

# Development and Analysis of Multi Copies Fuzzy Graphs

John Borg S<sup>1</sup>, Mayilvizhi <sup>2</sup> and Iruthayaraj S<sup>3</sup>

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## Abstract

This paper aims to explore and develop innovative multi copies fuzzy derived graphs and their corresponding product graphs. Derived graphs are characterized by vertices weighted based on the ratio of a generalized exponential function, while edges are identified through the differentiation and integration of this function. This study provides a comprehensive analysis of the regularity and irregularity properties of multi copies fuzzy product graphs, supported by illustrative examples. Key aspects investigated include the impact of various generalized exponential functions on vertex weighting, the influence of different differentiation and integration techniques on edge identification, and the topological properties of multi copies fuzzy derived graphs. Additionally, a comparative analysis is conducted to highlight the advantages and applications of multi copies fuzzy derived graphs in contrast to traditional graph structures. Through detailed case studies, the practical applications of multi copies fuzzy derived graphs in fields such as network analysis, data clustering and decision-making processes are demonstrated. This research contributes to the existing body of knowledge in fuzzy graph theory and lays the groundwork for future studies and applications in this domain.

**Key words:** Multi Copies Fuzzy Graphs, Derived Graphs, Generalized Exponential Function, Product Graphs

**AMS classification:** 05C72, 05C90, 68R10, 03E72.

## 1 Introduction

The authors, Santhimaheswari N.R and Sekar C (2016), focus on the behaviours of irregular fuzzy graphs and explore the concept of the  $d_m$  degree of vertices [16]. Moreover, the introduction of  $(m,k)$  regular fuzzy graphs and further investigations into totally  $(m,k)$  regular fuzzy graphs have opened avenues for comparing different classes of fuzzy graphs. By defining key concepts like

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<sup>1</sup>Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur, Tamil Nadu, India.  
Email:sjborg@gmail.com

<sup>2</sup>Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur, Tamil Nadu, India.  
Email:ashwinir452003@gmail.com

<sup>3</sup>Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur, Tamil Nadu, India.  
Email:iruthayarajsh@gmail.com

the  $d_m$  degree and exploring regularity conditions, the field continues to evolve, offering insights into the complexities of uncertain information in graph theoretical frameworks.

In[9], author clarifies how two different types of irregular fuzzy graphs neighbourly and highly irregular can be considered equivalent and establishes foundational results for the study of neighbourly irregular fuzzy graphs. To better understand, how these different types of fuzzy graphs interact and differ in their support neighbourly properties. The basic concepts of fuzzy graph is discussed in [11]. The transmission problem on graphs and digraphs are studied in[10]. In[4], the authors discussed how to investigate the solution of the difference equation. Also, the authors explore the properties of a newly defined extorial function and use it to solve a higher order difference equation with constant coefficients within the framework of discrete calculus.

The paper includes examples to validate the results. Furthermore, the authors introduce a novel function, called the extorial function, which is defined by polynomials with polynomial factorials. Here we introduce generalized extorial function by adding shift values. We derive identities involving difference operators and generalized extorial function. Some interesting results on the relationship among generalized extorial function, as well as its sums are obtained. This study also describes how to use the generalized extorial function to arrive at fuzzy delta graphs and their product graphs. By choosing the right value on  $x$ , such as for electron charge in molecular structures, we can obtain applications in the field of physical science because the generated graph's structure is similar to chemical bonds.

## 2 Preliminaries

In this section, we present basic concept of delta operator.

**Definition 2.1** The falling polynomial function  $t^{(n)} = t(t-1)(t-2)\cdots(t-(n-1))$ .

**Definition 2.2** The forward delta operator  $\Delta$  on real valued function  $f$  is defined as

$$\Delta f(t) = f(t+1) - f(t)$$

**Example 2.3** If  $f(t) = t^{(2)}$ , then

$$\Delta t^{(2)} = (t+1)^{(2)} - t^{(2)} = (t+1)(t) - t(t-1) = 2t^{(1)}.$$

In general,  $\Delta t^{(n)} = nt^{(n-1)}$

**Definition 2.4** The inverse delta operator, denoted as  $\Delta^{-1}$  is defined as if  $\Delta f(t) = g(t)$  then  $f(t) = \Delta^{-1} g(t) + c$ , where  $c$  is a constant

*Example:*  $\Delta t^{(2)} = 2t^{(1)} \Rightarrow \Delta^{-1} t^{(1)} = \frac{t^{(2)}}{2}$

**Theorem 2.5** If  $\Delta^{-1} f(t)$  exists, then  $\sum_{r=0}^m f(t+m-r) = \Delta^{-1} f(t+m+1) - \Delta^{-1} f(t)$ , for  $m = \{1, 2, 3, \dots\}$ .

*Proof:* From the definition of delta operator defined in definition (2.2), we have Let  $\Delta f(t) = f(t+1) - f(t)$

Taking  $\Delta^{-1} f(t) = g(t) \Rightarrow f(t) = \Delta g(t)$

which gives,  $f(t) = g(t+1) - g(t)$

Hence

$$f(t) + g(t) = g(t+1) \tag{1}$$

Replacing  $t$  by  $t+1$  in equation (1), we find

$$f(t+1) + g(t+1) = g(t+2)$$

Substituting  $g(t+1)$  given by equation (1) in the above expression, we get

$$f(t+1) + f(t) + g(t) = g(t+2)$$

Again replacing  $t$  by  $t+1$  gives

$$f(t+2) + f(t+1) + g(t+1) = g(t+3)$$

Substituting equation (1) in above relation, we arrive

$$f(t+2) + f(t+1) + f(t) + g(t) = g(t+3)$$

⋮

In general, we derive

$$f(t+m) + f(t+m-1) + \dots + f(t) + g(t) = g(t+m+1)$$

ie)  $f(t) + f(t+1) + \dots + f(t+m) = \Delta^{-1} f(t+m+1) - \Delta^{-1} f(t)$

The above equation can be expressed

$$\sum_{r=0}^m f(t+m-r) = \Delta^{-1}f(t+m+1) - \Delta^{-1}f(t) \quad (2)$$

In the following theorem we derive second order summation formula related to  $\Delta^{-2}$ .

**Theorem 2.6** Let  $\Delta^{-1}f(t)$  and  $\Delta^{-2}f(t)$  exists

$\sum_{r=0}^m (r+1)f(t+m-r) = \Delta^{-2}f(t+m+2) - \Delta^{-2}f(t) - (m+2)\Delta^{-1}f(t), \forall m \in \mathbb{Z}^+$  **Proof**  
 By equation (2), we have

$$\sum_{r=0}^m f(t+m-r) = \Delta^{-1}f(t+m+1) - \Delta^{-1}f(t)$$

Applying  $\Delta^{-1}$  on both sides,

$$\Delta^{-1}f(t) + \Delta^{-1}f(t+1) + \dots + \Delta^{-1}f(t+m) = \Delta^{-2}f(t+m+1) - \Delta^{-2}f(t)$$

Put  $m = 4$  in equation (2) to get

$$f(t) + f(t+1) + f(t+2) + f(t+3) + f(t+4) = \Delta^{-1}f(t+5) - \Delta^{-1}f(t)$$

Put  $m = 3$  in equation (2) to arrive

$$f(t) + f(t+1) + f(t+2) + f(t+3) = \Delta^{-1}f(t+4) - \Delta^{-1}f(t)$$

Put  $m = 2$  in equation (2), we find

$$f(t) + f(t+1) + f(t+2) = \Delta^{-1}f(t+3) - \Delta^{-1}f(t)$$

Put  $m = 1$  in equation (2), to derive

$$f(t) + f(t+1) = \Delta^{-1}f(t+2) - \Delta^{-1}f(t)$$

Put  $m = 0$  in equation (2)

$$f(t) = \Delta^{-1}f(t+1) - \Delta^{-1}f(t)$$

$$0 = \Delta^{-1}f(t) - \Delta^{-1}f(t)$$

Adding the above equations, we find

$$\begin{aligned} & 5f(t) + 4f(t+1) + 3f(t+2) + 2f(t+3) + f(t+4) \\ &= \Delta^{-2}f(t+6) - \delta^{-2}f(t) - 6\Delta^{-1}f(t)(m+1)f(t) + mf(t+1) + (m-1)f(t+2) + \dots + f(t+m) \\ &= \Delta^{-2}f(t+m+2) - \Delta^{-2}f(t) - (m+2)\Delta^{-1}f(t) \end{aligned}$$

Therefore, the above expression can be written as

$$\sum_{r=0}^m (r+1)f(t+m-r) = \Delta^{-2}f(t+m+2) - \Delta^{-2}f(t) - (m+2)\Delta^{-1}f(t) \quad (3)$$

In the following theorem we derive third order summation formula related to  $\Delta^{-3}$ .

