ISSN: 2456-8686, Volume 2, Issue 2, 2018:20-29
DOI : http://doi.org/10.26524/cm36

Journal of Computational Mathematica

Open Access

Fibonacci Sequence and its Sum by Second Order Difference Operator with Polynomial Factorial

Sandra Pinelas[1], Rajiniganth P[2*]
[1,2]Panimalar Institute of Technology, Chennai, Tamil Nadu 600123.

Abstract

In this paper, we introduce second order difference operator with polynomial factorial and its inverse by which we obtain advanced Fibonacci sequence and its sum. Some theorems and interesting results on the sum of the terms of second order Fibonacci sequence are derived. Suitable examples are provided to illustrate our results and verified by MATLAB.

Key words: Difference operator,polynomial factorial, Fibonacci sequence, Closed form solution, Fibonacci summation.

AMS classification: 39A70, 39A10, 47B39, 65J10, 65Q10.

1. Introduction

In 1984, Jerzy Popenda [5] introduced a particular type of difference operator on $u(k)$ as $\Delta_\alpha u(k) = u(k+1) - \alpha u(k)$. In 1989, Miller and Rose [8] introduced the discrete analogue of the Riemann-Liouville fractional derivative and its inverse $\Delta_h^{-\nu} f(t)$ ([1, 4]). The sum of m^{th} partial sums of products of higher powers of arithmetic and geometric progressions are derived in [9] by replacing h by ℓ, ν by m and $f(t)$ by $u(k)$ in $\Delta_h^{-\nu} f(t)$. In 2011, M.Maria Susai Manuel, et.al, [7] extended the operator Δ_α to generalized $\alpha-$difference operator as $\underset{\alpha(\ell)}{\Delta} v(k) = v(k+\ell) - \alpha v(k)$ for the real valued function $v(k)$. In 2014, G.Britto Antony Xavier, et.al, [2] introduced $q-$difference operator as $\Delta_q v(k) = v(qk) - v(k)$, $q \in (0, \infty)$ and obtained finite series solution to the corresponding generalized $q-$difference equation $\Delta_q v(k) = u(k)$. In this paper, we introduce second order difference operator by which we obtain second order polynomial factorial Fibonacci sequence and its sum. suitable examples verified by MATLAB are provided.

[1*]prajini.maths@gmail.com

ISSN: 2456-8686, Volume 2, Issue 2, 2018:20-29
DOI : http://doi.org/10.26524/cm36

2. Basic Definitions And Examples

Fibonacci and Lucas numbers cover a wide range of interest in modern mathematics as they appear in the comprehensive works of Koshy [6] and Vajda [10]. The $k-$Fibonacci sequence introduced by Falcon and Plaza [3] depends only on one integer parameter k and is defined as

$$F_{k,0} = 0, \quad F_{k,1} = 1 \quad \text{and} \quad F_{k,n+1} = kF_{k,n} + F_{k,n-1}, \quad \text{where} \quad n \geq 1,\, k \geq 1.$$

In particular, if $k = 2$, the Pell sequence is obtained as

$$P_0 = 0, \quad P_1 = 1 \quad \text{and} \quad P_{n+1} = 2P_n + P_{n-1} \quad \text{for} \quad n \geq 1.$$

Here we introduce second order difference operator with polynomial factorial $\underset{\overline{\alpha}(k)}{\Delta} v(k) = v(k) - \alpha_1 k^{(p)} v(k-1) - \alpha_2 k^{(q)} v(k-2)$, where $\overline{\alpha}(k) = \big(\alpha_1 k^{(p)}, \alpha_1 k^{(q)}\big)$ which generates $\overline{\alpha}(k)$-Fibonacci sequence and its sum.

