

A Score Function of Hexagonal Neutrosophic Number and its Application in Networking Problem Graphs

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Abstract

In this paper extended the version of single type neutrosophic neutrosophic number real life problems have different ways the problem is solving the networking problem the concept of Hexagonal neutrosophic number has been focused in a distinct framework here develop of a score function and its estimation have been formulated in perspective, networking problem considered here in Hexagonal neutrosophic number finally computation of total completion time of problem.

Key Words: Hexagonal neutrosophic number, Networking problem, Score function.

AMS Classification: 03E72

1 Introduction

Fuzzy set are some what like sets whose elements have degrees of membership Fuzzy sets were introduced independently by Lotfi A.Zadeh and Dieter Klaua in 1965 as a extension of the classical notion of set At the same time, Sali (1965) defined a more general kind of structure called an L-relation. Fuzzy relations which are now used throughout fuzzy mathematics and have applications in areas such as linguistics are special cases of L-relations when L is the unit interval $[0,1]$ Fuzzy set theory, permits the gradual assessment of the membership of elements in a set this is described with the aid of a membership function valued in the real unit interval $[0,1]$ Fuzzy sets theory is an extension of classical set theory. Elements having varying degree of membership A logic based on two truth values True and false is sometimes insufficient when describing human reasoning In real world there exist much fuzzy knowledge Human thinking and reasoning (analysis, logic, interpretation)

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frequently involved Fuzzy information Human can give satisfactory answer which are probably true. PERT (program evaluation and review technique) is available to assists the project manager in carrying out of these responsibilities.makes heavy use of network to help plane and display the condition of all the activities PERT is a method of analyzing the tasks involved in completing a given project,especially the time needed to complete to each task and to identify the minimum time needed to complete the project. PERT was developed primary to simplify the planning and scheduling of large complex project.It was developed for the U S NAVY special projects office in 1957 to support the U.S.Navy's polaris nuclear submarine project.The critical path is the longest sequence of activities in a project plane which must be completed on time for the project to complete on due date an activity on the critical path cannot be started until its predecessors activity is complete;if it is a delayed for a day the entire project will be delayed for a day unless the activity following the delayed activity following the delayed activity is completed a day earlier.The critical path is very useful in helping to manage any project.when the critical path has been identified it can clearly be seen where effort can be compromised.It any of activities on the critical path change,the date of the project will be affected.A critical path is determined by identifying the longest activities and measuring the time required to complete them from start to finish it has been superseded by the activity on node diagram where each activity is shows as a box or node and the arrows represent the logical relationship going from predecessors to successor. researchers though have various fields to work on but hesitant theory is one of the vital topics in today's world The theory of fuzziness has leading feature to solve clear soundly engineering and statistical problem applying uncertainty theory plentiful varieties of realistic problem can be solved,networking problem,decision making problem influence on science etc..This concept deals with hexagonal neutrosophic number in different aspects nowadays researchers are very much interested in doing networking problem in neutrosophic domain consider a networking based PERT/CPM problem in hexagonal neutrosophic where utilize the idea of our developed score function for solving problem.

2 Preliminaries

Definition 2.1. Fuzzy set: Set M called as a fuzzy set when represented by the pair $((x, \mu(x)))$ and thus stated as $M = \{x, \mu(x) : x \text{ belongs to } X, \mu(x) \text{ belongs to } [0, 1] \text{ where } x \text{ belongs to the crisp set } X \text{ and } \mu(x) \text{ belongs to the interval } [0, 1]\}$.