**Theorem 2.7** If  $\Delta^{-1}f(t)$ ,  $\Delta^{-2}f(t)$  and  $\Delta^{-3}f(t)$  exists, then for +ve integer m

$$\sum_{r=0}^m \frac{(r+2)^{(2)}}{2} f(t+m-r) = \Delta^{-3}f(t+m+3) - \Delta^{-3}f(t) - (m+3)\Delta^{-2}f(t) - \frac{(m+3)^{(2)}}{2}\Delta^{-1}f(t).$$

*Proof:* By equation (3), We have

$$\sum_{r=0}^m (r+1)f(t+m-r) = \Delta^{-2}f(t+m+2) - \Delta^{-2}f(t) - (m+2)\Delta^{-1}f(t)$$

Put  $m = 4$  in equation (3) to get

$$f(t+4) + 2f(t+3) + 3f(t+2) + 4f(t+1) + 5f(t) = \Delta^{-2}f(t+6) - \Delta^{-2}f(t) - (6)\Delta^{-1}f(t)$$

Put  $m = 3$  in equation (3) to arrive

$$f(t+3) + 2f(t+2) + 3f(t+1) + 4f(t) = \Delta^{-2}f(t+5) - \Delta^{-2}f(t) - (5)\Delta^{-1}f(t)$$

Put  $m = 2$  in equation (3), we find

$$f(t+2) + 2f(t+1) + 3f(t) = \Delta^{-2}f(t+4) - \Delta^{-2}f(t) - (4)\Delta^{-1}f(t)$$

Put  $m = 1$  in equation (3), to derive

$$f(t+1) + 2f(t) = \Delta^{-2}f(t+3) - \Delta^{-2}f(t) - (3)\Delta^{-1}f(t)$$

Put  $m = 0$  in equation (3)

$$f(t) = \Delta^{-2}f(t+2) - \Delta^{-2}f(t) - (2)\Delta^{-1}f(t)$$

$$0 = \Delta^{-2}f(t+1) - \Delta^{-2}f(t) - (1)\Delta^{-1}f(t)$$

$$0 = \Delta^{-2}f(t) - \Delta^{-2}f(t) - (0)\Delta^{-1}f(t)$$

Adding the above equations, we find

$$f(t+4) + (1+2)f(t+3) + (1+2+3)f(t+2) + (1+2+3+4)f(t+1) + (1+2+3+4+5)f(t) = \Delta^{-3}f(t+m+3) - \Delta^{-3}f(t) - (m+3)\Delta^{-2}f(t) - \frac{(7)^{(2)}}{2}\Delta^{-1}f(t)$$

Therefore, the above expression can be written as

$$\sum_{r=0}^m \frac{(r+2)^{(2)}}{2} f(t+m-r) = \Delta^{-3} f(t+m+3) - \Delta^{-3} f(t) - (m+3)\Delta^{-2} f(t) - \frac{(m+3)^{(2)}}{2} \Delta^{-1} f(t) \quad (4)$$

In the following theorem we derive fourth order summation formula related to  $\Delta^{-4}$ .

**Theorem 2.8** Let  $\Delta^{-1} f(t)$ ,  $\Delta^{-2} f(t)$ ,  $\Delta^{-3} f(t)$  and  $\Delta^{-4} f(t)$  exists then for positive integer  $m$ , we have

$$\sum_{r=0}^m \frac{(r+3)^{(3)}}{3^{(2)}} f(t+m-r) = \Delta^{-4} f(t+m+4) - \Delta^{-4} f(t) - (m+4)\Delta^{-3} f(t) - \frac{(m+4)^{(2)}}{2} \Delta^{-2} f(t) - \frac{(m+4)^{(3)}}{3^{(2)}} \Delta^{-1} f(t)$$

*Proof:* By equation 4, We have

$$\sum_{r=0}^m \frac{(r+2)^{(2)}}{2} f(t+m-r) = \Delta^{-3} f(t+m+3) - \Delta^{-3} f(t) - (m+3)\Delta^{-2} f(t) - \frac{(m+3)^{(2)}}{2} \Delta^{-1} f(t)$$

Put  $m = 4$  in equation (4) to get

$$\begin{aligned} \sum_{r=0}^3 \frac{(r+2)^{(2)}}{2} f(t+4-r) &= \Delta^{-3} f(t+4+3) - \Delta^{-3} f(t) - (4+3)\Delta^{-2} f(t) - \frac{(4+3)^{(2)}}{2} \Delta^{-1} f(t) \\ \frac{(2)^{(2)}}{2} f(t+4) + \frac{(3)^{(2)}}{2} f(t+3) + \frac{(4)^{(2)}}{2} f(t+2) + \frac{(5)^{(2)}}{2} f(t+1) + \frac{(6)^{(2)}}{2} f(t) &= \Delta^{-3} f(t+6) - \Delta^{-3} f(t) - (6)\Delta^{-2} f(t) - \frac{(6)^{(2)}}{2} \Delta^{-1} f(t) \\ f(t+4) + (3)f(t+3) + (6)f(t+2) + (10)f(t+1) + (15)f(t) &= \Delta^{-3} f(t+7) - \Delta^{-3} f(t) - (7)\Delta^{-2} f(t) - \frac{(7)^{(2)}}{2} \Delta^{-1} f(t) \end{aligned}$$

Put  $m = 3$  in equation (4) to arrive

$$\begin{aligned}
 \sum_{r=0}^3 \frac{(r+2)^{(2)}}{2} f(t+3-r) &= \Delta^{-3} f(t+3+3) - \Delta^{-3} f(t) \\
 &\quad - (3+3)\Delta^{-2} f(t) - \frac{(3+3)^{(2)}}{2} \Delta^{-1} f(t) \\
 &= \frac{(2)^{(2)}}{2} f(t+3) + \frac{(3)^{(2)}}{2} f(t+2) + \frac{(4)^{(2)}}{2} f(t+1) + \frac{(5)^{(2)}}{2} f(t) \\
 &= \Delta^{-3} f(t+6) - \Delta^{-3} f(t) - (6)\Delta^{-2} f(t) - \frac{(6)^{(2)}}{2} \Delta^{-1} f(t) \\
 &\quad f(t+3) + (3)f(t+2) + (6)f(t+1) + (10)f(t) \\
 &= \Delta^{-3} f(t+6) - \Delta^{-3} f(t) - (6)\Delta^{-2} f(t) - \frac{(6)^{(2)}}{2} \Delta^{-1} f(t)
 \end{aligned}$$

Put  $m = 2$  in equation (4), we find

$$\begin{aligned}
 \sum_{r=0}^2 \frac{(r+2)^{(2)}}{2} f(t+2-r) &= \Delta^{-3} f(t+2+3) - \Delta^{-3} f(t) - (2+3)\Delta^{-2} f(t) \\
 &\quad - \frac{(2+3)^{(2)}}{2} \Delta^{-1} f(t) \\
 &= \frac{(2)^{(2)}}{2} f(t+2) + \frac{(3)^{(2)}}{2} f(t+1) + \frac{(4)^{(2)}}{2} f(t) \\
 &= \Delta^{-3} f(t+5) - \Delta^{-3} f(t) - (5)\Delta^{-2} f(t) - \frac{(5)^{(2)}}{2} \Delta^{-1} f(t) \quad f(t+2) + (3)f(t+1) + (6)f(t) \\
 &= \Delta^{-3} f(t+5) - \Delta^{-3} f(t) - (5)\Delta^{-2} f(t) - \frac{(5)^{(2)}}{2} \Delta^{-1} f(t)
 \end{aligned}$$

