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Super Magic Labelling of Complete Bipartite Graphs

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Abstract

The study of graph labelling has focused on finding classes of graphs which admits a particular type of labelling. A graph is called super magic if it admits a labelling of the edges by pairwise different consecutive positive integers such that the sum of the labels of the edges incident with a vertex is independent of the particular vertex. Some constructions of super magic labelling of regular graphs are described. Super magic regular complete bipartite graphs are characterized.

Key Words: Magic squares, complete graphs, Super magic graphs, complete bipartite graphs.

AMS Classification: 05C78

1 Introduction

We consider finite undirected graphs without loops, multiple edges and isolated vertices. If G is a graph, then V(G) and E(G) stand for the vertex set and the edge set of G respectively. Cardinalities of these sets denoted by |V(G)| and |E(G)| are called the order and size of G. A magic labelling f(G) is called a super magic labelling of G if the set $\{ f(e) : e \in E(G) \}$ consists of consecutive positive integers. We say that a graph G is super magic (magic) if and only if there exist a super magic (magic) labelling of G.

The concept of magic graphs was introduced by Sedlacek. Super magic graphs were introduced by M. B. Stewart. It is easy to see that the classical concept of super magic square of n^2 boxes corresponds to the fact that the complete bipartite graph $K_{n,n}$ is super magic for every positive integer n not equal to 2.

M. B. Stewart proved that the complete graph K_n is super magic if and only if either $n \leq 6$ and $n \equiv 0 \mod 4$, or n=2.

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A special case of a magic labelling is the super magic labelling. Here the classic nxn magic corresponds to the super magic labelling of $K_{n,n}$. The existence of an super magic labelling is known for most basic families of graphs such as complete graphs, complete multipartite graphs, hypercubesetc,. In an antimagic labelling, we require the exact opposite such that all the weights have to be different. There is a large number of articles published on magic-type graph labellings.

The various labellings are obtained based on requirements put on the labellings. Magic graph labellings are a natural extension of the well-known magic squares and rectangles. The first magic square known to be recorded was a 3 by 3 (3x3) magic square called Loshu around 2200 BC.

2 Basic Definitions

Definition 2.1. A bipartite graph is said to be a complete bipartite graph if there is an edge between every pair of vertices in V_1 and V_2. A complete bipartite graph is denoted by $K_{m,n}$ where m is the number of vertices in V_1 and n is the number of vertices in V_2.

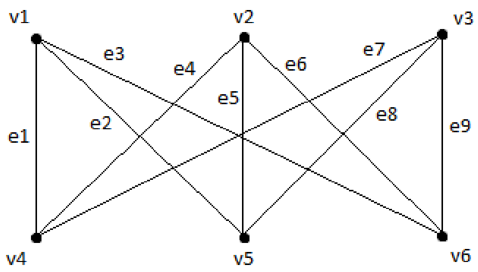

Figure 1: Complete bipartite graph

Definition 2.2. A connected graph G is called magic if there is a labelling of the edges with distinct positive integers such that, for each vertex v, the sum of the labels of all edges incident with v is the same for all v.

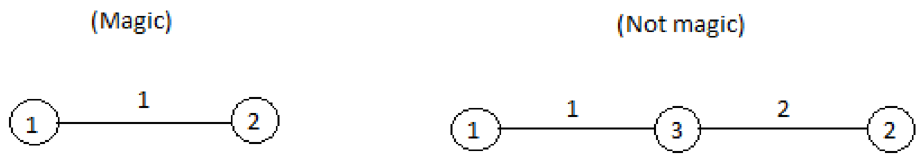

Figure 2: Magic labelling

Definition 2.3. A connected graph G is called super-magic if there is a magic labelling of the edges with consecutive positive integers.

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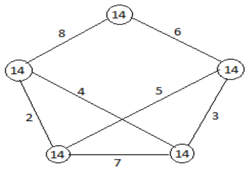

Figure 3: Supermagic labeling

3 Algorithm To Construct Supermagic Labeling Of Complete Bipartite Graph of Any Order

Step 1: Draw a complete bipartite graph $K_{n,n}$ (where $n \geq 3$).