Definition 2.2. Intuitionistic Fuzzy set (IFS): An fuzzy set S_F in the universal discourse X , symbolized widely by x is referred as intuitionistic set if $S_F = \{x; [\gamma(x), \delta(x)] : x \in X\}$ where $\gamma(x) : X \rightarrow [0, 1]$ is termed as the certainty membership function which specify the degree of confidence, $\delta(x) : X \rightarrow [0, 1]$ is termed as the uncertainty membership function which specify the degree of indistinctness. $\gamma(x), \delta(x)$ exhibits the following the relation $0 \leq \gamma(x) + \delta(x) \leq 1$

Definition 2.3. Neutrosophic set: A set A in the universal discourse X figuratively represented by x named as a neutrosophic set if $A = \{x; [T_A(x), I_A(x), F_A(x)] : x \in X\}$, Where $T_A(x) : X \rightarrow [0, 1]$ is stated as the certainty membership function, which designates the degree of confidence, $I_A(x) : X \rightarrow [0, 1]$ is stated as the uncertainty membership, which designates of indistinctness and $F_A(x) : X \rightarrow [0, 1]$ is stated as the untruthful membership, which designates the degree of deceptiveness on the decision taken by the decision maker. $T_A(x), I_A(x)$ and $F_A(x)$ display the following relation $-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.4. Single valued Neutrosophic Number: Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form

$A = \{x, [T_A(x), I_A(x), F_A(x)] : x \in X\}$ where $T_A(x) : X \rightarrow [0, 1], I_A(x) : X \rightarrow [0, 1]$ and $F_A(x) : X \rightarrow [0, 1]$ with $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in X$.

Definition 2.5. Single valued Hexagonal Neutrosophic Number: A single valued hexagonal neutrosophic number is defined as $H_N(x) = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$ is a set of real number in \mathfrak{R} , whose truth membership, indeterminacy membership and falsity membership function are defined as follows

$$T_A(x) = \begin{cases} 0 & \text{if } x \leq a_1 \\ \frac{1}{2} \left(\frac{x-a_1}{a_2-a_1} \right) & \text{if } a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{x-a_2}{a_3-a_2} \right) & \text{if } a_2 \leq x \leq a_3 \\ 1 & \text{if } a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2} \left(\frac{x-a_4}{a_5-a_4} \right) & \text{if } a_4 \leq x \leq a_5 \\ \frac{1}{2} \left(\frac{a_6-x}{a_6-a_5} \right) & \text{if } a_5 \leq x \leq a_6 \\ 0 & \text{otherwise} \end{cases}$$

$$I_A(x) = \begin{cases} 1 - \frac{1}{2} \left(\frac{x-b_1}{b_2-b_1} \right) & \text{if } b_1 \leq x \leq b_2 \\ \frac{1}{2} \left(\frac{b_3-x}{b_3-b_2} \right) & \text{if } b_2 \leq x \leq b_3 \\ 0 & \text{if } b_3 \leq x \leq b_4 \\ \frac{1}{2} \left(\frac{x-b_4}{b_5-b_4} \right) & \text{if } b_4 \leq x \leq b_5 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{x-b_5}{b_6-b_5} \right) & \text{if } b_5 \leq x \leq b_6 \\ 1 & \text{otherwise} \end{cases}$$

$$F_A(x) = \begin{cases} 1 - \frac{1}{2} \left(\frac{x-c_1}{c_2-c_1} \right) & \text{if } c_1 \leq x \leq c_2 \\ \frac{1}{2} \left(\frac{c_3-x}{c_3-c_2} \right) & \text{if } c_2 \leq x \leq c_3 \\ 0 & \text{if } c_3 \leq x \leq c_4 \\ \frac{1}{2} \left(\frac{x-c_4}{c_5-c_4} \right) & \text{if } c_4 \leq x \leq c_5 \\ \frac{1}{2} + \frac{1}{2} \left(\frac{x-c_5}{c_6-c_5} \right) & \text{if } c_5 \leq x \leq c_6 \\ 1 & \text{otherwise} \end{cases}$$

Definition 2.6. Proposed Score Function: Score function utterly relies upon the value of exact membership indicator degree, inexact membership indicator degree and hesitancy membership indicator degree of hexagonal neutrosophic number. The fundamental use of score function is to drag the judgment of conversion of hexagonal neutrosophic number to real number. A score function is developed for any hexagonal single type neutrosophic number (HSNN). $H_{hex} = (H_1 + H_2 + H_3 + H_4 + H_5 + H_6; \alpha, \beta, \gamma)$ score function is described as $S_{pen} = \frac{1}{18} \{(H_1 + H_2 + H_3 + H_4 + H_5 + H_6) \times (2 + \alpha - \beta - \gamma)\}$
 Here, $S_{pen} \in [1, 2]$