Definition 2.1 For $k \in [0, \infty)$, $\overline{\alpha}(k)$-Fibonacci sequence is defined as,

$$F_0 = 1,\ F_1 = \alpha_1 k^{(p)},\ F_n = \alpha_1[k-(n-1)]^{(p)} F_{n-1} + \alpha_2[k-(n-2)]^{(q)} F_{n-2},\ n \geq 2 \quad (1)$$

Example 2.2 (i) Taking $k = 3$,$\alpha_1 = 2$, $\alpha_2 = 3$, $p = 2$ and $q = 3$ in (1), we get a Fibonacci sequence $\{1, 12, 66, \cdots\}$.
(ii) When $k = 12$,$\alpha_1 = 2.22$, $\alpha_2 = 0.333$, $p = 3$ and $q = 2$ in (1), we have a Fibonacci sequence $\{1, 2930.40, 6440477.08, 10294565898.83, \cdots\}$.

Similarly, one can obtain Fibonacci sequences corresponding to each pair $(\alpha_1 k^{(p)}, \alpha_2 k^{(q)}) \in \mathbb{R}^2$.

Definition 2.3 A second order difference operator with polynomial factorial on $v(k)$, denoted as $\underset{\overline{\alpha}(k)}{\Delta} v(k)$, where $\overline{\alpha}(k) = \big(\alpha_1 k^{(p)}, \alpha_1 k^{(q)}\big)$, is defined as

$$\underset{\overline{\alpha}(k)}{\Delta} v(k) = v(k) - \alpha_1 k^{(p)} v(k-1) - \alpha_2 k^{(q)} v(k-2), \quad k \in [0, \infty), \quad (2)$$

and its inverse is defined as below;

$$\text{if} \quad \underset{\overline{\alpha}(k)}{\Delta} v(k) = u(k), \quad \text{then we write} \quad v(k) = \underset{\overline{\alpha}(k)}{\overset{-1}{\Delta}} u(k). \quad (3)$$

Lemma 2.4 Let $v(k)$ be a functions of $k \in (-\infty, \infty)$. Then we obtained

$$\underset{\overline{\alpha}(k)}{\overset{-1}{\Delta}} a^{sk}\big[1 - \frac{\alpha_1 k^{(p)}}{a} - \frac{\alpha_2 k^{(q)}}{a^{2s}}\big] = a^{sk}. \quad (4)$$

[1*]prajini.maths@gmail.com

ISSN: 2456-8686, Volume 2, Issue 2, 2018:20-29
DOI : http://doi.org/10.26524/cm36

Proof Taking $u(k) = a^{sk}\big[1 - \frac{\alpha_1 k^{(p)}}{a^s} - \frac{\alpha_2 k^{(q)}}{a^{2s}}\big]$ in (2), we obtained $\underset{\overline{\alpha}(k)}{\Delta} a^{sk} = a^{sk}\big[1 - \frac{\alpha_1 k^{(p)}}{a^s} - \frac{\alpha_2 k^{(q)}}{a^{2s}}\big]$. Now (4) follows from (3).

Corollary 2.5 If $\alpha_1 = 1 = \alpha_2$ in lemma 2.4, then we obtain

$$\overset{-1}{\underset{\mu}{\Delta}} a^{sk}\big[1 - \frac{k^{(p)}}{a^s} - \frac{k^{(q)}}{a^{2s}}\big] = a^{sk}. \tag{5}$$

Proof Taking $u(k) = a^{sk}\big[1 - \frac{k^{(p)}}{a^s} - \frac{k^{(q)}}{a^{2s}}\big]$ in (2), we have $\underset{\mu}{\Delta} a^{sk} = a^{sk}\big[1 - \frac{k^{(p)}}{a^s} - \frac{k^{(q)}}{a^{2s}}\big]$. Now (5) follows from (3).

Lemma 2.6 Let e^{-sk} be a function of $k \in (-\infty, \infty)$. Then we have

$$\overset{-1}{\underset{\overline{\alpha}(k)}{\Delta}} e^{-sk}\big[1 - \alpha_1 k^{(p)} e^s - \alpha_2 k^{(q)} e^{2s}\big] = e^{-sk}. \tag{6}$$

Proof The proof follows by assuming $a = e^{-1}$ in (4).