Put  $m = 1$  in equation (4) to derive

$$\begin{aligned}
 \sum_{r=0}^1 \frac{(r+2)^{(2)}}{2} f(t+1r) &= \Delta^{-3} f(t+1+3) - \Delta^{-3} f(t) - (1+3)\Delta^{-2} f(t) - \frac{(1+3)^{(2)}}{2} \Delta^{-1} f(t) \\
 &= \frac{(2)^{(2)}}{2} f(t+1) + \frac{(3)^{(2)}}{2} f(t) = \Delta^{-3} f(t+4) - \Delta^{-3} f(t) - (4)\Delta^{-2} f(t) - \frac{(4)^{(2)}}{2} \Delta^{-1} f(t) \\
 &\quad f(t+1) + (3)f(t) = \Delta^{-3} f(t+4) - \Delta^{-3} f(t) - (4)\Delta^{-2} f(t) - \frac{(4)^{(2)}}{2} \Delta^{-1} f(t)
 \end{aligned}$$

Put  $m = 0$  in equation (4)

$$\begin{aligned}
 \frac{(2)^{(2)}}{2} f(t) &= \Delta^{-3} f(t+3) - \Delta^{-3} f(t) - (3)\Delta^{-2} f(t) - \frac{(3)^{(2)}}{2} \Delta^{-1} f(t) \\
 f(t) &= \Delta^{-3} f(t+3) - \Delta^{-3} f(t) - (3)\Delta^{-2} f(t) - \frac{(3)^{(2)}}{2} \Delta^{-1} f(t)
 \end{aligned}$$

$$0 = \Delta^{-3}f(t+2) - \Delta^{-3}f(t) - (2)\Delta^{-2}f(t) - \frac{(2)^{(2)}}{2}\Delta^{-1}f(t)$$

$$0 = \Delta^{-3}f(t+1) - \Delta^{-3}f(t) - (1)\Delta^{-2}f(t) - \frac{(1)^{(2)}}{2}\Delta^{-1}f(t)$$

$$0 = \Delta^{-3}f(t) - \Delta^{-3}f(t)$$

Adding the above equations, we find

$$\begin{aligned} & f(t+4) + (1+3)f(t+3) + (1+3+6)f(t+2) + (1+3+6+10)f(t+1) \\ & + (1+3+6+10+15)f(t) = \Delta^{-4}f(t+m+4) - \Delta^{-4}f(t) - (m+4)\Delta^{-3}f(t) \\ & \quad - \frac{(m+4)^{(2)}}{2}\Delta^{-2}f(t) - (1+3+6+10+15+21)\Delta^{-1}f(t) \end{aligned}$$

$$\begin{aligned} & f(t+4) + (4)f(t+3) + (10)f(t+2) + (20)f(t+1) + (35)f(t) \\ & = \Delta^{-4}f(t+m+4) - \Delta^{-4}f(t) - (m+4)\Delta^{-3}f(t) - \frac{(m+4)^{(2)}}{2}\Delta^{-2}f(t) - (56)\Delta^{-1}f(t) \end{aligned}$$

Therefore, the above expression can be written as

$$\begin{aligned} \sum_{r=0}^m \frac{(r+3)^{(3)}}{3^{(2)}} f(t+m-r) &= \Delta^{-4}f(t+m+4) - \Delta^{-4}f(t) \\ &\quad - (m+4)\Delta^{-3}f(t) - \frac{(m+4)^{(2)}}{2}\Delta^{-2}f(t) - \frac{(m+4)^{(3)}}{3^{(2)}}\Delta^{-1}f(t) \end{aligned}$$

**Theorem 2.9** Let  $\Delta^{-n}f(t)$  exists, Then for +ve integer m, we have

$$\begin{aligned} \sum_{r=0}^m \frac{(r+n-1)^{(n-1)}}{(n-1)!} f(t+m-r) &= \Delta^{-n}f(t+m+n) - \Delta^{-n}f(t) - \frac{(m+n)^{(1)}}{1!}\Delta^{-(n-1)}f(t) \\ &\quad - \frac{(m+n)^{(2)}}{2!}\Delta^{-(n-2)}f(t) - \frac{(m+n)^{(n-1)}}{(n-1)!}\Delta^{-1}f(t) \end{aligned}$$

### 3 Delta Graph

In this section by employing fuzzy vertex values and difference of generalized extorial function, we developed new type of fuzzy delta graph and present basic concepts of the construction of delta graphs. Here we use the notation  $\mathbb{N}(a) = \{a, a+1, a+2 \dots\}$

**Lemma 3.1** For the positive integer's' we have

$$\Delta^s x^{(m)} = m^{(s)} x^{(m-s)}, \tag{5}$$

where  $m^{(s)} = m(m-1)(m-2)\dots(m-(s-1))$ .

**Example 3.2**  $\Delta^s x^{(m)} = \Delta^{s-1} (\Delta x^{(m)}) = \Delta^{s-1} (m)x^{(m-1)}$   
 $= \Delta^{s-2} (\Delta m x^{(m-1)}) = \Delta^{s-2} (m(m-1)x^{(m-2)})$

Continuing this process we get (5)

**Definition 3.3** For  $x \in \mathbb{R}$  and  $m, \kappa \in \mathbb{N}(0)$ , the generalized extorial function, denoted as  $e(\kappa, x^{(m)})$ , is defined as

$$\sum_{r=0}^{\infty} \frac{x^{(m+r\kappa)}}{(m+r\kappa)!} = e(x^{(m)}, \kappa) \tag{6}$$

Here,  $m$  and  $\kappa$  denote the initial power and the shift value of generalized extorial function(6)

**Lemma 3.4** For  $m \in \mathbb{N}(1)$ , we have  $\frac{x^{(m-1)}}{(m-1)!} = \Delta \frac{x^{(m)}}{m!}$ .

**Proof:** The proof follows from (5) and  $\frac{m}{m!} = \frac{1}{(m-1)!}$

**Lemma 3.5** For  $m = 0, 1, 2 \dots \kappa$ ,  $\kappa \in \mathbb{N}(2)$ , and  $0 \leq r \leq m$ ,  $r \in \mathbb{N}(0)$ , we have

$$\Delta^r e(x^{(m)}, \kappa) = \begin{cases} e(x^{((m-r) \bmod \kappa)}, \kappa), r > m, \\ e(x^{(m-r)}, \kappa), r \leq m \end{cases} \tag{7}$$

**Proof:** Since  $e(x^{(m)}, \kappa)$  is convergent for each  $x \in \mathbb{R}$  and  $(-\kappa)! = \infty$  for  $\mathbb{N}(1)$ , operating  $\Delta$  to each term of  $e(x^{(m)}, \kappa)$  in (6), we get

$$\Delta e(x^{(m)}, \kappa) = \left( \frac{x^{(m-1)}}{(m-1)!} + \frac{x^{(m+\kappa-1)}}{(m+\kappa-1)!} + \dots \right)$$

$$= e(\kappa, x^{(m-1)}).$$

Applying  $\Delta^2$  on  $e(x^{(m)}, \kappa)$ , we get

$$\Delta^2 e(x^{(m)}, \kappa) = \left( \frac{x^{(m-2)}}{(m-2)!} + \frac{x^{(m+\kappa-2)}}{(m+\kappa-2)!} + \dots \right)$$

$$= e(\kappa, x^{(m-2)})$$

$$\vdots$$

In general applying  $\Delta^r$  on  $e(x^{(m)}, \kappa)$ , we derive equation (7).

Note that  $e(1, x^{(0)})$  is given below

$$1 + \frac{x^{(1)}}{1!} + \frac{x^{(2)}}{2!} + \frac{x^{(4)}}{4!} + \frac{x^{(5)}}{5!} + \dots \infty = e(x^{(0)}, 1)$$

$$\frac{x^{(0)}}{0!} + \frac{x^{(5)}}{5!} + \frac{x^{(10)}}{10!} + \dots \infty = e(x^{(5)}, 5), \quad \frac{x^{(4)}}{4!} + \frac{x^{(9)}}{9!} + \dots \infty = e(x^{(4)}, 5),$$

$$\frac{x^{(3)}}{3!} + \frac{x^{(8)}}{8!} + \dots \infty = e(x^{(3)}, 5), \quad \frac{x^{(2)}}{2!} + \frac{x^{(7)}}{7!} + \dots \infty = e(x^{(2)}, 5),$$

$$\frac{x^{(1)}}{1!} + \frac{x^{(6)}}{6!} + \dots \infty = e(x^{(1)}, 5), \quad \frac{x^{(0)}}{0!} + \frac{x^{(5)}}{5!} + \dots \infty = e(x^{(0)}, 5)$$

Consider the extorial function

$$e(x^{(0)}, m) = \frac{x^{(0)}}{0!} + \frac{x^{(m)}}{m!} + \dots$$

Taking  $\Delta$  on bothsides and apply  $\Delta x^{(m)} = mx^{(m-1)}$ ,

$$\Delta e(x^0, m) = \Delta \left\{ \frac{x^{(0)}}{0!} + \frac{x^{(m)}}{m!} + \dots \right\}$$

$$= \frac{x^{(m-1)}}{m-1!} + \dots = e(x^{(m-1)}, m)$$

### 3.1 First Order Delta Graph

Consider the graph  $G = (V, E)$ , where  $V = \{e(x^{(0)}, m), e(x^{(2)}, m), \dots, e(x^{(m-1)}, m)\}$

and  $E = \{e(x^{(r)}, m) \rightarrow \Delta e(x^{(r)}, m) / r = 0, 1, 2, \dots, m-1\}$  is called first order graph.

