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Reliability Theory Applied to Medication Risk in Human Pathology

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Abstract

Patient safety within healthcare is a critical and complex issue, particularly concerning medication risks. This paper explores the application of reliability theory to human pathology, specifically focusing on the challenge of controlling a disease using multiple medicines. The goal is to determine the reliability of successfully managing a disease when a patient is treated with 'n' medicines, where success is defined as at least one medicine effectively controlling the condition. By adapting reliability concepts, traditionally used in engineering and other fields, this study aims to provide a quantitative framework for evaluating and improving the effectiveness of medical treatments in human systems.

Key words: Failure Rate, Reliability($\mathbb{R}$), Convolution Integral, Weibull Distribution

AMS classification: 62N05,90B25.

1 Introduction

Human pathology, a cornerstone of medical diagnostics, seeks to understand disease at the cellular and molecular levels. Despite its critical role, diagnostic accuracy and repeatability remain challenges. Reliability technology, traditionally used in engineering, offers innovative solutions by ensuring systems function reliably. Techniques like fault tree analysis (FTA) and failure mode effects analysis (FMEA) can identify errors in pathological workflows, while probabilistic reliability, a mathematical framework, assesses the likelihood of a system performing its intended function under specified conditions. This approach, utilizing models like Bayesian networks and Monte Carlo simulations, enhances diagnostic tool accuracy and workflow efficiency.[1]

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In pathology and medical sciences, reliability is paramount for accurate diagnoses and effective treatments. Pathological reliability focuses on diagnostic accuracy and repeatability, whereas medical reliability ensures consistent healthcare delivery [2]. Integrating these principles leads to robust, error-tolerant systems, particularly evident in AI-driven diagnostics within digital pathology. By combining probabilistic models and predictive analytics, healthcare reliability is significantly improved.

Human pathology aims to decipher disease causes and effects, facing the challenge of predicting biological system failures. Reliability technology, adapted from engineering, provides a framework for modelling biological systems and their failure probabilities. Treating the human body as a complex system, reliability technologies quantify functional decline in organs and predict life-threatening events. This integration enhances disease prediction, prevention, and treatment capabilities [3, 4].

The evolution of reliability technology in human pathology is intertwined with advancements in medicine and laboratory science. Early pathology relied on manual inspection, with pioneers like Rudolf Virchow laying the scientific groundwork. The Industrial Revolution brought mechanical and chemical advancements, improving laboratory procedure precision. The 20th century saw pathology's establishment as a medical pillar, with standardized methods and emerging subfields. World War II's demand for reliable systems influenced laboratory practices, leading to statistical quality control and external quality assurance programs. The introduction of automated analysers, like the Auto Analyzer, significantly enhanced dependability.

The late 20th century witnessed a technological revolution in pathology, with techniques like PCR enabling precise genetic material detection. Digitalization and AI-driven diagnostics further transformed the field, while regulatory bodies like the FDA and ISO enforced standards for precision and dependability. This historical progression highlights the continuous integration of reliability technology to improve accuracy, efficiency, and patient outcomes in human pathology.

This paper's structure is as follows: Section II lays the groundwork with essential definitions. Section III focuses on the specific Human Pathology Problem being addressed, detailing the reliability evaluation of a human system affected by a disease and treated with 'n' medicines. Section IV presents the algorithm developed for this problem. Finally, Section V summarizes the findings and conclusions.

2 Basic Definitions

2.1 Reliability

The Probability that a system or component will carry out its intended function without malfunctioning over a predetermined amount of time or under predetermined circumstances is known as Reliability

Reliability is defined as,

$$\begin{aligned}\mathbb{R}(t) &= 1 - F(t) \\ &= 1 - \int_{-\infty}^{t} f(x)dx\end{aligned}$$

$$\text{or}\mathbb{R}(t) = \int_{t}^{\infty} f(x)dx$$

Where the function R(t) is usually called reliability function and F(x) is the Probability density function of the system failure at time t.

2.2 Mean Time to Failure

A reliability metric called Mean Time to Failure (MTTF) calculates how long a system or component should typically last before failing. It is a crucial metric in reliability engineering that evaluates a systems or device's anticipated operating lifespan.

$$MTTF = \int_{0}^{\infty} R(t)dt$$

2.3 Failure Distribution: Weibull

A popular continuous probability distribution in survival research, life data analysis and engineering are the Weibull distributions. Reliability and risk analysis benefit greatly from these flexible distributions, which bears the name of its inventor, Waloddi Weibull, who first described it in 1951.