Corollary 2.7 Let e^{-sk} be a function of $k \in (-\infty, \infty)$, then we obtained

$$\overset{-1}{\underset{\mu}{\Delta}} e^{-sk}\big[1 - k^{(p)} e^s - k^{(q)} e^{2s}\big] = e^{-sk}. \tag{7}$$

Proof The proof follows by taking $\alpha_1 = 1 = \alpha_2$ in lemma 2.6.

3.$\overline{\alpha}(k)$-Fibonacci Sequence And its Sum

In this section, we derive sum of the value of $\overline{\alpha}(k)$-Fibonacci Sequence by inverse of $\overline{\alpha}(k)$ difference operator.

Theorem 3.1 If $v(k) = \overset{-1}{\underset{\overline{\alpha}(k)}{\Delta}} u(k)$, $F_0 = 1, F_1 = \alpha_1 k^{(p)}$ and $F_{n+1} = \alpha_1 (k-n)^{(r)} F_n + \alpha_2 (k-(n-1))^{(s)} F_{n-1}$, for $i = 0, 1, 2, ...$ then we have

$$v(k) - F_{n+1} v(k-(n+1)) - \alpha_2 (k-n)^{(q)} F_n v(k-(n+2)) = \sum_{i=0}^{n} F_i u(k-i). \tag{8}$$

1*prajini.maths@gmail.com

ISSN: 2456-8686, Volume 2, Issue 2, 2018:20-29
DOI : http://doi.org/10.26524/cm36

Proof From (2) and (3), we arrive

$$v(k) = u(k) + \alpha_1 k^{(p)} v(k-1) + \alpha_2 k^{(q)} v(k-2). \tag{9}$$

Replacing k by $k-1$ and then substituting the value of $v(k-1)$ in (9), we get
$v(k) = u(k) + \alpha_1 k^{(p)} \big[u(k-1) + \alpha_1 (k-1)^{(r)} v(k-2) + \alpha_2 (k-1)^{(s)} v(k-3)\big] + \alpha_2 k^{(p)} v(k-3)$
which gives

$$v(k) = F_0 u(k) + F_1 u(k-1) + F_2 v(k-2) + \alpha_2 (k-1)^{(s)} F_1 v(k-3), \tag{10}$$

where F_0, F_1 and F_2 are given in (1).
Replacing k by $k-2$ in (9) and then substituting $v(k-2)$ in (10), we obtain
$$v(k) = F_0 u(k) + F_1 u(k-1) + F_2 u(k-2) + F_3 v(k-3) + \alpha_2 (k-2)^{(s)} F_2 v(k-4),$$
where F_3 is given in (1).
Repeating this process again and again, we get (8).

Corollary 3.2 If $\overset{-1}{\underset{\mu}{\Delta}} u(k) = v(k)$, $\mu = (k^{(p)}, k^{(q)})$, $F_0 = 1, F_1 = k^{(p)}$ and $F_{n+1} = (k-n)^{(r)} F_n + (k-(n-1))^{(s)} F_{n-1}$, for $i = 0, 1, 2, ...$ then

$$v(k) - F_{n+1} v(k-(n+1)) - (k-n)^{(q)} F_n v(k-(n+2)) = \sum_{i=0}^{n} \underset{i}{F}\, u(k-i). \tag{11}$$

Proof The proof follows by taking $\alpha_1 = 1 = \alpha_2$ in Theorem 3.1.

Corollary 3.3 If $v(k)$ is a closed form solution of the second order difference equation with polynomial factorial $\underset{\overline{\alpha}(k)}{\Delta} v(k) = a^{sk}\big[1 - \frac{\alpha_1 k^{(p)}}{a^s} - \frac{\alpha_2 k^{(q)}}{a^{2s}}\big]$,
then we obtain $a^{sk} - F_{n+1} a^{s(k-(n+1))} - \alpha_2 (k-n)^{(q)} F_n a^{s(k-(n+2))}$

$$= \sum_{i=0}^{n} F_i a^{s(k-i)} \big[1 - \frac{\alpha_1 (k-i)^{(p)}}{a^s} - \frac{\alpha_2 (k-i)^{(q)}}{a^{2s}}\big]. \tag{12}$$

Proof The proof follows by choosing $v(k) = a^{sk}$ in (8) and using (4).