**Example 3.6**  $\Delta e(x^{(0)}, 5) = \frac{x^{(4)}}{4!} + \frac{x^{(9)}}{9!} + \dots$

$$\Delta e(x^{(0)}, 5) = e(x^{(4)}, 5), \quad \Delta e(x^{(1)}, 5) = e(x^{(0)}, 5), \quad \Delta e(x^{(2)}, 5) = e(x^{(1)}, 5),$$

$$\Delta e(x^{(3)}, 5) = e(x^{(2)}, 5), \quad \Delta e(x^{(4)}, 5) = e(x^{(3)}, 5)$$

$$E = \{e(x^{(0)}, 5) \rightarrow e(x^{(4)}, 5), e(x^{(1)}, 5) \rightarrow e(x^{(0)}, 5), e(x^{(2)}, 5) \rightarrow e(x^{(1)}, 5), \\ e(x^{(3)}, 5) \rightarrow e(x^{(2)}, 5), e(x^{(4)}, 5) \rightarrow e(x^{(3)}, 5)\}$$

$G = (V, E)$  is called first order delta graph

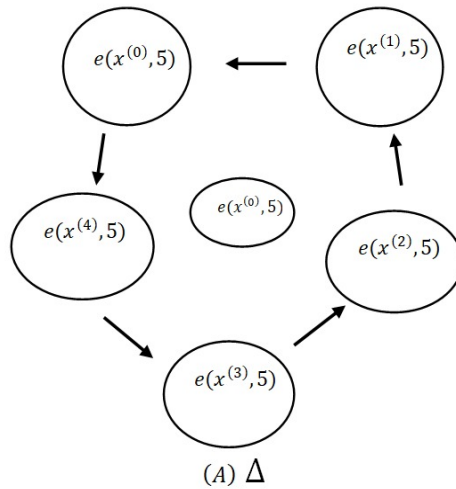


Figure 1: First Order Delta Graph

### 3.2 Second Order Delta Graph

Consider the graph  $G^{(2)} = (V^{(1)}, E^{(1)})$ , where  $V = \{e(x^{(r)}, m) / r = 1, 2, 3, \dots\}$  and  $E = \{e(x^{(r)}, m) \rightarrow \Delta^2 e(x^{(r)}, m)\}$  is called second order delta graph.

**Example 3.7**  $\Delta^2 e(x^{(0)}, 5) = \frac{x^{(3)}}{3!} + \frac{x^{(8)}}{8!} + \dots$

$$\Delta^2 e(x^{(0)}, 5) = e(x^{(3)}, 5), \Delta^2 e(x^{(1)}, 5) = e(x^{(4)}, 5), \Delta^2 e(x^{(2)}, 5) = e(x^{(0)}, 5),$$

$$\Delta^2 e(x^{(3)}, 5) = e(x^{(1)}, 5), \Delta e(x^{(4)}, 5) = e(x^{(2)}, 5)$$

Consider the edge set  $E = \{e(x^{(0)}, 5) \rightarrow e(x^{(3)}, 5), e(x^{(1)}, 5) \rightarrow e(x^{(4)}, 5),$

$$e(x^{(2)}, 5) \rightarrow e(x^{(0)}, 5), e(x^{(3)}, 5) \rightarrow e(x^{(1)}, 5), e(x^{(4)}, 5) \rightarrow e(x^{(2)}, 5)\}$$

$G = (V, E)$  is called second order delta graph.

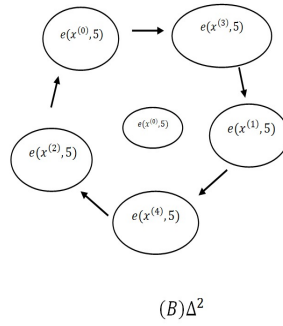


Figure 2: Second Order Delta Graph

### 3.3 Third Order Delta Graph

Consider the graph  $G^{(3)} = (V^{(2)}, E^{(2)})$ , where  $V = \{e(x^{(r)}, m) / r = 1, 2, 3, \dots\}$  and  $E^{(3)} = \{e(x^{(r)}, m) \rightarrow \Delta^3 e(x^{(r)}, m)\}$  is called third order delta graph.

**Example 3.8**  $\Delta^3 e(x^{(0)}, 5) = \frac{3x^{(2)}}{3!} + \frac{8x^{(7)}}{8!} + \dots = \frac{x^{(2)}}{2!} + \frac{x^{(7)}}{7!} + \dots$

$\Delta^3 e(x^{(0)}, 5) = e(x^{(2)}, 5), \Delta^3 e(x^{(1)}, 5) = e(x^{(3)}, 5), \Delta^3 e(x^{(2)}, 5) = e(x^{(4)}, 5),$

$\Delta^3 e(x^{(3)}, 5) = e(x^{(0)}, 5), \Delta^3 e(x^{(4)}, 5) = e(x^{(1)}, 5)$

$E = \{e(x^{(0)}, 5) \rightarrow e(x^{(2)}, 5), e(x^{(1)}, 5) \rightarrow e(x^{(3)}, 5), e(x^{(2)}, 5) \rightarrow e(x^{(4)}, 5),$   
 $e(x^{(3)}, 5) \rightarrow e(x^{(0)}, 5), e(x^{(4)}, 5) \rightarrow e(x^{(1)}, 5)\}$

$G = (V, E)$  is called third order delta graph

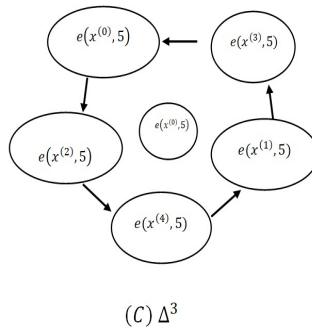


Figure 3: Third Order Delta Graph

### 3.4 Forth Order Delta Graph

Consider the graph  $G^{(4)} = (V^{(3)}, E^{(3)})$ , where  $V = \{e(x^{(r)}, m) / r = 1, 2, 3, \dots\}$  and  $E^{(4)} = \{e(x^{(r)}, m) \rightarrow \Delta^4 e(x^{(r)}, m)\}$  is called fourth order delta graph.

**Example 3.9**  $\Delta^4 e(x^{(0)}, 5) = \frac{2x^{(1)}}{2!} + \frac{7x^{(6)}}{7!} + \dots = \frac{x^{(1)}}{1!} + \frac{x^{(6)}}{6!} + \dots$   
 $\Delta^4 e(x^{(0)}, 5) = e(x^{(1)}, 5)$ ,  $\Delta^4 e(x^{(1)}, 5) = e(x^{(2)}, 5)$ ,  $\Delta^4 e(x^{(2)}, 5) = e(x^{(3)}, 5)$ ,  
 $\Delta^4 e(x^{(3)}, 5) = e(x^{(4)}, 5)$ ,  $\Delta^4 e(x^{(4)}, 5) = e(x^{(0)}, 5)$   
 $E = \{e(x^{(0)}, 5) \rightarrow e(x^{(1)}, 5), e(x^{(1)}, 5) \rightarrow e(x^{(2)}, 5), e(x^{(2)}, 5) \rightarrow e(x^{(3)}, 5),$   
 $e(x^{(3)}, 5) \rightarrow e(x^{(4)}, 5), e(x^{(4)}, 5) \rightarrow e(x^{(0)}, 5)\}$   
 $G = (V, E)$  is called fourth order delta graph

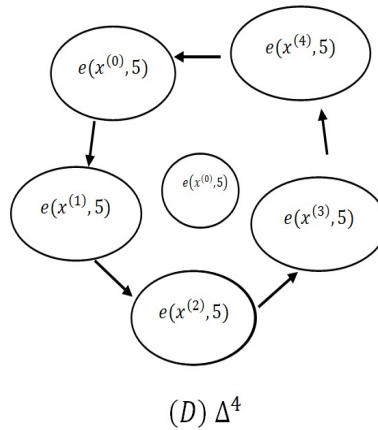


Figure 4: Fourth Order Delta Graph

In general for the function  $e(x^{(r)}, m)$  the  $m^{th}$  order delta graph is spinning graph.

