2\lambda}{0.3\lambda^2 + 0.3\lambda + 0.2};\ f\left(\frac{S_3}{R_3}\right) = 0$$

$$\therefore \text{Comparison matrix: } R = \begin{bmatrix} 0 & \frac{0.1\lambda^2 - 0.2\lambda}{0.3\lambda^2 + 0.3\lambda + 0.2} & 0 \\ \frac{0.1\lambda^2 + 0.2\lambda}{0.3\lambda^2 + 0.3\lambda + 0.2} & 0 & \frac{0.1\lambda^2 + 0.2\lambda}{0.3\lambda^2 + 0.3\lambda + 0.2} \\ 0 & \frac{0.1\lambda^2 - 0.2\lambda}{0.3\lambda^2 + 0.3\lambda + 0.2} & 0 \end{bmatrix}$$

Maximum of $i^{th}\ row$ (i=1, 2, 3) = $\begin{bmatrix} R_2 \\ R_1, R_3 \\ R_2 \end{bmatrix}$

It is found from this analysis that, reason for elephant-human conflict in Kerala and Karnataka States is mostly due to anti-social activities and that of Tamil Nadu are habitat loss and encroachment. Being counted as an animal of IUCN red list, each of the governments should have to take necessary actions to protect this endangered species and have to give awareness among young generations to attain sustainability goal 15 (Year - 2023).

**8 Conclusion**

Real life situations can be easily converted to polynomials. Matrices are the simple tools which could be made use in computer languages. Being a hybrid representation of both these, polynomial matrices which serve more efficiently in day-to-day life situations. Polynomial matrices in fuzzy matrix theory is a novel concept. This paper aims to introduce the idea of centro-symmetry in polynomial hesitant fuzzy matrix. Its determinant is discussed with examples. A real life application is also provided.

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