The following example is an verification of (12).

Example 3.4 Taking $k = 3$, $n = 2$, $a = 2$, $\alpha_1 = 2$, $\alpha_2 = 3$, $s = 3$, $p = 2$ and

1*prajini.maths@gmail.com

ISSN: 2456-8686, Volume 2, Issue 2, 2018:20-29
DOI : http://doi.org/10.26524/cm36

$q = 3$ in (12), we get $2^9 - F_3 2^0 - 3F_2 2^{-1} 1^3 = \sum_{i=0}^{2} F_i a^{(3-i)} \big[1 - \frac{2(3-i)^{(2)}}{2^3} - \frac{3(3-i)^{(3)}}{2^6}\big] = 512$, where $F_0 = 1,\ F_1 = 12,\ F_2 = 66,\ F_3 = 0$.

MATLAB coding for the verification of Example 3.4
$syms\ k; k = 3; syms\ n; n = 2; syms\ p; p = 2; syms\ q; q = 3;$
$syms\ s; s = 3; syms\ a1; a1 = 2; syms\ a2; a2 = 3; syms\ a; a = 2;$
$format\ bank$
$f1 = k * (k - 1)$
$f2 = f1 * (k - 1) * (k - 2) + k * (k - 1) * (k - 2)$
$f3 = f2 * (k - 2) * (k - 3) + f1 * (k - 1) * (k - 2) * (k - 3)$
$s1 = a\hat{(}s * k)$
$s2 = f3 * a\hat{(}s * (k - 3))$
$s3 = f2 * (k - 2) * (k - 3) * (k - 4) * a\hat{(}s * (k - 4))$
$LHS = s1 - s2 - s3$
$u0 = a\hat{(}s * k) - k * (k - 1) * a\hat{(}s * (k - 1)) - k * (k - 1) * (k - 2) * a\hat{(}s * (k - 2))$
$u1 = a\hat{(}s*(k-1)) - (k-1)*(k-2)*a\hat{(}s*(k-2)) - (k-1)*(k-2)*(k-3)*a\hat{(}s*(k-3))$
$u2 = a\hat{(}s*(k-2)) - (k-2)*(k-3)*a\hat{(}s*(k-3)) - (k-2)*(k-3)*(k-4)*a\hat{(}s*(k-4))$
$RHS = u0 + f1 * u1 + f2 * u2$

Corollary 3.5 Let e^{-sk} be a function of $k \in (-\infty, \infty)$. Then we have
$e^{-sk} - F_{n+1} e^{-s(k-(n+1))} - \alpha_2 (k-n)^{(q)} F_n e^{-s(k-(n+2))}$

$$= \sum_{i=0}^{n} F_i e^{-s(k-i)} \big[1 - \alpha_1 (k-i)^{(p)} e^s - \alpha_2 (k-i)^{(q)} e^{2s}\big]. \tag{13}$$

Proof Taking $v(k) = e^{-sk}$ and applying (6) in (8), we get (13).

Example 3.6 Taking $k = 2.88,\ n = 3,\ s = 3,\ \alpha_1 = 0.7,\ \alpha_2 = 0.5,\ p = 2$ and $q = 2$ in (13), then we obtain
$e^{-8.64} - F_4 e^{3.36} - (0.5)(-0.12)^{(2)} F_3 e^{6.36} = \sum_{i=0}^{3} F_i e^{-3(2.88-i)}$
$\big[1 - (0.7)(2.88-i)^{(2)} e^3 - (0.5)(2.88-i)^{(2)} e^6\big] = -97.43,$
where $F_0 = 1,\ F_1 = 3.79,\ F_2 = 7.10,\ F_3 = 2.61$ and $F_4 = -0.13$.