The Weibull distribution is defined by its probability density function (PDF):

$$f(t;k,\lambda) = \frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1} e^{\left(-\frac{t}{\lambda}\right)^{k}}, t \geq 0$$

It has two parameters.A scale parameter λ and shape parameter k.

2.4 Hazard Rate

The hazard rate function is given by

$$h(t) = \frac{f(t)}{R(t)}$$

$$h(t) = \left(\frac{k}{\lambda^k}\right) t^{k-1}$$

Thus,hazard rate for Weibull distribution varies with time.

3 Human Pathology System

Think of a human system impacted by a certain illness.A different medication is employed when the first one is unable to treat a certain illness.Let X be the illness. First, medicine 1 is used. In the event that medicine 1 is unable to control the disease, the doctor will prescribed medicine 2 to the patient who has tat specific ailment. Medicine 2 is only utilized in such cases. The challenge is to determine the likelihood that a specific illness will be managed by 'n' medications for a human body. When one medication makes it through the control system, success is achieved. Presume that a given medication's time to failure has a Weibull distribution with a parameter (failure rate).

The Weibull distribution is a continuous probability distribution used in statistics and probability theory. Mostly in the form of a time to failure or time between events, it models a wide variety of random factors.

The probability density function is provided by

$$f(t;k,\lambda) = \frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1} e^{\left(-\frac{t}{\lambda}\right)^k}, t \geq 0$$

$$g(t) = \frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1} e^{\left(-\frac{t}{\lambda}\right)^k}$$

Which represent the probability mass function for the amount of time before a specific medication fails to control the illness. The i-fold convolution of the probability function equals the probability function of the system failure time.

$$g_i(t) = \frac{k}{\lambda_i}\left(\frac{t}{\lambda_i}\right)^{k-1} e^{-\left(\frac{t}{\lambda_i}\right)^k}, i = 1,2,3,\cdots,n$$

For the second medication, the probability function of the system failure at time t is,

$$f_2(t) = \int_0^t g_1(x) g_2(t-x)dx$$

Where,
The probability mass function of the first medicine's time to failure up to time x is represented by $g_1(x)$. The probability mass function of the second medicine's time to failure for the time interval t-x is $g_2(t-x)$.
2-fold convolution is given by

$$f_2(t) = \int_0^t g_1(x) g_2(t-x)dx$$

$$f_2(t) = \int_0^t \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{\left(-\frac{x}{\lambda}\right)^k} \frac{k}{\lambda}\left(\frac{t-x}{\lambda}\right)^{k-1} e^{-\left(\frac{t-x}{\lambda}\right)^k} dx$$

$$= \frac{k^2}{\lambda^2}\int_0^t \left(\frac{x}{\lambda}\right)^{k-1}\left(\frac{t-x}{\lambda}\right)^{k-1} e^{\left(-\frac{x}{\lambda}\right)^k} e^{-\left(\frac{t-x}{\lambda}\right)^k} dx$$

$$= \frac{k^2}{\lambda^{2k}} e^{-\left(\frac{t}{\lambda}\right)^k}\int_0^t (x)^{k-1}(t-x)^{k-1}dx$$

$$= \frac{k^2}{\lambda^{2k}} e^{-\left(\frac{t}{\lambda}\right)^k}\int_0^1 (tu)^{k-1}(t-tu)^{k-1}t\,du$$

$$= \frac{k^2 t^{2k-1}}{\lambda^{2k}} e^{-\left(\frac{t}{\lambda}\right)^k}\int_0^1 u^{k-1}(1-u)^{k-1}du$$

$$= \frac{k^2 t^{2k-1}}{\lambda^{2k}} e^{-\left(\frac{t}{\lambda}\right)^k}\beta(k,k)$$

$$f_2(t) = \frac{k^2 t^{2k-1}}{\lambda^{2k}} e^{-\left(\frac{t}{\lambda}\right)^k}\frac{\Gamma(k)^2}{\Gamma(2k)}$$

