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On the positive integral solutions to the Diophantine equation of n-variables $(x_1 + x_2 + x_3 + \cdots + x_n)^2 = x_1x_2x_3\cdots x_n$

Sadhasivam V[1*], Nagajothi N [2] and Vimala G [3]
[1,2,3]PG & Research Department of Mathematics, Thiruvalluvar Govt. Arts College, Rasipuram - 637401, Namakkal Dt, Tamilnadu, India.

Abstract

In this paper, we discuss solvability of the Diophantine equation of the form $(x_1+x_2+x_3+\cdots+x_n)^2 = x_1x_2x_3\cdots x_n$. An explicit closed form solutions are obtained by using the theory of Pell's equations which generate the different families of positive integral solutions to this equation. Some illustrative examples are inserted in order to explain the effectiveness of our results.

Key words: Diophantine equation, Pell's equation, General Pell's equations.

AMS classification: 11D09, 11D25, 11D72.

1. Introduction

A Diophantine equation is a polynomial equation in any number of variables whose unknowns are allowed to be integers only. The study of Diophantine equations goes back to ancients; indeed, they are named after Diophantus of Alexandria (c.200-284 AD), who made a study of such equations and was one of the first mathematician to introduce symbolism into Algebra. In more technical language, they define an algebraic curve, algebraic surface or more general object, and ask about the lattice points on it. The more general, the Diophantine equation in two variables is of the form

$$ax^2 + bxy + cy^2 + dx + ey + f \ = 0 \tag{1}$$

Integer solutions to equation of three or more variables are given in various parametric forms see [4] the reference sited there in. A special class of Diophantine Equation known as Pell's equation is an active of area of research for long time and many researchers have been investigated the solvability problem through many different methods see [3],[5]-[9],[11]-[18], the reference sited there in. In[10], it is proved that

[1*]ovsadha@gmail.com,[2]nagajothi006@gmail.com

the Diophantine equation $x+y+z=xyz$ has solutions to the quadratic field $Q(\sqrt{d})$ if and only if $d=-1,2$ or 5 and in these cases all solutions are given.
In 2012, Titu Andreescu [1], studied the solvability problem of the Diophantine equation of the form

$$(x+y+z)^2 = xyz \tag{2}$$

And in 2013 [2], the author also investigated, the solutions to the Diophantine equation

$$(x+y+z+t)^2 = xyzt \tag{3}$$

It seems that there has been no work published for n-variable case. Motivated by above observation we have proposed the following Diophantine equation of the form

$$(x_1+x_2+x_3+\cdots+x_n)^2 = x_1x_2x_3\cdots x_n. \tag{4}$$

and investigated the solvability problem, by using the theory of Pell's equation. Also we will indicate the a general method of generating such families of solutions by using the theory of Pell's equations.

This work is planned as follows: Section 2, presents the fundamentals which are required throughout this paper. In Section 3, we discuss the solvability problem of the Diophantine equation of n-variables. In Section 4, we present some examples to illustrate our main results..

2. The General Pell's Equation $Au^2 - Bv^2 = C$

Recall that the equation

$$r^2 - Ds^2 = 1, \tag{5}$$

where D is a positive integer that is not a perfect square is called Pell's equation.

Denoting by $(r_0, s_0) = (1, 0)$ its trivial solution, the main result concerning equation (5) is the following. There are infinitely many solutions in positive integers to (5) and all solutions to equation (5) are given by $(r_n, s_n)_{(n\geq 0)}$, where

$$\begin{cases} r_{n+1} = r_1r_n + Ds_1s_n \\ s_{n+1} = s_1r_n + r_1s_n \end{cases} \tag{6}$$

Here (r_1, s_1) represents the fundamental solution to (5), that is minimal solution different from (r_0, s_0).
It is not difficult to see that (6)is equivalent to

$$r_n + s_n\sqrt{D} = (r_1 + s_1\sqrt{D})^n, \qquad n \geq 0 \tag{7}$$

[1*]ovsadha@gmail.com,[2]nagajothi006@gmail.com

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Also remains (7) could be written in the following useful matrix form:

$$\begin{pmatrix} r_{n+1} \\ s_{n+1} \end{pmatrix} = \begin{pmatrix} r_1 & Ds_1 \\ s_1 & r_1 \end{pmatrix} \begin{pmatrix} r_n \\ s_n \end{pmatrix}$$

from where

$$\begin{pmatrix} r_n \\ s_n \end{pmatrix} = \begin{pmatrix} r_1 & Ds_1 \\ s_1 & r_1 \end{pmatrix}^n \begin{pmatrix} r_0 \\ s_0 \end{pmatrix}$$

$$\begin{pmatrix} r_n \\ s_n \end{pmatrix} = \begin{pmatrix} r_1 & Ds_1 \\ s_1 & r_1 \end{pmatrix}^n \begin{pmatrix} 1 \\ 0 \end{pmatrix}, n \geq 0 \tag{8}$$