Relation Between Two Hexagonal Neutrosophic Number: Let us consider any two hexagonal neutrosophic fuzzy number defined as follows $H_{hex1} = (\alpha_{hex1}, \beta_{hex1}, \gamma_{hex1})$

$$H_{hex2} = (\alpha_{hex2}, \beta_{hex2}, \gamma_{hex2})$$

$$1) S_{hex1} > S_{hex2}, H_{hex1} > H_{hex2}$$

$$2) S_{hex1} < S_{hex2}, H_{hex1} < H_{hex2}$$

$$3) S_{hex1} = S_{hex2}, H_{hex1} = H_{hex2}$$

3 Numerical Example:

Hexagonal neutrosophic number H_{hex}	Score value S_{hex}	Ordering
$H_{hex1} = \langle (1.3, 1.4, 1.5, 1.6, 1.7, 1.8; 1.4, 1.7, 1.6) \rangle$	0.0597	$H_{hex5} > H_{hex1} > H_{hex4}$ $> H_{hex6} > H_{hex3} > H_{hex2}$
$H_{hex2} = \langle (1.3, 1.35, 1.45, 1.55, 1.7, 1.75; 1.6, 1.5, 1.4) \rangle$	0.0101	
$H_{hex3} = \langle (1.25, 1.3, 1.4, 1.5, 1.6, 1.7; 1.5, 1.6, 1.4) \rangle$	0.0126	
$H_{hex4} = \langle (1.4, 1.45, 1.5, 1.6, 1.7, 1.75; 1.3, 1.5, 1.6) \rangle$	0.0295	
$H_{hex5} = \langle (1.6, 1.3, 1.4, 1.5, 1.7, 1.85; 1.6, 1.7, 1.8) \rangle$	0.5694	
$H_{hex6} = \langle (1.3, 1.45, 1.5, 1.55, 1.6, 1.7; 1.4, 1.6, 1.5) \rangle$	0.0203	

PERT in Hexagonal Neutrosophic Environment and The Proposed Model

PERT system or Project Evaluation and Review Techniques are a project managing scheme which used to plan, arrange, systemize and equalize tasks amongst a project. these technique scheme which is basically examine the minimum time required in finishing the total task and also calculate the time required in completion of each task for the given project. PERT arrangement entails the specified steps:

1. Identification of specified activities and

2. In determination of accurate sequence of the activities
3. In construction of a network map.
4. Evaluation of time needed for ac task.
5. Determine of the critical path.
6. Updating the PERT chart on progression with the project.

The three time estimation for activity duration are:

Optimistic time(o_t):

In general, the optimistic time requires minimum time for completing the activities and it is considered with three standards deviations from mean and approximately there is 1% chance for the activity to complete within time.

Pessimistic time(p_t):

It known for tasks taking the longest time. here also the three standards deviations are used.

Most likely time(m_t):