3-fold convolution is given by

$$f_3(t) = \int_0^t g_2(x) g_3(t-x)dx$$

$$= \frac{k^2 t^{2k-1}}{\lambda^{2k}} e^{-\left(\frac{t}{\lambda}\right)^k}\frac{\Gamma(k)^2}{\Gamma(2k)}\frac{k}{\lambda}\left(\frac{t-x}{\lambda}\right)^{k-1} e^{-\left(\frac{t-x}{\lambda}\right)^k} dx$$

$$= \frac{k^3}{\lambda^{3k}} e^{-\left(\frac{t}{\lambda}\right)^k}\frac{\Gamma(k)^2}{\Gamma(2k)}\int_0^t (x)^{2k-1}(t-x)^{k-1}dx$$

$$= \frac{k^3}{\lambda^{3k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^2}{\Gamma(2k)} \int_0^1 (tu)^{2k-1}(t-tu)^{k-1} t\,du$$

$$= \frac{k^3}{\lambda^{3k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^2}{\Gamma(2k)} t^{3k-1} \int_0^1 u^{2k-1}(t-u)^{k-1} du$$

$$= \frac{k^3}{\lambda^{3k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^2}{\Gamma(2k)} t^{3k-1} \beta(2k,k)$$

$$= \frac{k^3}{\lambda^{3k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^2}{\Gamma(2k)} t^{3k-1} \frac{\Gamma(2k)\Gamma(k)}{\Gamma(3k)}$$

$$f_3(t) = \frac{k^3 t^{3k-1}}{\lambda^{3k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^3}{\Gamma(3k)}$$

4-fold convolution is given by

$$f_4(t) = \int_0^t g_3(x) g_4(t-x)\,dx$$

$$f_4(t) = \int_0^t \frac{k^4 t^{3k-1}}{\lambda^{3k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^3}{\Gamma(3k)} \frac{k}{\lambda} \left(\frac{t-x}{\lambda}\right)^{k-1} e^{-\left(\frac{t-x}{\lambda}\right)^k} dx$$

$$= \frac{k^4}{\lambda^{4k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^3}{\Gamma(3k)} \int_0^t (x)^{3k-1}(t-x)^{k-1} dx$$

$$= \frac{k^4}{\lambda^{4k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^3}{\Gamma(3k)} \int_0^1 (tu)^{3k-1}(t-tu)^{k-1} t\,du$$

$$= \frac{k^4}{\lambda^{4k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^3}{\Gamma(3k)} t^{4k-1} \int_0^1 u^{3k-1}(1-u)^{k-1} du$$

$$= \frac{k^4}{\lambda^{4k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^3}{\Gamma(3k)} t^{4k-1} \beta(3k,k)$$

$$= \frac{k^4}{\lambda^{4k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^3}{\Gamma(3k)} t^{4k-1} \frac{\Gamma(3k)\Gamma(k)}{\Gamma(4k)}$$

$$f_4(t) = \frac{k^4 t^{4k-1}}{\lambda^{4k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^4}{\Gamma(4k)}$$

In general,

$$f_n(t) = R_1(t) + R_2(t)$$

Where

$R_1(t)$=Probability of Medicine 1 working successfully at time t.

$R_2(t)$=Probability of Medicine 1 failed prior to t and Medicine 2 working successfully at time t ie., for the remaining period t-x.

$$R_1(t) = \int_t^\infty g_1(x)dx$$

$$= \int_t^\infty \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} dx$$

$$= 1 - \int_0^t \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} dx$$

$$= 1 - \frac{k}{\lambda_k}\int_0^t (x)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} dx$$

$$= 1 - \frac{k}{\lambda_k}\int_0^{\left(\frac{t}{\lambda}\right)^k} (\lambda u^{\frac{1}{k}})^{k-1} e^{-u}\frac{\lambda}{k}u^{\frac{1}{k}-1}du$$

$$= 1 - \frac{k}{\lambda_k}\int_0^{\left(\frac{t}{\lambda}\right)^k} \frac{\lambda^k}{k}u^{\frac{k-1}{k}} e^{-u}u^{\frac{1-k}{k}}du$$

$$= 1 - \int_0^{\left(\frac{t}{\lambda}\right)^k} e^{-u}du$$

$$R_1(t) = e^{-\left(-\frac{t}{\lambda}\right)^k}$$

$$R_2(t) = \int_0^t f_1(x)R(t-x)dx$$

$$= \int_t^\infty \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^k} e^{-\left(\frac{t-x}{\lambda}\right)^k} dx$$