From (7) or (8) it follows immediately that

$$\begin{cases} r_n = \frac{1}{2}\left[(r_1 + s_1\sqrt{D})^2 + (r_1 - s_1\sqrt{D})^2\right] \\ s_n = \frac{1}{2\sqrt{D}}\left[(r_1 + s_1\sqrt{D})^2 - (r_1 - s_1\sqrt{D})^2\right]. \end{cases} \tag{9}$$

The main method of determining the fundamental solution (r_1, s_1) involves continued fractions.

In what follows we consider the general Pell's equation,

$$Au^2 - Bv^2 = C, \tag{10}$$

where A, B, C are positive integers with $\gcd(A, B) = 1$ and A and B are free squares. See [3] the reference cited there in.

Theorem 2.1 If the equation (10) is solvable in positive integers, then it has infinitely many positive integral solutions.
Proof: Since the proof is similar to that of [1], so we omit it.

3. Main Results

In this section, we give the infinite families of solutions to the equation of the form

$$(x_1 + x_2 + x_3 + \cdots + x_n)^2 = x_1 x_2 x_3 \cdots x_n. \tag{11}$$

1*ovsadha@gmail.com,2nagajothi006@gmail.com

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Using the transformations

$$x_1 = \frac{u+v}{2} + a_1,\ x_2 = \frac{u-v}{2} + a_2,\ x_3 = a_2,\ x_4 = a_3, \cdots, x_n = a_{n-1}, \tag{12}$$

where $a_1, a_2, a_3, \cdots, a_{n-1}$ are position integers.
Subtitude (12) in (11), we have

$$\left[\left(\frac{u+v}{2} + a_1\right) + \left(\frac{u-v}{2} + a_1\right) + a_2 + a_3 + \cdots + a_{n-1}\right]^2$$

$$= \left[\left(\frac{u+v}{2} + a_1\right)\ \left(\frac{u-v}{2} + a_1\right) = a_2 a_3 \cdots a_{n-1}\right]$$

$$[u^2 + (2a_1 + a_2 + a_3 + \cdots + a_{n-1})^2 + 2u(2a_1 + a_2 + \cdots + a_{n-1})]$$

$$= \frac{a_2\ a_3 \cdots a_{n-1}}{4}(u^2 - v^2) + a_1\ a_2\ a_3 \cdots a_{n-1} + a_1^2 a_2\ a_3 \cdots a_{n-1}$$

Setting the condition $2(2a_1 + a_2 + \cdots + a_{n-1}) = a_1\ a_2\ a_3 \cdots a_{n-1}$ and $a_2\ a_3 \cdots a_{n-1} > 4$. We obtain the following general Pell's equation

$$(a_2\, a_3 \cdots a_{n-1} - 4)u^2 - a_2\, a_3 \cdots a_{n-1} v^2 = 4[(2a_1 + a_2 + a_3 + \cdots + a_{n-1})^2 - a_1\, a_2\, a_3 \cdots a_{n-1}]. \tag{13}$$

The corresponding Pell's resolvent Equation of (13) is

$$r^2 - (a_2\ a_3 \cdots a_{n-1})(a_2\ a_3 \cdots a_{n-1})s^2 = 1 \tag{14}$$

By Theorem (2.1), we have infinitely many solution of the equation (13), we get

$$\begin{cases} u_m = u_0 r_m + (a_2\ a_3 \cdots a_{n-1}) v_0 s_m \\ v_m = v_0 r_m + (a_2\ a_3 \cdots a_{n-1} - 4) u_0 s_m \end{cases} \tag{15}$$

where, $(r_m + \sqrt{AB} s_m) = (r_1 + s_1\sqrt{AB})^m, m \geq 1$.
Here (u_0, v_0) is a minimal solution of (3) and (r_1, s_1) is the fundamental solution for (4). The Equation (2) yields the following families of positive integers solutions to

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the equation (1).