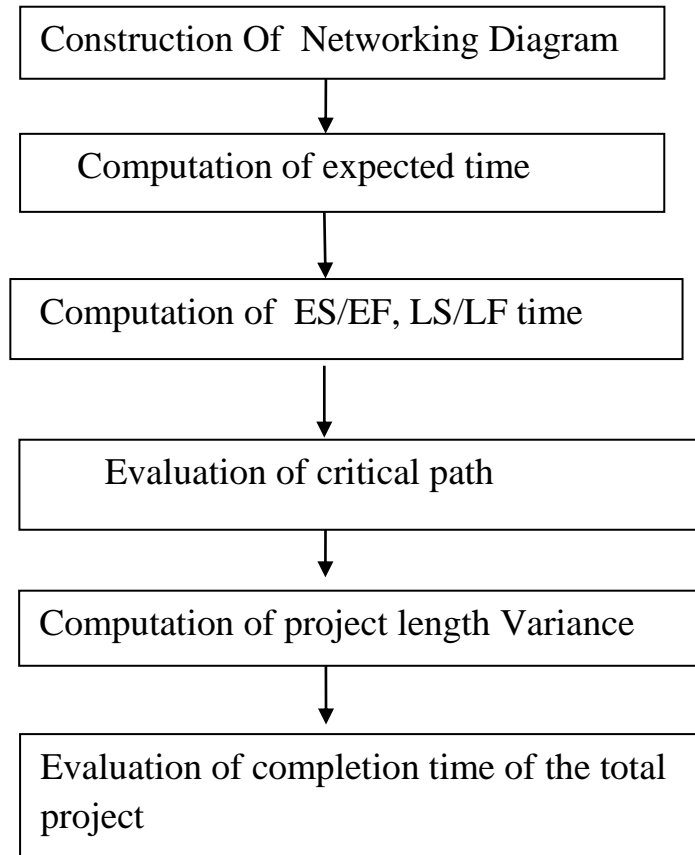
The completion time in general status for most likely have the highest probability and is absolutely different from the project time. Score value $R(S_{hex}, 0) = 1/18(H_1, H_2, H_3, H_4, H_5, H_6) \times (2 + \alpha, \beta, \gamma)$ is presented to attain the hexagonal neutrosophic number $(H_1, H_2, H_3, H_4, H_5, H_6)$ By the use of formulas the Expected time $E_t = \frac{(o_t + 4m_t + p_t)}{6}$ Standard deviation $\sigma_t = \frac{(p_t - o_t)}{6}$ is calculated where o_t, p_t, m_t denotes the optimistic time, pessimistic time and most likely time respectively for all crisp value. For the add on calculations of latest time, critical path method is used (CPM).

Considering the forward pass with zero starting time, the first event progresses from left to right and reaches to final event. Let us assume j, k for an activity, the earliest time event of j is ES_j therefore $S_k = ES_j + t_{jk}$ there might be case where in an event more than one activity enters then the earliest time is calculated as $ES_k = \max ES_j + t_j$ for all activities radiating from node j to k. Backward pass starts from with the final node and calculation progresses from right to left till the initial event.

Let us assume j, k for any activity, the latest finished time event of j is LF_j therefore $F_j = LF_k - t_{jk}$. there might ne case where in an event more than one activity enters then the latest finish time is calculated as $LF_j = \min LF_k - t_{kj}$ for all activities radiating from node k to i.

Once critical path is calculated, computation of project length variance is done which is sum of the variances of all critical activities. After that standard normal variable $Z = \frac{T_{sd} - T_{ex}}{\sigma}$ is computed where T_{sd} is the schedules time given for a project to complete and, T_{ex} is the expected project length duration. By the use of normal curve, the probability of project completion within the definite time can be approximated.