### 3.5 Fifth Order Delta Graph

Consider the graph  $G^{(5)} = (V^{(4)}, E^{(4)})$ , where  $V = \{e(x^{(r)}, m) / r = 1, 2, 3, \dots\}$  and  $E^{(5)} = \{e(x^{(r)}, m) \rightarrow \Delta^5 e(x^{(r)}, m)\}$  is called fifth order delta graph.

**Example 3.10**  $\Delta^5 e(x^{(0)}, 5) = \frac{1x^{(0)}}{1!} + \frac{6x^{(5)}}{6!} + \dots = \frac{x^{(0)}}{0!} + \frac{x^{(5)}}{5!} + \dots$   
 $\Delta^5 e(x^{(0)}, 5) = e(x^{(0)}, 5)$ ,  $\Delta^5 e(x^{(1)}, 5) = e(x^{(1)}, 5)$ ,  $\Delta^5 e(x^{(2)}, 5) = e(x^{(2)}, 5)$ ,

$\Delta^5 e(x^{(3)}, 5) = e(x^{(3)}, 5), \Delta^5 e(x^{(4)}, 5) = e(x^{(4)}, 5)$   
 $E = \{e(x^{(0)}, 5) \rightarrow e(x^{(0)}, 5), e(x^{(1)}, 5) \rightarrow e(x^{(1)}, 5), e(x^{(2)}, 5) \rightarrow e(x^{(2)}, 5),$   
 $e(x^{(3)}, 5) \rightarrow e(x^{(3)}, 5), e(x^{(4)}, 5) \rightarrow e(x^{(4)}, 5)\}$   
 $G = (V, E)$  is called fifth order delta graph

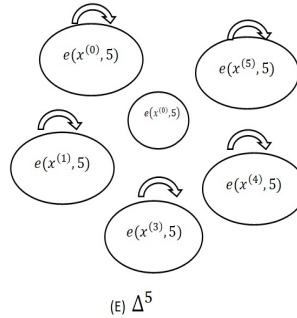


Figure 5: Fifth Order Delta Graph

## 4 Multi Copies Fuzzy Graph

In this section we apply fuzzy concept to multi copies of delta graph.

**Definition 4.1** Let  $G_1 = (V_1, E_1)$  be a graph with vertex set  $V_1 = \{v_0, v_1, v_2, \dots, v_n\}$  and  $G_2 = (V_2, E_2)$  be another graph of with vertex set. The  $m$ -multi-copies of  $G_1$  with  $G_2$  is a graph  $G_m = (V_m, E_m)$  where  $V_m = V_2 \cup m \times V_1$  and  $E_m = E_2 \cup m \times E_1 \cup \{u_0 v_0, u_1 v_0, u_2 v_0 \dots u_m v_0\}$ .

**Definition 4.2** Let  $\bar{G}_1 = (V_1, E_1, \sigma_1, \mu_1)$  be the fuzzy graph and  $\bar{G}_2 = (V_2, E_2, \sigma_2, \mu_2)$  be the fuzzy graph. Let  $G_m = (V_m, E_m)$  be multi-copies graph of  $G_1$  with respect to  $G_2$ . Let  $\sigma_m = \sigma_1 \cup \sigma_2$  and  $\mu_m = \mu_1 \cup \mu_2$  then  $\bar{G}_m(V_m, E_m, \sigma_m, \mu_m)$  is called multi-copies fuzzy graph.

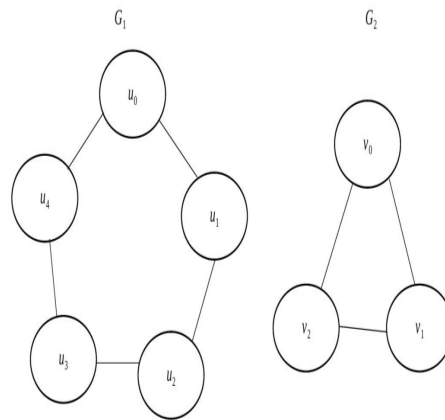


Figure 6:  $G_1$  and  $G_2$  Graph

$\sigma_1$ -values,  $\mu_1$  - values of  $G_1$  is given below,

$\sigma_1$	$u_0$	$u_1$	$u_2$	$u_3$	$u_4$
	$\frac{e(x^{(0)}, 5)}{e}$	$\frac{e(x^{(1)}, 5)}{e}$	$\frac{e(x^{(2)}, 5)}{e}$	$\frac{e(x^{(3)}, 5)}{e}$	$\frac{e(x^{(4)}, 5)}{e}$
$\mu_1$	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$
	$\frac{e(x^{(1)}, 5)}{e}$	$\frac{e(x^{(2)}, 5)}{e}$	$\frac{e(x^{(3)}, 5)}{e}$	$\frac{e(x^{(4)}, 5)}{e}$	$\frac{e(x^{(4)}, 5)}{e}$

$\sigma_2$ -values,  $\mu_2$  - values of  $G_2$  is given below,

$\sigma_2$	$v_0$	$v_1$	$v_2$
	$\frac{e(x^{(0)}, 5)}{e}$	$\frac{e(x^{(1)}, 5)}{e}$	$\frac{e(x^{(2)}, 5)}{e}$
$\mu_1$	$e_1$	$e_2$	$e_3$
	$\frac{e(x^{(1)}, 5)}{e}$	$\frac{e(x^{(2)}, 5)}{e}$	$\frac{e(x^{(3)}, 5)}{e}$

Using the following formula we get suzzy values for multi graph

(i)  $\sigma(u_0) =$   
(ii)  $\mu(u_0, u_1) = \min \left\{ \frac{e(x^{(0)}, 5)}{e}, \frac{e(x^{(1)}, 5)}{e} \right\}$

Vertex values of multi copies are calculated as follows:  $\sigma(u_0) = 0.3679$ ,