$$\begin{aligned}
(x_1)_m^{(k)} &= \frac{(u_m^{(k)} + v_m^{(k)}}{2} + a_1^{(k)}, \\
(x_2)_m^{(k)} &= u_m^{(k)} - v_m^{(k)}2 + a_1^{(k)}, \\
(x_3)_m^{(k)} &= a_2^{(k)}, \\
&. \\
&. \\
&. \\
(x_n)_m^{(k)} &= a_{(n-1)}^{(k)}.\text{for} m \geq 1.
\end{aligned}$$

4. Examples

Example 4.1 Let us assume that the different families of solution for the Diophantine Equation

$$(x_1 + x_2 + x_3 + x_4 + x_5)^2 = x_1\ x_2\ x_3\ x_4\ x_5. \tag{16}$$

Given $(x_1 + x_2 + x_3 + x_4 + x_5)^2 = x_1\ x_2\ x_3\ x_4\ x_5$
Using the transformations

$$x_1 = \frac{(u+v)}{2} + a_1,\ x_2 = \frac{(u-v)}{2} + a_1,\ x_3 = a_2,\ x_4 = a_3,\ x_5 = a_4, \tag{17}$$

where $a_1,\ a_2,\ a_3,\ a_4$ are positive integers, bring the equation (16) to the form

$$(u + 2a_1 + a_2 + a_3 + a_4)^2 = \frac{(a_2\ a_3\ a_4)}{4}(u^2 - v^2)\ + a_1\ a_2\ a_3\ a_4 u + a_1^2 a_2\ a_3 a_4.$$

Setting the conditions $2(2a_1 + a_2 + a_3 + a_4) = a_1 a_2 a_3 a_4$ and $a_2 a_3 a_4 > 4$, we obtain the following general Pell's equation

$$(a_2\ a_3\ a_4 - 4)u^2 - a_2\ a_3\ a_4\ v^2 = 4[(2a_1 + a_2 + a_3 + a_4) - a_1^2\ a_2\ a_3\ a_4]. \tag{18}$$

There are eight $(a_1,\ a_2,\ a_3,\ a_4)$ up to permutations satisfying the above conditions: (2,1,3,4), (1,12,1,3), (4,1,10,1), (5,8,1,1), (2,2,1,7), (3,2,2,2), (1,7,4,1), (6,7,1,1).
The following table contains the general Pell's equation (18) corresponding to the above $(a_1,\ a_2,\ a_3,\ a_4)$, their Pell's resolvent, both equations with their fundamental solutions.

(a_1, a_2, a_3, a_4)	General Pell's Equation (18) and its fundamental solution	Pell's resolvent equation and its fundamental solution
$(2,1,3,4)$	$2u^2 - 3v^2 = 96$ $(12,8)$	$r^2 - 6s^2 = 1$ $(5,2)$
$(1,12,1,3)$	$8u^2 - 9v^2 = 288$ $(18,16)$	$r^2 - 72s^2 = 1$ $(17,2)$
$(4,1,10,1)$	$3u^2 - 5v^2 = 480$ $(20,12)$	$r^2 - 15s^2 = 1$ $(4,1)$
$(5,8,1,1)$	$u^2 - 2v^2 = 200$ $(20,10)$	$r^2 - 2s^2 = 1$ $(3,2)$
$(2,2,1,7)$	$5u^2 - 7v^2 = 280$ $(14,10)$	$r^2 - 35s^2 = 1$ $(6,1)$
$(3,2,2,2)$	$u^2 - 2v^2 = 72$ $(12,6)$	$r^2 - 2s^2 = 1$ $(3,2)$
$(1,7,4,1)$	$6u^2 - 7v^2 = 168$ $(14,12)$	$r^2 - 42s^2 = 1$ $(13,2)$
$(6,7,1,1)$	$3u^2 - 7v^2 = 756$ $(84,54)$	$r^2 - 21s^2 = 1$ $(55,12)$