$$= \frac{k}{\lambda^k} e^{-\left(\frac{t}{\lambda}\right)^k} \int_0^t (x)^{k-1} dx$$

$$R_2(t) = \frac{kt^k}{k\lambda^k} e^{-\left(\frac{t}{\lambda}\right)^k}$$

The Reliability of the system is given by

$$R(t) = R_1(t) + R_2(t) = e^{-\left(\frac{t}{k}\right)^k} + \frac{kt^k}{k\lambda^k} e^{-\left(\frac{t}{\lambda}\right)^k}$$

$$R(t) = e^{-\left(\frac{t}{k}\right)^k}\left[1 + \frac{kt^k}{k\lambda^k}\right]$$

Mean Time to Failure is given by

MTTF=$\int_0^\infty R(t)dt$

$$=\int_0^\infty e^{-\left(\frac{t}{k}\right)^k}\left[1+\frac{kt^k}{k\lambda^k}\right]dt$$

$$=\int_0^\infty e^{-\left(\frac{t}{k}\right)^k}dt+\frac{k}{k\lambda^k}\int_0^\infty t^k e^{-\left(\frac{t}{\lambda}\right)^k}dt$$

$$=\int_0^\infty e^{-u}\frac{\lambda}{k}u^{\frac{1}{k}-1}du+\frac{1}{\lambda^k}\int_0^\infty(\lambda u^{\frac{1}{k}})^k e^{-u}\frac{\lambda}{k}u^{\frac{1}{k}-1}du$$

$$=\frac{\lambda}{k}\int_0^\infty e^{-u}u^{\frac{1}{k}-1}du+\frac{\lambda^{k+1}}{k\lambda^k}\int_0^\infty u^{\frac{k}{k}}e^{-u}u^{\frac{1}{k}-1}du$$

$$=\frac{\lambda}{k}\Gamma\left(\frac{1}{k}\right)+\frac{\lambda}{k}\int_0^\infty u^{\frac{1}{k}}e^{-u}du$$

$$=\frac{\lambda}{k}\Gamma\left(\frac{1}{k}\right)+\frac{\lambda}{k}\Gamma\left(\frac{1}{k}+1\right)$$

MTTF=$\frac{\lambda}{k}\left[\Gamma\left(\frac{1}{k}\right)+\Gamma\left(\frac{1}{k}+1\right)\right]$

If we give the values of λ and k at the time t, we get the values of R(t) and MTTF

t	λ	k	R(t)	MTTF
1	0.2	0.6	0.2831	0.8424
2	0.3	0.8	0.0875	0.7649
3	0.4	1	0.0045	0.8
4	0.5	1.2	0.0003675	0.8722
5	0.6	1.4	0.0000000109	0.9684

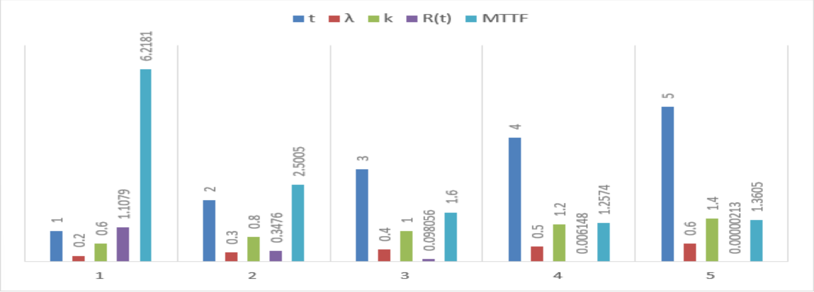

Figure 1: Illustrates how the parameters change when two medicines are used

In case 1, reliability declines as time, scale and shape parameters grow. The shape parameter determines MTTF:if $0 < k < 1$ MTTF rises and if $k > 1$ MTTF increases.