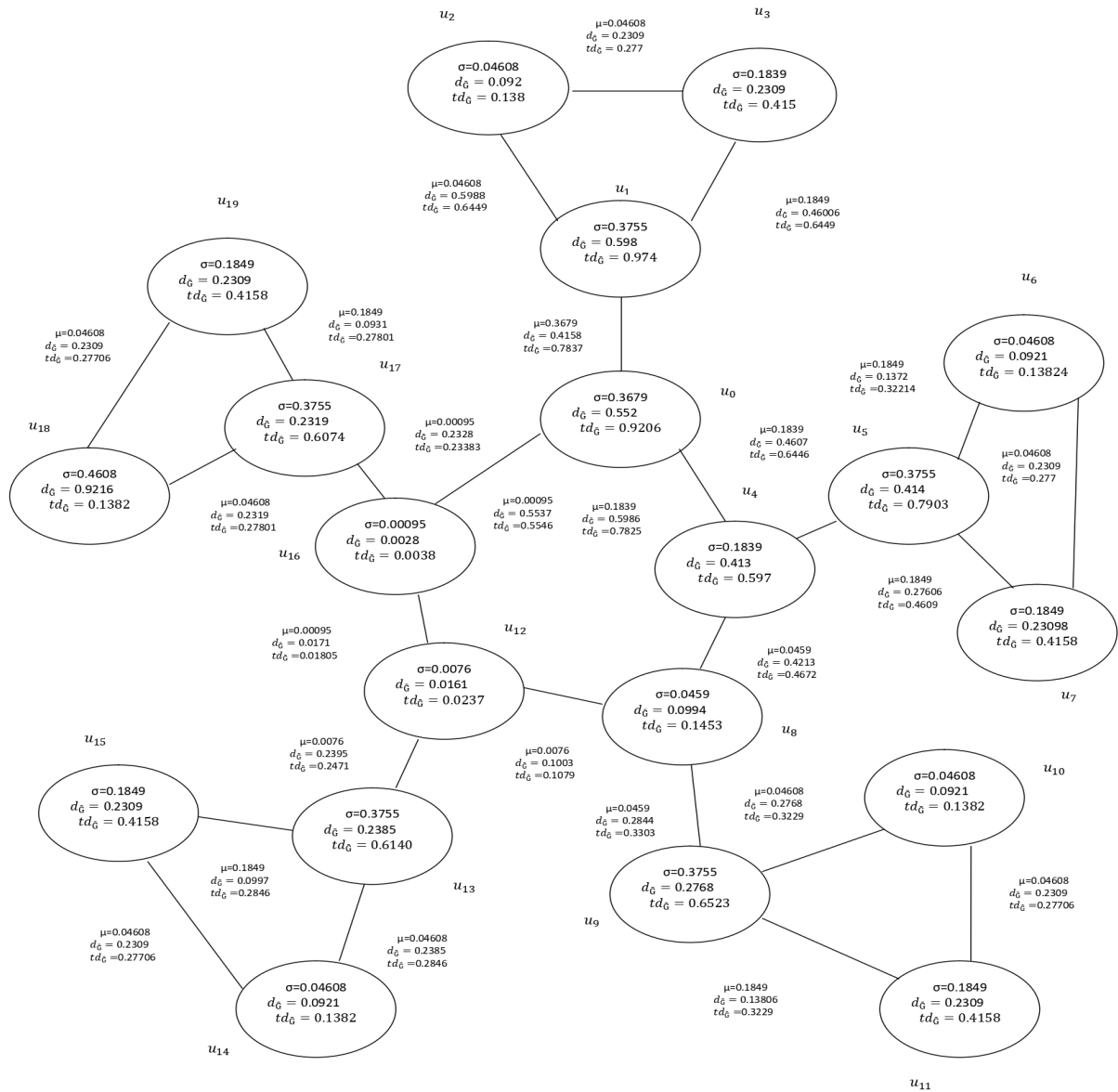


Figure 7: Multi Copies Fuzzy Graph

$$\begin{aligned} \sigma(u_1) &= 0.3755, \sigma(u_2) = 0.04608, \sigma(u_3) = 0.1849, \sigma(u_4) = 0.1839, \sigma(u_5) = 0.3755, \\ \sigma(u_6) &= 0.04608, \sigma(u_7) = 0.1849, \sigma(u_8) = 0.0459, \sigma(u_9) = 0.3755, \sigma(u_{10}) = 0.04608, \\ \sigma(u_{11}) &= 0.1849, \sigma(u_{12}) = 0.0076, \sigma(u_{13}) = 0.3755, \sigma(u_{14}) = 0.04608, \\ \sigma(u_{15}) &= 0.1849, \sigma(u_{16}) = 0.00095, \sigma(u_{17}) = 0.3755, \sigma(u_{18}) = 0.04608, \\ \sigma(u_{19}) &= 0.1849 \end{aligned}$$

**degree of a vertex:**

Let  $\tilde{G} = (\sigma, \mu)$  be a multi copy fuzzy graph. The degree of a vertex  $u$  is

$$d_{\tilde{G}}(u) = \sum_{u \neq v \in E} \mu(u, v), \text{ for } (u, v) \in E \text{ and } \mu(u, v) = 0, \text{ for } (u, v) \text{ not in } E.$$

$$\begin{aligned} \text{This is equivalent to } d_{\tilde{G}}(u) &= \sum_{u=v \in E} \mu(u, v). \quad d(u_0) = 0.55275, d(u_1) = 0.59888, \\ d(u_2) &= 0.09216, d(u_3) = 0.23098, d(u_4) = 0.4137, d(u_5) = 0.41488, d(u_6) = 0.09216, \\ d(u_7) &= 0.23098, d(u_8) = 0.0994, d(u_9) = 0.27688, d(u_{10}) = 0.09216, d(u_{11}) = 0.23098, \\ d(u_{12}) &= 0.01615, d(u_{13}) = 0.23858, \end{aligned}$$

$$d(u_{14}) = 0.09216, d(u_{15}) = 0.23098, d(u_{16}) = 0.00285, d(u_{17}) = 0.23193,$$

$$d(u_{18}) = 0.09216, d(u_{19}) = 0.23098$$

**Total degree of a vertex:**

Let  $\tilde{G} = (\sigma, \mu)$  be a multi copy fuzzy graph.  $td_{\tilde{G}}(u) = d_{\tilde{G}}(u) + \sigma(u), (u, v) \in E$

$$\begin{aligned} td_{\tilde{G}}(u_0) &= 0.92065, td_{\tilde{G}}(u_1) = 0.97438, td_{\tilde{G}}(u_2) = 0.13824, td_{\tilde{G}}(u_3) = 0.41588, \\ td_{\tilde{G}}(u_4) &= 0.5976, td_{\tilde{G}}(u_5) = 0.79038, td_{\tilde{G}}(u_6) = 0.13824, td_{\tilde{G}}(u_7) = 0.41588, \\ td_{\tilde{G}}(u_8) &= 0.1453, td_{\tilde{G}}(u_9) = 0.65238, td_{\tilde{G}}(u_{10}) = 0.13824, td_{\tilde{G}}(u_{11}) = 0.41588, \\ td_{\tilde{G}}(u_{12}) &= 0.02375, td_{\tilde{G}}(u_{13}) = 0.61408, td_{\tilde{G}}(u_{14}) = 0.13824, td_{\tilde{G}}(u_{15}) = 0.41588, \\ td_{\tilde{G}}(u_{16}) &= 0.0038, td_{\tilde{G}}(u_{17}) = 0.60743, td_{\tilde{G}}(u_{18}) = 0.13824, td_{\tilde{G}}(u_{19}) = 0.41588 \end{aligned}$$

**Irregular Fuzzy Graph**

Let  $\tilde{G} = (\sigma, \mu)$  be a fuzzy graph on  $G = (V, E)$ . Then  $\tilde{G}$  is said to be irregular fuzzy graph if there exists a vertex which is adjacent to be vertices with distinct degrees.

From the graph 7 we have the vertex values

$$\begin{aligned} \sigma(u_0) &= 0.3679, \sigma(u_1) = 0.3755, \sigma(u_2) = 0.04608, \sigma(u_3) = 0.1849, \\ \text{and } \mu(u_0, u_1) &= 0.3679, \mu(u_1, u_2) = 0.04608, \mu(u_2, u_3) = 0.04608, \mu(u_3, u_1) = 0.1849, \\ \text{such that } u_0, u_1 \in \tilde{G} &\text{ such that } d_{\tilde{G}}(u_0) = 0.3679 + 0.00095 + 0.1839 = 0.55275 \text{ and} \\ d_{\tilde{G}}(u_1) &= 0.3679 + 0.04608 + 0.1849 = 0.59888 \text{ such that } u_0, u_1 \text{ are adjacent to a} \\ \text{vertices but } d_{\tilde{G}}(u_0) &\neq d_{\tilde{G}}(u_1). \text{ So, } \tilde{G} \text{ is said to be a irregular fuzzy graph.} \end{aligned}$$

### Totally Irregular Fuzzy Graph

Let  $\bar{G} = (\sigma, \mu)$  be a fuzzy graph on  $G = (V, E)$ . Then  $\bar{G}$  is said to be totally irregular fuzzy graph if there exists a vertex which is adjacent to a vertices with distinct total degrees. From the graph 7 we have  $td_{\bar{G}}(u_1) = 0.97438$ ,  $td_{\bar{G}}(u_2) = 0.13824$ ,  $td_{\bar{G}}(u_3) = 0.41588$ ,  $td_{\bar{G}}(u_4) = 0.5976$  such that  $u_0, u_1 \in \bar{G}$  such that  $td_{\bar{G}}(u_0) = 0.92065$ ,  $td_{\bar{G}}(u_1) = 0.97438$ , and  $u_0, u_1$  are adjacent vertices but  $td_{\bar{G}}(u_0) \neq td_{\bar{G}}(u_1)$

So  $\bar{G}$  is a totally irregular fuzzy graph.