Theorem 3.7 Let $t \in \mathbb{N}(0)$. Then a closed form solution of the second order difference equation with polynomial factorial
$v(k) - \alpha_1 k^{(p)} v(k-1) - \alpha_2 k^{(q)} v(k-2) = \big[k^t - \alpha_1 k^{(p)} (k-1)^t - \alpha_2 k^{(q)} (k-2)^t\big]$ is

$$\underset{\overline{\alpha}(k)}{\overset{-1}{\Delta}} \big[k^t - \alpha_1 k^{(p)} (k-1)^t - \alpha_2 k^{(q)} (k-2)^t\big] = k^t \tag{14}$$

ISSN: 2456-8686, Volume 2, Issue 2, 2018:20-29
DOI : http://doi.org/10.26524/cm36

Proof Taking $v(k) = k^t$ in (2) and using (3), we get (14).

Corollary 3.8 Taking $t = 2$ in Theorem 3.7, we have

$$\mathop{\Delta}_{\overline{\alpha}(k)}^{-1}\left[k^2 - \alpha_1 k^{(p)}(k-1)^2 - \alpha_2 k^{(q)}(k-2)^2\right] = k^2 \tag{15}$$

which is a closed form solution of the difference equation $\mathop{\Delta}_{\overline{\alpha}(k)} v(k) = k^2 - \alpha_1 k^{(p)}(k-1)^2 - \alpha_2 k^{(q)}(k-2)^2$. Proof From (14), replacing $t = 2$, we get 15

Corollary 3.9 If $v(k) = \mathop{\Delta}_{\overline{\alpha}(k)}^{-1}\left[k^t - \alpha_1 k^{(p)}(k-1)^t - \alpha_2 k^{(q)}(k-2)^t\right]$ is the closed form solution given in (14), then $v(k) - F_{n+1}v(k-(n+1) - \alpha_2(k-n)^{(q)}F_n v(k-(n+2))$

$$= \sum_{i=0}^{n} F_i\left[(k-i)^t - \alpha_1(k-i)^{(p)}[k-(i+1)]^t - \alpha_1(k-i)^{(q)}[k-(i+2)]^t\right]. \tag{16}$$

Proof Taking $u(k) = k^t - \alpha_1 k^{(p)}(k-1)^t - \alpha_2 k^{(q)}(k-2)^t$ in Theorem 3.1, we have 16.

Example 3.10 Let $k = 11$, $n = 3$, $t = 3$, $p = 3$, $q = 4$ $\alpha_1 = 3$, $\alpha_2 = 2$ in Corollary (3.9). Then $\sum_{i=0}^{3} F_i u(11-i) = v(11) - F_4 v(7) - \alpha_2 8^{(2)} F_3 v(6) = -10464437466361548$. where $u(k) = k^t - \alpha_1 k^{(p)}(k-1)^t - \alpha_2 k^{(q)}(k-2)^t$, $F_0 = 1$, $F_1 = 2970$, $F_2 = 6431040$, $F_3 = 9753670080$ and $F_4 = 9753670080$.

Theorem 3.11 If $v(k)$ is a closed form solution of second order difference equation

$$v(k) - \alpha_1 k^{(p)}v(k-1) - \alpha_2 k^{(q)}v(k-2) = k^t a^{sk} - \alpha_1 k^{(p)}(k-1)^t a^{s(k-1)} - \alpha_2 k^{(q)}(k-2)^t a^{s(k-2)},$$

then we have

$$v(k) - F_{n+1}v(k-[n+1]) - \alpha_2(k-n)^{(q)}v(k-[n+2]) = \sum_{i=0}^{n} F_i\Big[(k-i)^t a^{s(k-i)}$$

$$- \alpha_1(k-i)^{(p)}[k-(i+1)]^t a^{s(k-(i+1))} - \alpha_2(k-i)^{(q)}[k-(i+2)]^t a^{s(k-(i+2))}\Big]. \tag{17}$$

Proof Taking $u(k) = \left[k^t a^{sk} - \alpha_1 k^{(p)}(k-1)^t a^{s(k-1)} - \alpha_2 k^{(q)}(k-2)^t a^{s(k-2)}\right]$ in Theorem 3.1 and using (4), we get 17.