Step 2: Label the top n vertices of the graph by R1, R2, R3, . . . (where Ri's are considered as row).

Step 3: Label the bottom n vertices of the graph by C1, C2, C3, . . . (where Ci's are considered as column).

Step 4: Now, construct a magic square of order n x n (where $n \geq 3$) using rules for constructing magic square which is given below.

Step 5: There are n-rows and n-columns in magic square filled with numbers 1 to n^2. Name the rows and columns as R1, R2, . . . and C1, C2, . . . respectively.

Step 6: Allocate each Ri and Cj values in the respective complete bipartite graph as per the order in the magic square without any changes.

Step 7: Sum up every edges of the respective vertices of complete bipartite graph which yields a constant sum. We have labeled the edges with consecutive integers.

Hence, we have constructed super magic labelling of complete bipartite graph.

4 Rules For Creating Magic Square Of Any Order

Let 0, 1, 2, . . . be the index for rows and columns of matrix which we are going to consider. For a magic square of order n x n, there are numbers from 1 to n^2 and each box in the magic square is filled with one of these numbers.

Now, we are going to start allocate the numbers by using below rules.

Rule 1: The starting number 1 is stored at position $(n , n - 1)$ in the cell.

Rule 2: For next position,

$$(i, j) \rightarrow (i - 1, j + 1)$$

Now search for the position. If new j^{th} column is not present, that is if $j = n$ move on to rule 3.

Rule 3: If $j = n$ then consider the value of j to be zero.

Rule 4: If the value of $i = -1$ then consider the value of i to be n-1.

Rule 5: Suppose if the position is occupied already then the value of i and j are i+1 and j-2 respectively.

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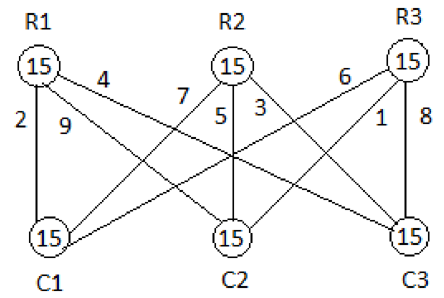
Rule 6: If we have (-1,n) in any cases then change it to the form (0,n-2).

Rule 7: Repeat the rules from 2 to 6 to fill the numbers 1 to n^2 in the magic square of order n x n.

Super magic labelling of $K_{3,3}$



2	9	4	15
7	5	3	15
6	1	8	15
15	15	15	

Figure 4: SML of $K_{3,3}$Figure 5: 3x3 - Magic square

Super magic labeling of $K_{4,4}$

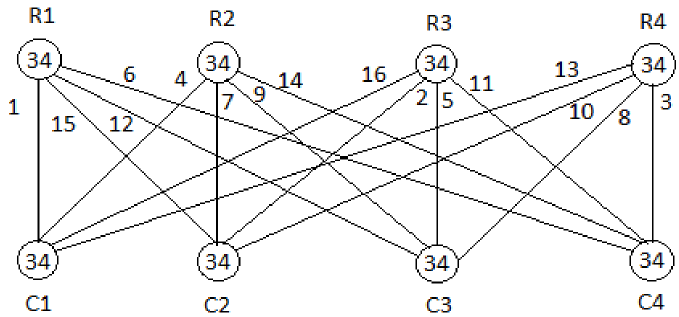


Figure 6: SML of $K_{4,4}$

1	15	12	6	34
4	7	9	14	34
16	2	5	11	34
13	10	8	3	34
34	34	34	34	

5 Conclusion

In Graph theory, super magic labelling is a very active field of research. I have studied and presented some important theorems based on super magic labelling of graphs with some results were discussed here. Through this paper, one can able to do super magic labelling of complete bipartite graphs Kn,n of any order n easily with the help of magic square technique.

In this, we hope that the intest in super magic labellings of complete bipartite graphs will be aroused among those who study graph labellings.

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