Case 2: When 3 medicines are applied.

$$R_3(t) = \int_0^t f_2(x) R(t-x)dx$$

$$= \int_0^t \frac{k^2 x^{2k-1}}{\lambda^{2k}} e^{-\left(\frac{x}{\lambda}\right)^k} \frac{\Gamma(k)^2}{\Gamma(2k)} e^{-\left(\frac{t-x}{\lambda}\right)^k} dx$$

$$= \frac{k^2}{\lambda^{2k}} \frac{\Gamma(k)^2}{\Gamma(2k)} e^{-\left(\frac{x}{\lambda}\right)^k} \int_0^t x^{2k-1} dx$$

$$= \frac{k^2}{\lambda^{2k}} \frac{\Gamma(k)^2}{\Gamma(2k)} e^{-\left(\frac{x}{\lambda}\right)^k} \frac{t^{2k}}{2k}$$

$$= \frac{k^2 t^{2k}}{\lambda^{2k} \lambda^{2k}} \frac{\Gamma(k)^2}{\Gamma(2k)} e^{-\left(\frac{x}{\lambda}\right)^k}$$

The Reliability of the system is given by

$$R(t) = R_1(t) + R_2(t) + R_3(t)$$

$$= e^{-\left(\frac{t}{k}\right)^k} + \frac{kt^k}{k\lambda^k} e^{-\left(\frac{t}{\lambda}\right)^k} + \frac{k^2 t^{2k}}{\lambda^{2k} \lambda^{2k}} \frac{\Gamma(k)^2}{\Gamma(2k)} e^{-\left(\frac{x}{\lambda}\right)^k}$$

$$R(t) = e^{-\left(\frac{t}{k}\right)^k} \left[1 + \frac{kt^k}{k\lambda^k} + \frac{k^2 t^{2k}}{\lambda^{2k} \lambda^{2k}} \frac{\Gamma(k)^2}{\Gamma(2k)} \right]$$

Mean Time to Failure is given by

$$\text{MTTF} = \int_0^\infty R(t)dt$$

$$= \int_0^\infty e^{-\left(\frac{t}{k}\right)^k} \left[1 + \frac{kt^k}{k\lambda^k} + \frac{k^2 t^{2k}}{\lambda^{2k} \lambda^{2k}} \frac{\Gamma(k)^2}{\Gamma(2k)} \right] dt$$

$$=\int_0^\infty e^{-\left(\frac{t}{k}\right)^k}dt+\frac{k}{k\lambda^k}\int_0^\infty t^k e^{-\left(\frac{t}{\lambda}\right)^k}dt+\frac{k^2}{2k\lambda^{2k}}\frac{\Gamma(k)^2}{\Gamma(2k)}\int_0^\infty t^{2k}e^{-\left(\frac{t}{\lambda}\right)^k}$$

$$=\frac{\lambda}{k}\Gamma\left(\frac{1}{k}\right)+\frac{\lambda}{k}\Gamma\left(\frac{1}{k}+1\right)+\frac{k}{2\lambda^{2k}}\frac{\Gamma(k)^2}{\Gamma(2k)}\int_0^\infty\left(\lambda u^{\frac{1}{k}}\right)^{2k}e^{-u}\frac{\lambda}{k}u^{\frac{1}{k}-1}du$$

$$=\frac{\lambda}{k}\Gamma\left(\frac{1}{k}\right)+\frac{\lambda}{k}\Gamma\left(\frac{1}{k}+1\right)+\frac{k\lambda^{2k+1}}{2\lambda^{2k}}\frac{\Gamma(k)^2}{\Gamma(2k)}\int_0^\infty u^2e^{-u}\frac{\lambda}{k}u^{\frac{1}{k}-1}du$$

$$=\frac{\lambda}{k}\Gamma\left(\frac{1}{k}\right)+\frac{\lambda}{k}\Gamma\left(\frac{1}{k}+1\right)+\frac{\lambda k}{2k}\frac{\Gamma(k)^2}{\Gamma(2k)}\int_0^\infty u^{1+\frac{1}{k}}e^{-u}du$$

$$\text{MTTF}=\frac{\lambda}{k}\Gamma\left(\frac{1}{k}\right)+\frac{\lambda}{k}\Gamma\left(\frac{1}{k}+1\right)+\frac{\lambda}{2}\frac{\Gamma(k)^2}{\Gamma(2k)}\Gamma\left(\frac{1}{k}+2\right)$$

If we give the values of λ and k at the time t,we get the values of $R(t)$ and MTTF

t	λ	k	R(t)	MTTF
1	0.2	0.6	0.7261	2.4663
2	0.3	0.8	0.2950	1.4553
3	0.4	1	0.0202	1.2
4	0.5	1.2	0.002025	1.0983
5	0.6	1.4	0.00000167	1.1786