### Strongly Irregular Fuzzy Graph

Let  $\bar{G} = (\sigma, \mu)$  be a fuzzy graph on  $G = (V, E)$ . Then  $\bar{G}$  is said to be a strongly irregular fuzzy graph if every pair of vertices have distinct degrees.

From the graph 7 we have, Every pair of vertices have distinct degrees then  $\bar{G}$  is strongly irregular fuzzy graph.

### Highly Irregular Fuzzy Graph

Let  $\bar{G} = (\sigma, \mu)$  be a fuzzy graph on  $G = (V, E)$ . Then  $\bar{G}$  is said to be a highly irregular fuzzy graph if every vertex in  $\bar{G}$  is adjacent to the vertices having distinct degrees.

From the graph 7 we have  $\sigma(u_0) = 0.3679$ ,  $\sigma(u_1) = 0.3755$ ,  $\sigma(u_2) = 0.04608$ ,  $\sigma(u_3) = 0.1849$ , and  $\mu(u_0, u_1) = 0.3679$ ,  $\mu(u_1, u_2) = 0.04608$ ,  $\mu(u_2, u_3) = 0.04608$ ,  $\mu(u_3, u_1) = 0.1849$ ,

$d_{\bar{G}}(u_0) = 0.55275$ ,  $d_{\bar{G}}(u_1) = 0.59888$ ,  $d_{\bar{G}}(u_2) = 0.04608$ ,  $d_{\bar{G}}(u_3) = 0.23098$ ,

$d_{\bar{G}}(u_4) = 0.4137$  From the graph every vertex in  $\bar{G}$  is adjacent to the vertices having distinct degrees. So,  $\bar{G}$  is highly irregular fuzzy graph.

### Neighbourly irregular fuzzy graph

Let  $\bar{G} = (\sigma, \mu)$  be a fuzzy graph on  $G = (V, E)$ . Then  $\bar{G}$  is said to be a neighbourly irregular fuzzy graph if every two adjacent vertices of a fuzzy graph  $\bar{G}$  have distinct degree.

From the graph 7 Every two adjacent vertices of  $\bar{G}$  have distinct degree.

Then  $\bar{G}$  neighbourly irregular fuzzy graph.

Here edge values of fuzzy multi copies graph calculated as,

$\mu(u_0, u_1) = 0.3679$   $\mu(u_1, u_2) = 0.04608$ ,  $\mu(u_2, u_3) = 0.04608$ ,  $\mu(u_3, u_1) = 0.1849$ ,  
 $\mu(u_0, u_4) = 0.1839$ ,  $\mu(u_4, u_5) = 0.1839$ ,  $\mu(u_5, u_6) = 0.1849$ ,  $\mu(u_6, u_7) = 0.04608$ ,

$$\begin{aligned} \mu(u_7, u_5) &= 0.1849, \mu(u_4, u_8) = 0.0459, \mu(u_8, u_9) = 0.0459, \mu(u_9, u_{10}) = 0.04608, \\ \mu(u_{10}, u_{11}) &= 0.04608, \mu(u_{11}, u_9) = 0.1849, \mu(u_8, u_{12}) = 0.0076, \mu(u_{12}, u_{13}) = 0.0076, \\ \mu(u_{13}, u_{14}) &= 0.04608, \mu(u_{14}, u_{15}) = 0.04608, \mu(u_{15}, u_{13}) = 0.1849, \\ \mu(u_{12}, u_{16}) &= 0.00095, \mu(u_{16}, u_{17}) = 0.00095, \mu(u_{17}, u_{18}) = 0.04608, \\ \mu(u_{18}, u_{19}) &= 0.04608, \mu(u_{19}, u_{17}) = 0.1849, \mu(u_{16}, u_0) = 0.00095, \end{aligned}$$

**Degree of an edge:**

$\bar{G} = (\sigma, \mu)$  be a fuzzy graph. The degree of an edge  $(u, v)$  is defined as

$$d_{\bar{G}}(u, v) = d_{\bar{G}}(u) + d_{\bar{G}}(v) - 2\mu(u, v).$$

$$d_{\bar{G}}(u_0, u_1) = 0.55275 + 0.59888 - 2(0.3679) = 0.41583$$

$$d_{\bar{G}}(u_1, u_2) = 0.59888, d_{\bar{G}}(u_2, u_3) = 0.23098, d_{\bar{G}}(u_3, u_1) = 0.46006,$$

$$d_{\bar{G}}(u_0, u_4) = 0.59865, d_{\bar{G}}(u_4, u_5) = 0.46078, d_{\bar{G}}(u_5, u_6) = 0.13724,$$

$$d_{\bar{G}}(u_6, u_7) = 0.23098, d_{\bar{G}}(u_7, u_5) = 0.27606, d_{\bar{G}}(u_4, u_8) = 0.4213,$$

$$d_{\bar{G}}(u_8, u_9) = 0.28448, d_{\bar{G}}(u_9, u_{10}) = 0.27688, d_{\bar{G}}(u_{10}, u_{11}) = 0.23098,$$

$$d_{\bar{G}}(u_{11}, u_9) = 0.13806, d_{\bar{G}}(u_8, u_{12}) = 0.10035, d_{\bar{G}}(u_{12}, u_{13}) = 0.23953,$$

$$d_{\bar{G}}(u_{13}, u_{14}) = 0.23858, d_{\bar{G}}(u_{14}, u_{15}) = 0.23098, d_{\bar{G}}(u_{15}, u_{13}) = 0.09976,$$

$$d_{\bar{G}}(u_{12}, u_{16}) = 0.0171, d_{\bar{G}}(u_{16}, u_{17}) = 0.23288, d_{\bar{G}}(u_{17}, u_{18}) = 0.23193,$$

$$d_{\bar{G}}(u_{18}, u_{19}) = 0.23098, d_{\bar{G}}(u_{19}, u_{17}) = 0.09311, d_{\bar{G}}(u_{16}, u_0) = 0.5537$$

**Minimum degree of an edge**

The minimum degree of an edge is  $\delta_E(\bar{G}) = \wedge\{d_{\bar{G}}(u, v) : (u, v) \in E\}$ .

$$\delta_E(\bar{G}) = 0.0171$$

**Maximum degree of an edge**

The maximum degree of an edge is  $\Delta_E(\bar{G}) = \vee\{d_{\bar{G}}(u, v) : (u, v) \in E\}$ .

$$\Delta_E(\bar{G}) = 0.5988$$

**Total vertex degree of an edge:**

Let  $\bar{G} = (\sigma, \mu)$  be a fuzzy graph. The total degree of an edge  $(u, v)$  is defined as  $td_{\bar{G}}(u, v) = d_{\bar{G}}(u) + d_{\bar{G}}(v) - \mu(u, v)$

$$td_{\bar{G}}(u_0, u_1) = 0.78373, td_{\bar{G}}(u_1, u_2) = 0.64496, td_{\bar{G}}(u_2, u_3) = 0.27706,$$

$$td_{\bar{G}}(u_3, u_1) = 0.64496, td_{\bar{G}}(u_0, u_4) = 0.78255, td_{\bar{G}}(u_4, u_5) = 0.6446,$$

$$\begin{aligned}td_{\bar{G}}(u_5, u_6) &= 0.32214, td_{\bar{G}}(u_6, u_7) = 0.27706, td_{\bar{G}}(u_7, u_5) = 0.46096, \\td_{\bar{G}}(u_4, u_8) &= 0.4672, td_{\bar{G}}(u_8, u_9) = 0.33038, td_{\bar{G}}(u_9, u_{10}) = 0.32296, \\td_{\bar{G}}(u_{10}, u_{11}) &= 0.27706, td_{\bar{G}}(u_{11}, u_9) = 0.32296, td_{\bar{G}}(u_8, u_{12}) = 0.10795, \\td_{\bar{G}}(u_{12}, u_{13}) &= 0.24713, td_{\bar{G}}(u_{13}, u_{14}) = 0.28466, td_{\bar{G}}(u_{14}, u_{15}) = 0.27706, \\td_{\bar{G}}(u_{15}, u_{13}) &= 0.28466, td_{\bar{G}}(u_{12}, u_{16}) = 0.01805, td_{\bar{G}}(u_{16}, u_{17}) = 0.23383, \\td_{\bar{G}}(u_{17}, u_{18}) &= 0.27801, td_{\bar{G}}(u_{18}, u_{19}) = 0.27706, td_{\bar{G}}(u_{19}, u_{17}) = 0.27801, \\td_{\bar{G}}(u_{16}, u_0) &= 0.55465\end{aligned}$$