[1]*prajini.maths@gmail.com

ISSN: 2456-8686, Volume 2, Issue 2, 2018:20-29
DOI : http://doi.org/10.26524/cm36

Corollary 3.12 A closed form solution of the second order difference equation with polynomial factorial $\underset{\overline{\alpha}(k)}{\Delta} v(k) = k^3a^{sk} - \alpha_1 k^{(p)}(k-1)^3 a^{s(k-1)} - \alpha_2 k^{(p)}(k-2)^3 a^{s(k-2)}$ is k^3a^{sk} and hence we have

$$v(k) - F_{n+1}v(k-(n+1)) - \alpha_2(k-n)^{(q)}v(k-(n+2)) = \sum_{i=0}^{n} F_i[(k-i)^3 a^{s(k-i)} \times$$

$$- \alpha_1(k-i)^{(p)}[k-(i+1)]^3 a^{s(k-(i+1))} - \alpha_2(k-i)^{(q)}[k-(i+2)]^3 a^{s(k-(i+2))}]. \quad (18)$$

Proof The proof follows by taking $t = 3$ in Theorem 3.11.

Example 3.13 Let $k = 13$, $a = 2$, $n = 3$,$s = 0.35$ $\alpha_1 = 0.8$, $\alpha_2 = 0.4$, $p = 3$, $q = 2$ in Corollary (3.12). Then we obtain
$v(13) - F_4 v(9) - F_3(0.4)(10)^{(2)}v(8) = \sum_{i=0}^{3} F_i[(13-i)^3 2^{0.35(13-i)} - (0.8)(13-i)^{(3)} \times [13 - (i+1)]^3 2^{0.35(13-(i+1))} - (0.4)(13-i)^{(2)}[13-(i+2)]^3 2^{0.35(13-(i+2))}] = -4427746334499175$, where $F_0 = 1$, $F_1 = 1372.80$, $F_2 = 1449739.20$, $F_3 = 1148265930.24$ and $F_4 = 661464964343.04$.

Corollary 3.14 A closed form solution of the second order difference equation
$v(k) - \alpha_1 k^{(p)} v(k-1) - \alpha_2 k^{(q)} v(k-2) = k^t e^{-sk} - \alpha_1 k^{(p)} \frac{(k-1)^t}{e^{s(k-1)}} - \alpha_2 k^{(q)} \frac{(k-2)^t}{e^{s(k-2)}}$
is given by

$$v(k) - F_{n+1}v(k-(n+1)) - \alpha_2 F_n (k-n)^{(q)} v(k-(n+2)) = \sum_{i=0}^{n} F_i e^{-sk} \times$$

$$\big[(k-i)^t e^{is} - \alpha_1(k-i)^{(p)}[k-(i+1)]^t e^{(i+1)s} - \alpha_2(k-i)^{(q)}[k-(i+2)]^t e^{(i+2)s}\big]. \quad (19)$$

Proof Taking $a = e^{-1}$ in (17), we get (19).

Corollary 3.15 If $v(k) = \overset{-1}{\underset{\mu}{\Delta}}[ke^{-sk} - \alpha_1 k^{(p)}(k-1)e^{-s(k-1)} - \alpha_2 k^{(q)}(k-2)e^{-s(k-2)}]$ is the closed form solution given in (19), then

$$v(k) - F_{n+1}v(k-(n+1)) - \alpha_2 F_n (k-n)^{(q)} v(k-(n+2)) = \sum_{i=0}^{n} F_i e^{-sk} \times$$

$$\big[(k-i)e^{i} - \alpha_1(k-i)^{(p)}[k-(i+1)]e^{s(i+1)} - \alpha_2(k-i)^{(q)}[k-(i+2)]e^{s(i+2)}\big]. \quad (20)$$

Proof The proof follows by taking $t = 1$ in Corollory 3.14.