By using the formula (15) we obtain the following sequences of solutions to equation (18):

$$u_m^{(1)} = 12\ r_m^{(1)} + 24\ s_m^{(1)}, v_m^{(1)} = 8\ r_m^{(1)} + 24\ s_m^{(1)},$$

where $(r_m^{(1)} + s_m^{(1)}\sqrt{6}) = (5 + 2\sqrt{6})^m$, and $m \geq 1$;

$$u_m^{(2)} = 18\ r_m^{(2)} + 144\ s_m^{(2)}, v_m^{(2)} = 16\ r_m^{(2)} + 144\ s_m^{(2)},$$

where $(r_m^{(2)} + s_m^{(2)}\sqrt{72}) = (17 + 2\sqrt{72})^m$, and $m \geq 1$;

$$u_m^{(3)} = 20\ r_m^{(3)} + 60\ s_m^{(3)}, v_m^{(3)} = 12\ r_m^{(3)} + 60\ s_m^{(3)},$$

where $(r_m^{(3)} + s_m^{(3)}\sqrt{15}) = (4 + \sqrt{15})^m$, and $m \geq 1$;

$$u_m^{(4)} = 12\ r_m^{(4)} + 20\ s_m^{(4)}, v_m^{(1)} = 10\ r_m^{(4)} + 20\ s_m^{(4)},$$

where $(r_m^{(4)} + s_m^{(4)}\sqrt{2}) = (3 + 2\sqrt{2})^m$, and $m \geq 1$;

[1*]ovsadha@gmail.com,[2]nagajothi006@gmail.com

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$$u_m^{(5)} = 14\ r_m^{(5)} + 70\ s_m^{(5)}, v_m^{(5)} = 10\ r_m^{(5)} + 70\ s_m^{(5)},$$
where $(r_m^{(5)} + s_m^{(5)}\sqrt{35}) = (5 + 2\sqrt{35})^m,$ and $m \geq 1;$
$$u_m^{(6)} = 12\ r_m^{(6)} + 12\ s_m^{(6)}, v_m^{(6)} = 6\ r_m^{(6)} + 12\ s_m^{(6)},$$
where $(r_m^{(6)} + s_m^{(6)}\sqrt{2}) = (3 + 2\sqrt{2})^m,$ and $m \geq 1;$
$$u_m^{(7)} = 14\ r_m^{(7)} + 84\ s_m^{(7)}, v_m^{(7)} = 12\ r_m^{(7)} + 84\ s_m^{(7)},$$
where $(r_m^{(7)} + s_m^{(7)}\sqrt{42}) = (13 + 2\sqrt{42})^m,$ and $m \geq 1;$
$$u_m^{(8)} = 84\ r_m^{(8)} + 378\ s_m^{(8)}, v_m^{(8)} = 54\ r_m^{(8)} + 252\ s_m^{(8)},$$
where $(r_m^{(8)} + s_m^{(8)}\sqrt{21}) = (55 + 12\sqrt{21})^m,$ and $m \geq 1;$

Formulas (17) yield the following nine families of positive integer solutions to the equation (16):

$$(x_1)_m^{(1)} = 10\ r_m^{(1)} + 24\ s_m^{(1)} + 2,\ (x_2)_m^{(1)} = 2\ r_m^{(1)} + 2,\ (x_3)_m^{(1)} = 1,\ (x_4)_m^{(1)} = 3,$$
$$(x_5)_m^{(1)} = 4,\ m \geq 1.$$
$$(x_1)_m^{(1)} = 10\ r_m^{(1)} + 24\ s_m^{(1)} + 2,\ (x_2)_m^{(1)} = 2\ r_m^{(1)} + 2,\ (x_3)_m^{(1)} = 1,\ (x_4)_m^{(1)} = 3,$$
$$(x_5)_m^{(1)} = 4,\ m \geq 1.$$
$$(x_1)_m^{(2)} = 17\ r_m^{(2)} + 144\ s_m^{(2)} + 1\ (x_2)_m^{(2)} = r_m^{(2)} + 1,\ (x_3)_m^{(2)} = 12,\ (x_4)_m^{(2)} = 1,$$
$$(x_5)_m^{(2)} = 3,\ m \geq 1.$$
$$(x_1)_m^{(3)} = 16\ r_m^{(3)} + 60\ s_m^{(3)} + 4,\ (x_2)_m^{(3)} = 4\ r_m^{(3)} + 4,\ (x_3)_m^{(3)} = 1,\ (x_4)_m^{(3)} = 10,$$
$$(x_5)_m^{(3)} = 1,\ m \geq 1.$$
$$(x_1)_m^{(4)} = 15\ r_m^{(4)} + 20\ s_m^{(4)} + 5,\ (x_2)_m^{(4)} = 5\ r_m^{(4)} + 5,\ (x_3)_m^{(4)} = 8,\ (x_4)_m^{(4)} = 1,$$
$$(x_5)_m^{(4)} = 1,\ m \geq 1.$$
$$(x_1)_m^{(5)} = 12\ r_m^{(5)} + 70\ s_m^{(5)} + 2,\ (x_2)_m^{(5)} = 2\ r_m^{(5)} + 2,\ (x_3)_m^{(5)} = 2,\ (x_4)_m^{(5)} = 1,$$
$$(x_5)_m^{(5)} = 7,\ m \geq 1.$$
$$(x_1)_m^{(6)} = 9\ r_m^{(6)} + 12\ s_m^{(6)} + 3,\ (x_2)_m^{(6)} = 3\ r_m^{(6)} + 3,\ (x_3)_m^{(6)} = 2,\ (x_4)_m^{(6)} = 2,$$
$$(x_5)_m^{(6)} = 2,\ m \geq 1.$$
$$(x_1)_m^{(7)} = 13\ r_m^{(7)} + 84\ s_m^{(7)} + 1,\ (x_2)_m^{(7)} = \ r_m^{(7)} + 1,\ (x_3)_m^{(7)} = 7,\ (x_4)_m^{(7)} = 4,$$
$$(x_5)_m^{(7)} = 1,\ m \geq 1.$$
$$(x_1)_m^{(8)} = 69\ r_m^{(8)} + 315\ s_m^{(8)} + 6,\ (x_2)_m^{(8)} = 15\ r_m^{(8)} + 163\ s_m^{(8)} + 6,\ (x_3)_m^{(8)} = 1,$$
$$(x_4)_m^{(8)} = 3, (x_5)_m^{(8)} = 7,\ m \geq 1.$$