Flow chart



Illustrative example

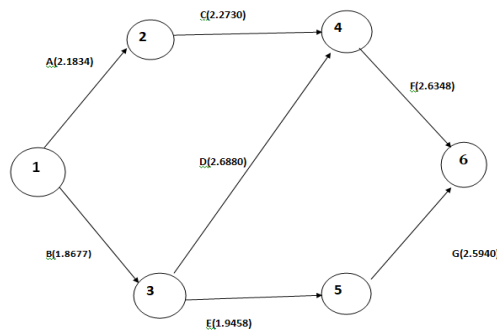
ACTIVITY	DESCRIPTION	PREDECESSORS	OPTIMISTIC TIME	PESSIMISTIC TIME	MOST LIKELY TIME
A	Finish component development	_____	(0.6,1.9,2.6,4.4,5.5; 0.6,0.6,0.9)	(2.4,3.3,6.4,5.5,6; 0.5,0.6,0.8)	(3.2,3.8,4.8,5.6,6.5,7.5; 0.8,0.6,0.7)
B	Design marketing programme	_____	(0.8,2.6,3.8,4.5,5.9; 0.7,0.8,0.3)	(3,4,4.5,6,7,7.5; 0.7,0.8,0.7)	(2.5,3,3.5,4.5,5.5; 0.7,0.6,0.8)
C	Design production system	A	(2,2.5,3,4,5,5.6; 0.8,0.7,0.6)	(0.8,1.9,2.5,3.6, 4.5,5.8; 0.9,0.8,0.7)	(2,3.5,4.7,5,6.8,7, 8;0.7,0.6,0.5)
D	Select advertising media	B	(3.5,4,5,5.5,6,6.5; 0.7,0.6,0.8)	(2,3,4,5,6,5.7;0.7 0.6,0.7)	(2,2.7,3.5,4.5,5,6.5, 7;0.9,0.5,0.7)
E	Initial production	B	(2.5,3.5,4,5,6,5.7; 0.8,0.7,0.5)	(1.8,2.7,3,4,7,5,6; 0.9,0.7,0.8)	(2,4,3,2,3.8,4.5,5.5, 6;0.7,0.8,0.6)
F	Delayed times	C,D	(3,3.5,4.5,5,6,5.7; 0.8,0.7,0.5)	(2.5,3,3.7,4.8,5,5.8; 0.6,0.4,0.5)	(3,2,3.8,4.5,5.5,6.5, 7;0.8,0.5,0.7)
G	Release component to market	E	(2.6,3,3.5,4,5,5,6; 0.6,0.5,0.4)	(3,3.5,4,4.7,5,5,6; 0.5,0.4,0.7)	(3,6,4,5,5,8,6,6.5; 0.8,0.6,0.6)

Draw the project network and find the probability that the projects is completed in 7.2 days.

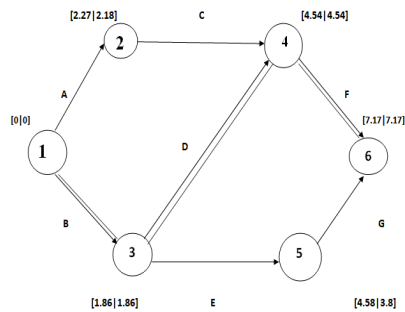
STEP1

Optimistic Time(o_t)	Pessimistic Time(p_t)	Most likely Time(m_t)	$E_{ik} = \frac{o_t + 4m_t + p_t}{6}$	$\sigma_{ik}^2 = \left[\frac{p_t - o_t}{6} \right]^2$
1.1367	1.4972	2.6167	2.1837	0.0036
2.2844	2.1333	1.6972	1.8677	0.0006
1.8417	1.4855	2.5778	2.2730	0.0035
2.2027	2.1389	2.9467	2.6880	0.0001
2.5333	1.8044	1.8344	1.9458	0.0131
2.6222	2.3422	2.7111	2.6348	0.0021
2.3233	2.0767	2.7911	2.5940	0.0016

Step2



Step3



Expected project duration: 7.17 days

Line denote critical path -----

Critical path: 1 → 3 → 4 → 6

Project length variance $\sigma^2 = 0.0093$

Standard deviation = 0.0964

Probability that the project will be finished with in 7.2 days is

$$P(Z \leq (7.2 - 7.17) / 0.0964) = P(Z \leq 0.3)$$

4 Conclusion

In this paper, The idea of hexagonal neutrosophic number is intriguing, competent and has an ample scope of utilization in various research domains. In this research article, we vigorously erect the perception of hexagonal neutrosophic number from different aspects. We introduced a score function here in hexagonal neutrosophic domain. Additionally, we consider a networking problem in neutrosophic environment and solve the problem utilizing the idea of score function. Since, there is no such articles is till now established in hexagonal networking neutrosophic arena, thus we cannot compare our work with other methods. Further, researchers can immensely apply this idea of neutrosophic number in numerous flourishing research fields like engineering problem, mobile computing problems, diagnoses problem, realistic mathematical modeling, social media problem etc.

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