1*prajini.maths@gmail.com

ISSN: 2456-8686, Volume 2, Issue 2, 2018:20-29
DOI : http://doi.org/10.26524/cm36

Theorem 3.16 Let $v(k)$ be a solution of the second order difference equation with polynomial factorial $v(k) - \alpha_1 k^{(p)} v(k-1) - \alpha_2 k^{(q)} v(k-2)$
$= k^{(t)} a^{sk} - \alpha_1 k^{(p)} (k-1)^{(t)} a^{s(k-1)} - \alpha_2 k^{(q)} (k-2)^{(t)} a^{s(k-2)}$, then we have
$v(k) - F_{n+1} v(k-[n+1]) - \alpha_2 (k-n)^{(q)} v(k-[n+2]) = \sum_{i=0}^{n} F_i a^{s(k-i)} \times$

$$\Big[(k-i)^{(t)} - \alpha_1 (k-i)^{(p)} \frac{[k-(i+1)]^{(t)}}{a^s} - \alpha_2 (k-i)^{(q)} \frac{[k-(i+2)]^{(t)}}{a^{2s}}\Big]. \tag{21}$$

Proof Taking $v(k) = k^{(t)} a^{sk}$ in Theorem 3.1 and using (4), we get 21.

Corollary 3.17 If $v(k)$ is the closed form solution given of (21), then
$k^{(2)} a^{sk} - F_{n+1}(k-(n+1))^{(2)} a^{s(k-(n+1))} - \alpha_2 (k-n)^{(q)} (k-(n+2))^{(2)} a^{s(k-(n+2))} =$

$$\sum_{i=0}^{n} F_i a^{s(k-i)} \Big[(k-i)^{(2)} - \alpha_1 (k-i)^{(p)} \frac{(k-(i+1))^{(2)}}{a^s} - \alpha_2 (k-i)^{(q)} \frac{(k-(i+2))^{(2)}}{a^{2s}}\Big] \tag{22}$$

Proof The proof follows by taking $t = 2$ in Theorem 3.16.

Example 3.18 Let $k = 7$, $a = 4$, $n = 2$, $s = 0.55$, $\alpha_1 = 0.5$, $\alpha_2 = 0.0.08$, $p = 3$, $q = 2$ in Corollary (3.17). Then we obtain
$v(7) - F_3 v(4) - (0.5) 5^{(2)} F_2 v(3) = \sum_{i=0}^{2} F_i [(7-i)^{(2)} 4^{0.55(7-i)} - (0.5)(7-i)^{(3)} \times$
$[7-(i+1)]^2 4^{0.55(7-(i+1))} - (.08)(7-i)^{(2)} [7-(i+2)]^2 4^{0.55(7-(i+2))}] = -48558944.63$,
where $F_0 = 1$, $F_1 = 105$, $F_2 = 6303.36$, $F_3 = 189352.80$.

Corollary 3.19 Let $v(k)$ be a solution of second order difference equation
$v(k) - \alpha_1 k^{(p)} v(k-1) - \alpha_2 k^{(q)} v(k-2) = e^{-sk} \big[k^{(t)} - \alpha_1 k^{(p)} (k-1)^{(t)} e^s - \alpha_2 k^{(q)} (k-2)^{(t)} e^2\big]$.
Then we have
$v(k) - F_{n+1} v(k-[n+1]) - \alpha_2 (k-n)^{(q)} v(k-[n+2]) = \sum_{i=0}^{n} F_i e^{-s(k-i)} \times$

$$\big[(k-i)^{(t)} - \alpha_1 (k-i)^{(p)} [k-(i+1)]^{(t)} e^s - \alpha_2 (k-i)^{(q)} [k-(i+2)]^{(t)} e^{2s}\big]. \tag{23}$$

Proof Taking $a = e^{-1}$ in (3.16), we get (23).