Example 4.2 Let us assume that the different families of solution for the

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Diophantine Equation

$$(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)^2 = x_1\, x_2\, x_3\, x_4\, x_5\, x_6. \tag{19}$$

Given $(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)^2 = x_1\, x_2\, x_3\, x_4\, x_5\, x_6$

Using the transformations

$$x_1 = \frac{(u+v)}{2} + a_1,\ x_2 = \frac{(u-v)}{2} + a_1,\ x_3 = a_2,\ x_4 = a_3,\ x_5 = a_4,\ x_6 = a_5 \tag{20}$$

where $a_1,\ a_2,\ a_3,\ a_4\ a_5$ are positive integers, bring the equation (19) to the form

$$(u+2a_1+a_2+a_3+a_4+a_5)^2 = \frac{(a_2\ a_3\ a_4\ a_5)}{4}(u^2-v^2) + a_1\ a_2\ a_3\ a_4\ a_5 u + a_1^2 a_2\ a_3\ a_4\ a_5.$$

Setting the conditions $2(2a_1 + a_2 + a_3 + a_4 + a_5) = a_1a_2a_3a_4a_5$ and $a_2a_3a_4a_5 > 4$, we obtain the following general Pell's equation

$$(a_2\ a_3\ a_4\ a_5 - 4)u^2 - a_2\ a_3\ a_4\ a_5 v^2 = 4[(2a_1+a_2+a_3+a_4+a_5) - a_1^2\ a_2\ a_3\ a_4\ a_5]. \tag{21}$$

There are five $(a_1, a_2, a_3, a_4, a_5)$ up to permutations satisfying the above conditions:

$(7, 1, 3, 2, 1), (2, 1, 8, 1, 2), (1, 4, 3, 2, 1), (2, 3, 1, 2, 2), (3, 5, 1, 2, 1).$

The following table contains the general Pell's equation (21) corresponding to the above $(a_1, a_2, a_3, a_4, a_5)$, their Pell's resolvent, both equations with their fundamental solutions.

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$(a_1, a_2, a_3, a_4, a_5)$	General Pell's Equation (21) and its fundamental solution	Pell's resolvent equation and its fundamental solution
$(7,1,3,2,1)$	$u^2 - 3v^2 = 294$ $(21,7)$	$r^2 - 7s^2 = 1$ $(2,1)$
$(2,1,8,2,1)$	$3u^2 - 4v^2 = 192$ $(16,12)$	$r^2 - 12s^2 = 1$ $(7,2)$
$(1,4,3,2,1)$	$5u^2 - 6v^2 = 120$ $(12,10)$	$r^2 - 30s^2 = 1$ $(11,2)$
$(2,3,1,2,2)$	$2u^2 - 3v^2 = 96$ $(12,8)$	$r^2 - 6s^2 = 1$ $(5,2)$
$(3,5,1,2,1)$	$3u^2 - 5v^2 = 270$ $(15,9)$	$r^2 - 15s^2 = 1$ $(4,1)$