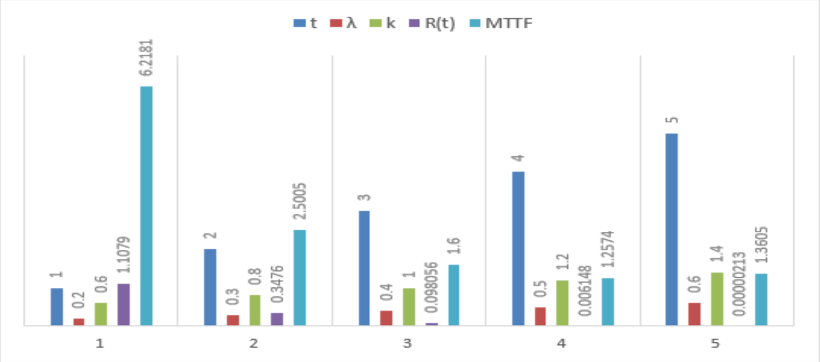

Figure 2: Illustrates how the parameters change when three medicines are used

In case 2,reliability declines as time, scale and shape parameter rise.The shape parameter determines MTTF: if $0 < k \leq 1$ MTTF falls and if $k > 1$ MTTF increase.

Case 2: When 3 medicines are applied.

$$R_4(t) = \int_0^t f_3(x) R(t-x)dx$$

$$= \int_0^t \frac{k^3 x^{3k-1}}{\lambda^{3k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^3}{\Gamma(3k)} e^{-\left(\frac{t-x}{\lambda}\right)^k} dx$$

$$= \frac{k^3}{\lambda^{3k}} \frac{\Gamma(k)^3}{\Gamma(3k)} e^{-\left(\frac{t}{\lambda}\right)^k} \int_0^t x^{3k-1} dx \frac{k^3}{\lambda^{3k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^2}{\Gamma(3k)} \frac{t^{3k}}{3k}$$

$$R_4(t) = \frac{k^3 t^{3k}}{3k\lambda^{3k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^2}{\Gamma(3k)}$$

The Reliability of the system is given by

$$R(t) = R_1(t) + R_2(t) + R_3(t) + R_4(t)$$

$$R(t) = e^{-\left(\frac{t}{\lambda}\right)^k} + \frac{kt^k}{k\lambda^k} e^{-\left(\frac{t}{\lambda}\right)^k} + \frac{k^2 t^{2k}}{2k\lambda^{2k}} \frac{\Gamma(k)^2}{\Gamma(2k)} e^{-\left(\frac{t}{\lambda}\right)^k} + \frac{k^3 t^{3k}}{3k\lambda^{3k}} e^{-\left(\frac{t}{\lambda}\right)^k} \frac{\Gamma(k)^3}{\Gamma(3k)}$$

$$R(t) = e^{-\left(\frac{t}{\lambda}\right)^k} \left[1 + \frac{kt^k}{k\lambda^k} + \frac{k^2 t^{2k}}{2k\lambda^{2k}} \frac{\Gamma(k)^2}{\Gamma(2k)} + \frac{k^3 t^{3k}}{3k\lambda^{3k}} \frac{\Gamma(k)^3}{\Gamma(3k)}\right]$$

Mean Time to Failure is given by

$$\text{MTTF} = \int_0^\infty R(t)dt$$

$$= \int_0^\infty e^{-\left(\frac{t}{\lambda}\right)^k} \left[1 + \frac{kt^k}{k\lambda^k} + \frac{k^2 t^{2k}}{2k\lambda^{2k}} \frac{\Gamma(k)^2}{\Gamma(2k)} + \frac{k^3 t^{3k}}{3k\lambda^{3k}} \frac{\Gamma(k)^3}{\Gamma(3k)}\right] dt$$

$$= \int_0^\infty e^{-\left(\frac{t}{\lambda}\right)^k} dt + \frac{k}{k\lambda^k} \int_0^\infty t^k e^{-\left(\frac{t}{\lambda}\right)^k} dt + \frac{k^2}{2k\lambda^{2k}} \frac{\Gamma(k)^2}{\Gamma(2k)} \int_0^\infty t^{2k} e^{-\left(\frac{t}{\lambda}\right)^k} dt$$