1*prajini.maths@gmail.com

Corollary 3.20 A closed form solution of second order difference equation with polynomial factorial $\underset{\overline{\alpha}(k)}{\Delta} v(k) = e^{-sk}[k^{(2)} - \alpha_1 k^{(p)}(k-1)^{(2)}e^{s} - \alpha_2 k^{(q)}(k-2)^{(2)}e^{2s}]$ is $k^{(2)}e^{-sk}$ and hence we have

$$v(k) - F_{n+1}v(k-(n+1)) - \alpha_2(k-n)^{(q)}v(k-(n+2)) = \sum_{i=0}^{n} F_i e^{-s(k-i)}$$

$$\big[(k-i)^{(2)} - \alpha_1(k-i)^{(p)}[k-(i+1)]^{(2)}e^{s} - \alpha_2(k-i)^{(q)}[k-(i+2)]^{(2)}e^{2s}\big]. \quad (24)$$

Proof The proof follows by taking $t = 2$ in Corollory (3.19), we get (24).

Example 3.21 Let $k = 12$, $n = 2$, $s = 0.64$, $a = 0.25$, $\alpha_1 = 2.22$, $\alpha_2 = 0.333$, $p = 3$, $q = 2$ in Corollary (3.20). Then we obtain

$v(12) - F_3 v(9) - (0.333)F_2 v(8) = \sum_{i=0}^{2} F_i[(12-i)^{(2)}(0.25)^{0.64(12-i)} - (2.22)(12-i)^{(3)}$
$\times[12-(i+1)]^{(2)}(0.25)^{0.64(12-(i+1))} - (0.333)(12-i)^{(2)}[12-(i+2)]^{(2)}(0.25)^{0.64(12-(i+2))}]$
$= -2400227388.32$, where $F_0 = 1$, $F_1 = 2930.40$, $F_2 = 6440477.08$ and $F_3 = 10294565898.83$.

4. Conclusion

We obtained summation formula to $\overline{\alpha}(k)$-Fibonacci sequence by introducing $\overline{\alpha}(k)$-difference operator and have derived certain results on closed and summation form solution of second order difference equation which will be used to our further research.

References

[1] Bastos NRO, Ferreira RAC, and Torres DFM, Discrete-Time Fractional Variational Problems, Signal Processing, 91(3), 2011, 513-524.

[2] Britto Antony Xavier G, Gerly TG and Nasira Begum H, Finite Series of Polynomials and Polynomial Factorials arising from Generalized q-Difference operator, Far East Journal of Mathematical Sciences, 94(1), 2014, 47-63.

[3] Falcon S and Plaza A, On the Fibonacci $k-$numbers, Chaos, Solitons and Fractals , 32(5), 2007, 1615-1624.

[4] Ferreira RAC and Torres DFM, Fractional h-difference equations arising from the calculus of variations, Applicable Analysis and Discrete Mathematics, 5(1), 2011, 110-121.

1*prajini.maths@gmail.com

ISSN: 2456-8686, Volume 2, Issue 2, 2018:20-29
DOI : http://doi.org/10.26524/cm36

[5] Jerzy Popenda and Blazej Szmanda, On the Oscillation of Solutions of Certain Difference Equations, Demonstratio Mathematica, XVII(1), 1984, 153-164.

[6] Koshy T, Fibonacci and Lucas Numbers with Applications, Wiley-Interscience, New York, NY, USA, (2001).

[7] Maria Susai Manuel M, Chandrasekar V and Britto Antony Xavier G, Solutions and Applications of Certain Class of α-Difference Equations, International Journal of Applied Mathematics, 24(6), 2011, 943-954.

[8] Miller KS and Ross B, Fractional Difference Calculus in Univalent Functions, Horwood, Chichester, UK, 139-152, (1989).

[9] Susai Manuel M, Britto Antony Xavier G, Chandrasekar V and Pugalarasu R, Theory and application of the Generalized Difference Operator of the n^{th} kind(Part I), Demonstratio Mathematica, 45(1), 2012, 95-106.

[10] Vajda S, Fibonacci and Lucas Numbers and the Golden Section, Ellis Horwood, Chichester, UK, (1989).

[1*]prajini.maths@gmail.com