### Minimum total degree of an edge

The minimum degree of an edge is  $\delta_{tE}(\bar{G}) = \wedge\{td_{\bar{G}}(u, v) : (u, v) \in E\}$ .

$$\delta_{tE}(\bar{G}) = 0.01805$$

### Maximum total degree of an edge

The maximum degree of an edge is  $\Delta_{tE}(\bar{G}) = \vee\{td_{\bar{G}}(u, v) : (u, v) \in E\}$ .

$$\Delta_{tE}(\bar{G}) = 0.78375$$

### Neighbourly edge irregular fuzzy graph

Let  $\bar{G} = (\sigma, \mu)$  be a connected fuzzy graph on  $G = (V, E)$ . Then  $\bar{G}$  is said to be a neighbourly edge irregular fuzzy graph if every adjacent edges having distinct degree. From the graph 7 we have  $\sigma(u_0) = 0.3679$ ,  $\sigma(u_1) = 0.3755$ ,  $\sigma(u_2) = 0.04608$ ,  $\sigma(u_3) = 0.1849$  and  $\mu(u_0, u_1) = 0.3679$ ,  $\mu(u_1, u_2) = 0.04608$ ,  $\mu(u_2, u_3) = 0.04608$ ,  $\mu(u_3, u_1) = 0.1849$ ,  $d_{\bar{G}}(u_0, u_1) = 0.41583$ ,  $d_{\bar{G}}(u_1, u_2) = 0.59888$ ,  $d_{\bar{G}}(u_2, u_3) = 0.23098$ ,  $d_{\bar{G}}(u_3, u_4) = 0.46006$ ,  $d_{\bar{G}}(u_4, u_0) = 0.59865$ .

Every adjacent edges having distinct degree.

Then  $\bar{G}$  neighbourly edge irregular fuzzy graph.

### Neighbourly edge totally irregular fuzzy graph

Let  $\bar{G} = (\sigma, \mu)$  be a connected fuzzy graph on  $G = (V, E)$ . Then  $\bar{G}$  is said to be a neighbourly edge totally irregular fuzzy graph if every adjacent edges having distinct total degree.

$$d_{\bar{G}}(u_0, u_1) = 0.41583, d_{\bar{G}}(u_1, u_2) = 0.59888, d_{\bar{G}}(u_2, u_3) = 0.23098,$$

$$d_{\bar{G}}(u_3, u_4) = 0.46006, d_{\bar{G}}(u_4, u_0) = 0.59865$$

$$td_{\bar{G}}(v_0, v_1) \neq td_{\bar{G}}(u_1, u_2), td_{\bar{G}}(u_1, u_2) \neq td_{\bar{G}}(u_2, u_3), td_{\bar{G}}(u_2, u_3) \neq td_{\bar{G}}(u_3, u_4),$$

$$td_{\bar{G}}(u_3, u_4) \neq td_{\bar{G}}(u_4, u_0), td_{\bar{G}}(u_4, u_0) \neq td_{\bar{G}}(u_0, u_1)$$

Every adjacent edges having distinct total degree.

Then  $\bar{G}$  neighbourly edge totally irregular fuzzy graph.

### Complete Fuzzy Graph

A fuzzy graph  $\bar{G} = (\sigma, \mu)$  is said to be complete if  $\mu(u, v) = \sigma(u) \wedge \sigma(v)$  for all  $u, v \in V$  and is denoted by  $K_{\sigma}$ .

### Strongly edge irregular fuzzy graph

Let  $\bar{G} = (\sigma, \mu)$  be a fuzzy graph on  $\bar{G} = (V, E)$  then  $\bar{G}$  is said to be strongly edge irregular fuzzy graph if every pair of edges having distinct degrees.

From the graph 7 we have, every pair of edges having distinct degrees. so,  $\bar{G}$  is strongly edge irregular fuzzy graph

### Strongly edge totally irregular fuzzy graph

Let  $\bar{G} = (\sigma, \mu)$  be a fuzzy graph on  $\bar{G} = (V, E)$  then  $\bar{G}$  is said to be strongly edge totally irregular fuzzy graph if every pair of edges having distinct total degrees.

From the graph 7 we have, every pair of edges having distinct degrees. So,  $\bar{G}$  is strongly edge totally irregular fuzzy graph.

### Regular fuzzy graph

Let  $\bar{G} = (\sigma, \mu)$  be a fuzzy graph on  $\bar{G} = (V, E)$  if  $d_G(v) = k$  for all  $v \in V$  that is if each vertex has the same degrees  $k$ , then  $\bar{G}$  is said to be regular fuzzy graph of degree  $k$ .

Let  $\bar{G} = (V, E)$  where  $V = \{v_0, v_1, v_2, v_3, v_4\}$   
 and  $E = \{(v_0, v_1), (v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_0)\}$

Therefore The vertex of degree

$$d_{\bar{G}}(u_0) = 0.55275, d_{\bar{G}}(u_1) = 0.59888, d_{\bar{G}}(u_2) = 0.04608, d_{\bar{G}}(u_3) = 0.23098,$$

$d_{\bar{G}}(u_4) = 0.4137$ . From the graph we have, each vertex have distinct degree

Hence  $\bar{G}$  is not regular fuzzy graphs.

### Totally regular fuzzy graph

If each vertex  $\bar{G}$  have the distinct total degree

$$td_{\bar{G}}(u_1) = 0.97438, td_{\bar{G}}(u_2) = 0.13824, td_{\bar{G}}(u_3) = 0.41588, td_{\bar{G}}(u_4) = 0.5976$$

Hence  $\bar{G}$  is not totally regular fuzzy graph.

### Degree of an vertex:

#### $d_2^{th}$ degree of a vertex:

$$d_2(u_0) = 0.04608 + 0.1849 + 0.0459 + 0.1839 + 0.00095 + 0.00095 = 0.4626$$

$$d_2(u_1) = 0.18485, d_2(u_2) = 0.04608, d_2(u_3) = 0.1849, d_2(u_4) = 0.46833, \\ d_2(u_5) = 0.2298, d_2(u_6) = 0.1839, d_2(u_7) = 0.1839, d_2(u_8) = 0.19215, d_2(u_9) = 0.0535, \\ d_2(u_{10}) = 0.0459, d_2(u_{11}) = 0.0459, d_2(u_{12}) = 0.0323, d_2(u_{13}) = 0.00855, \\ d_2(u_{14}) = 0.0076, d_2(u_{15}) = 0.0076, d_2(u_{16}) = 0.0057, d_2(u_{17}) = 0.0019, \\ d_2(u_{18}) = 0.00095, d_2(u_{19}) = 0.00095$$

#### $d_3^{th}$ degree of a vertex:

$$d_3(u_0) = 0.04608 + 0.1839 + 0.0459 + 0.00095 + 0.00095 + 0.00095 = 0.2787 \\ d_3(u_1) = 0.2317, d_3(u_2) = 0.04703, d_3(u_3) = 0.1848, d_3(u_4) = 0.3303, d_3(u_5) = 0.2383, \\ d_3(u_6) = 0.0919, d_3(u_7) = 0.2298, d_3(u_8) = 0.1538, d_3(u_9) = 0.1003, d_3(u_{10}) = 0.0535, \\ d_3(u_{11}) = 0.0535, d_3(u_{12}) = 0.0256, d_3(u_{13}) = 0.0171, d_3(u_{14}) = 0.0085, \\ d_3(u_{15}) = 0.0085, d_3(u_{16}) = 0.0057, d_3(u_{17}) = 0.0038, d_3(u_{18}) = 0.0019, \\ d_3(u_{19}) = 0.0019$$

The fuzzy graph concept can be applied for chemical graphs for atoms and molecules.

## 5 Conclusion

By defining the generalized extorial function and a new type delta graph have been introduced. After applying fuzzy concept on these delta graphs, product of fuzzy graph is established with fuzzy vertex and edge degrees. The regularity and irregularity behaviour like neighbourly edge, neighbourly irregular, strongly irregular, highly irregular, totally irregular, strongly edge irregular, strongly edge totally irregular are discussed. The fuzzy values assigned to with example each vertex may be replaced by change of electrons in the chemical graphs for getting applications in physical science.

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