$$+ \frac{k^3}{3k\lambda^{3k}} \frac{\Gamma(k)^3}{\Gamma(3k)} \int_0^\infty t^{3k} e^{-\left(\frac{t}{\lambda}\right)^k} dt$$

$$= \frac{\lambda}{k}\Gamma\left(\frac{1}{k}\right) + \frac{\lambda}{k}\Gamma\left(\frac{1}{k}+1\right) + \frac{\lambda}{2}\frac{\Gamma(k)^2}{\Gamma(2k)}\Gamma\left(\frac{1}{k}+2\right) + \frac{k^3}{3k\lambda^{3k}} \frac{\Gamma(k)^3}{\Gamma(3k)} \int_0^\infty \left(\lambda u^{\frac{1}{k}}\right)^{3k} e^{-u} \frac{\lambda}{k} u^{\frac{1}{k}-1} du$$

$$=\frac{\lambda}{k}\Gamma\left(\frac{1}{k}\right)+\frac{\lambda}{k}\Gamma\left(\frac{1}{k}+1\right)+\frac{\lambda}{2}\frac{\Gamma(k)^2}{\Gamma(2k)}\Gamma\left(\frac{1}{k}+2\right)+\frac{k^3\lambda^{3k+1}}{3k^2\lambda^{3k}}\frac{\Gamma(k)^3}{\Gamma(3k)}\int_0^\infty u^3e^{-u}\frac{\lambda}{k}u^{\frac{1}{k}-1}du$$

$$=\frac{\lambda}{k}\Gamma\left(\frac{1}{k}\right)+\frac{\lambda}{k}\Gamma\left(\frac{1}{k}+1\right)+\frac{\lambda}{2}\frac{\Gamma(k)^2}{\Gamma(2k)}\Gamma\left(\frac{1}{k}+2\right)+\frac{k\lambda}{3}\frac{\Gamma(k)^3}{\Gamma(3k)}\int_0^\infty u^{2+\frac{1}{k}}e^{-u}du$$

$$\text{MTTF}=\frac{\lambda}{k}\Gamma\left(\frac{1}{k}\right)+\frac{\lambda}{k}\Gamma\left(\frac{1}{k}+1\right)+\frac{\lambda}{2}\frac{\Gamma(k)^2}{\Gamma(2k)}\Gamma\left(\frac{1}{k}+2\right)+\frac{k\lambda}{3}\frac{\Gamma(k)^3}{\Gamma(3k)}\Gamma\left(\frac{1}{k}+3\right)$$

If we give the value of λ and k at the time t,we get the values of R(t) and MTTF

t	λ	k	R(t)	MTTF
1	0.2	0.6	1.1079	6.2181
2	0.3	0.8	0.3476	2.5005
3	0.4	1	0.098056	1.6
4	0.5	1.2	0.006148	1.2574
5	0.6	1.4	0.00000213	1.3605

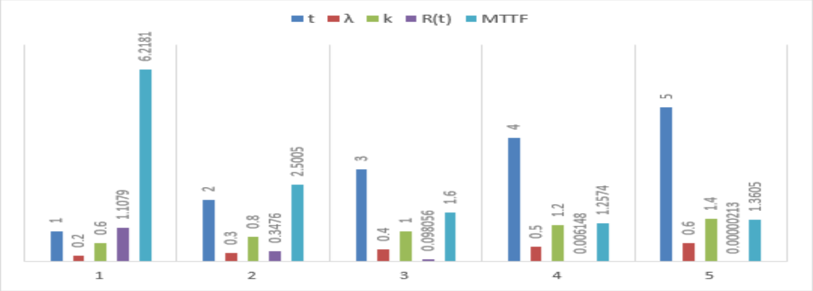

Figure 3: Illustrates how the parameters change when four medicines are used

In case 3,reliability declines as time, scale and shape parameters rise.The shape parameter determines MTTF:if$0 < k \leq 1$ MTTF drops and if $k > 1$ MTTF increases.

4 Conclusion

This paper addresses the issue of human pathology by framing it in terms of reliability analysis. The failure distribution is modelled using a Weibull distribution; however, this approach can be extended to other types of failure distributions to evaluate the reliability of the human system under study.

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