By using the formula (15) we obtain the following sequences of solutions to equation (21):

$$u_m^{(1)} = 21\ r_m^{(1)} + 21\ s_m^{(1)}, v_m^{(1)} = 7\ r_m^{(1)} + 21\ s_m^{(1)},$$

where $(r_m^{(1)} + s_m^{(1)}\sqrt{3}) = (2+\sqrt{3})^m$, and $m \geq 1$;

$$u_m^{(2)} = 16\ r_m^{(2)} + 48\ s_m^{(2)}, v_m^{(2)} = 12\ r_m^{(2)} + 48\ s_m^{(2)},$$

where $(r_m^{(2)} + s_m^{(2)}\sqrt{12}) = (7+2\sqrt{12})^m$, and $m \geq 1$;

$$u_m^{(3)} = 12\ r_m^{(3)} + 60\ s_m^{(3)}, v_m^{(3)} = 10\ r_m^{(3)} + 60\ s_m^{(3)},$$

where $(r_m^{(3)} + s_m^{(3)}\sqrt{30}) = (11+2\sqrt{30})^m$, and $m \geq 1$;

$$u_m^{(4)} = 12\ r_m^{(4)} + 24\ s_m^{(4)}, v_m^{(1)} = 8\ r_m^{(4)} + 24\ s_m^{(4)},$$

where $(r_m^{(4)} + s_m^{(4)}\sqrt{6}) = (5+2\sqrt{6})^m$, and $m \geq 1$;

$$u_m^{(5)} = 15\ r_m^{(5)} + 45\ s_m^{(5)}, v_m^{(5)} = 9\ r_m^{(5)} + 45\ s_m^{(5)},$$

where $(r_m^{(5)} + s_m^{(5)}\sqrt{15}) = (4+\sqrt{15})^m$, and $m \geq 1$;

Formulas (20) yield the following nine families of positive integer solutions to the equation (19):

$$(x_1)_m^{(1)} = 14\ r_m^{(1)} + 21\ s_m^{(1)} + 7,\ (x_2)_m^{(1)} = 7\ r_m^{(1)} + 7,\ (x_3)_m^{(1)} = 1,\ (x_4)_m^{(1)} = 3,$$
$$(x_5)_m^{(1)} = 2,\ (x_6)_m^{(1)} = 1,\ m \geq 1.$$

$$(x_1)_m^{(2)} = 16\ r_m^{(2)} + 48\ s_m^{(2)} + 2,\ (x_2)_m^{(2)} = 4\ r_m^{(2)} + 2,\ (x_3)_m^{(2)} = 1,\ (x_4)_m^{(2)} = 8,$$
$$(x_5)_m^{(2)} = 1,\ (x_6)_m^{(2)} = 2,\ m \geq 1.$$
$$(x_1)_m^{(3)} = 11\ r_m^{(3)} + 60\ s_m^{(3)} + 1\ (x_2)_m^{(3)} = r_m^{(3)} + 1,\ (x_3)_m^{(3)} = 4,\ (x_4)_m^{(3)} = 3,$$
$$(x_5)_m^{(3)} = 2,\ (x_6)_m^{(3)} = 1,\ m \geq 1.$$
$$(x_1)_m^{(4)} = 10\ r_m^{(4)} + 24\ s_m^{(4)} + 2,\ (x_2)_m^{(4)} = 2\ r_m^{(4)} + 2,\ (x_3)_m^{(4)} = 3,\ (x_4)_m^{(4)} = 1,$$
$$(x_5)_m^{(4)} = 2,\ (x_6)_m^{(4)} = 2,\ m \geq 1.$$
$$(x_1)_m^{(5)} = 12\ r_m^{(5)} + 45\ s_m^{(5)} + 3,\ (x_2)_m^{(5)} = 3\ r_m^{(5)} + 3,\ (x_3)_m^{(5)} = 5,\ (x_4)_m^{(5)} = 1,$$
$$(x_5)_m^{(5)} = 2,\ (x_6)_m^{(5)} = 1,\ m \geq 1.$$

5. Conclusion

Although we have succeeded in indicating a general method of generating different classes of families of solutions by using the theory of Pell's equation, as per as our knowledge goes the problem of determining all class of solutions to the equation (4)is still open. The existing methods in the literature are not adequate to determine the all classes of solutions to the general Diophantine equation of n variables.

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[1*]ovsadha@gmail.com,[2]nagajothi006@gmail.com