2456-8686, ix(i), 2025:011-035
https://doi.org/10.26524/cm203

ORIGINAL RESEARCH Open Access

Analysis of Fuzzy Derivative Graphs and Their Product Graphs

John Borg S[1], Ashwini R[2] and Iruthayaraj S [3]

Received: 29 April 2025/ Accepted: 20 May 2025 / Published online: 16 June 2025

Abstract

This paper aims to explore and develop innovative multi copies fuzzy derived graphs and their corresponding product graphs. Derived graphs are characterized by vertices weighted based on the ratio of a generalized exponential function, while edges are identified through the differentiation and integration of this function. This study provides a comprehensive analysis of the regularity and irregularity properties of multi copies fuzzy product graphs, supported by illustrative examples. Key aspects investigated include the impact of various generalized exponential functions on vertex weighting, the influence of different differentiation and integration techniques on edge identification, and the topological properties of multi copies fuzzy derived graphs. Additionally, a comparative analysis is conducted to highlight the advantages and applications of multi copies fuzzy derived graphs in contrast to traditional graph structures. Through detailed case studies, the practical applications of multi copies fuzzy derived graphs in fields such as network analysis, data clustering and decision-making processes are demonstrated. This research contributes to the existing body of knowledge in fuzzy graph theory and lays the groundwork for future studies and applications in this domain.

Key words: Multi Copies Fuzzy Graphs, Derived Graphs, Generalized Exponential Function, Product Graphs.

AMS classification: 05C72, 05C90, 68R10, 03E72.

1 Introduction

The graph is a mathematical structure that models relationships between objects, represented by vertices (nodes) and edges connecting them. Graphs are used in fields like computer science, social networks, and transportation to model networks and systems. In [6], the authors propose globally consistent anisotropic kernels

---

[1] Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur, Tamil Nadu, India.
Email:sjborg@gmail.com
[2] Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur, Tamil Nadu, India.
Email:ashwinir452003@gmail.com
[3] Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur, Tamil Nadu, India.
Email: iruthayarajsh@gmail.com

for graph neural networks (GNNs), addressing issues such as over-smoothing by incorporating directional derivatives through Laplacian eigenvectors. This approach enhances the expressiveness more effectively in anisotropic features.

The Harary and Schwenk in [11] addresses the communication problem in graph theory, providing insights into the minimum number of communications required for information exchange among individuals in various network structures. It establishes key theorems for both complete graphs and connected graphs, highlighting the importance of specific configurations, such as quadrilaterals, in optimizing communication efficiency. This work contributes to the understanding of network dynamics and has implications for fields such as computer science, telecommunications, and social network analysis.

The relationship between graph covers using non-overlapping complete graphs and a class of connected even graphs are discussed in [18]. It observes that the relational product of an admissible even graph results in two components, each covered by non-overlapping complete graphs. Futhermore, the authors in [4] investigates the isomorphism of circulant graphs and digraphs. This work contributes to the understanding of graph theory, particularly in the context of algebraic structures and their applications. In fuzzy graphs, the strength of relationships between vertices is represented by membership values.

2 Preliminaries

The project consists of five chapters. In the introduction we present the literature review in the successive two chapters, we dedicate to the study of fuzzy derivative graphs and extend it to the product graphs. As we conclude, we present the summary and awerness for future engagements.

In this section, by employing fuzzy vertex values and derivatives of generalized exponential function, we develop new type of fuzzy derived graph and present basic concepts of the construction of derived graphs. Here we use the notation $\mathbb{N}(a) = \{a, a+1, a+2 \cdots\}$

Lemma 2.1 For a positive integer's', from the basic theory of differential calculus, by taking $D^s = \frac{d^s}{dx^s}$, $x > 0$,we have

$$D^s x^m = m^{(s)} x^{m-s}, \tag{1}$$

where $m^{(s)} = m(m-1)(m-2)\dots(m-(s-1))$.

Proof: The proof follows from $\frac{d}{dx}(x^m) = mx^{m-1}$ and the definition of of $m^{(s)}$.

Definition 2.2 For $x > 0$ and $m, \kappa \in \mathbb{N}(0)$, the generalized exponential function, denoted as $e(\kappa, x^m)$, is defined as

$$\sum_{r=0}^{\infty} \frac{x^{m+r\kappa}}{(m+r\kappa)!} = e(x^m, \kappa) \tag{2}$$

Here, m and κ denote the initial power and the shift value of generalized exponential function defined in (2).

Lemma 2.3 Let κ be any positive integer. Then,

$$\sum_{k=0}^{s-1} e(x^k, \kappa) = e(1, x^0) \tag{3}$$

Proof: By taking $m = 0, 1, 2 \cdots \kappa - 1$ in 2 we arrive

$$e(x^0, \kappa) = 1 + \frac{x^\kappa}{\kappa!} + \frac{x^{2\kappa}}{(2\kappa)!} + \cdots$$

$$e(x^1, \kappa) = \frac{x^1}{1!} + \frac{x^{\kappa+1}}{(\kappa+1)!} + \cdots$$

$$e(x^{\kappa-1}, \kappa) = \frac{x^{\kappa-1}}{(\kappa-1)!} + \frac{x^{2\kappa-1}}{(2\kappa-1)!} + \cdots$$

Now the proof follows by adding all the above expansions and the expansion of $e(x^0, \kappa)$ we have $e(1, x^0) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$

Theorem 2.4 For any positive integer 'n'

$$e(x^0, n) + e(x^1, n) + e(x^2, n) + e(x^3, n) + \cdots e(x^{n-1}, n) = e(x^0, 1).$$

Proof: The proof follows from adding the expansion of all the terms of LHS and RHS.

Example 2.5 $e(x^0, 5) + e(x^1, 5) + e(x^2, 5) + e(x^3, 5) + e(x^4, 5) = e(x^0, 1)$

$$e(x^0, 5) = \frac{x^0}{0!} + \frac{x^5}{5!} + \dots \tag{4}$$

$$e(x^1,5)=\frac{x}{1!}+\frac{x^6}{6!}+\ldots \tag{5}$$

$$e(x^2,5)=\frac{x^2}{2!}+\frac{x^7}{7!}+\ldots \tag{6}$$

$$e(x^3,5)=\frac{x^3}{3!}+\frac{x^8}{8!}+\ldots \tag{7}$$

$$e(x^4,5)=\frac{x^4}{4!}+\frac{x^9}{9!}+\ldots \tag{8}$$

Adding $(4)+(5)+(6)+(7)+(8)$ we have

$$1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots=e^x$$

Theorem 2.6 $\sum\limits_{r=0}^{\infty}\int\limits_{(r)n}^{t} 1dx=1+\frac{t^n}{n!}+\frac{t^{2n}}{(2n)!}+\ldots=e(t^0,n)$

Proof:First Integration

$\int\limits_{(0)0}^{t} 1=1,\ \int\limits_{(1)0}^{t} dx=[x]_0^t=t,\ \int\limits_{(2)0}^{t} xdx=[\frac{x^2}{2}]_0^t=\frac{t^2}{2!},$

$\int\limits_{(3)0}^{t} x^2dx=[\frac{x^3}{3}]_0^t=\frac{t^3}{3!},\ \int\limits_{(n)0}^{t} x^{n-1}dx=[\frac{x^n}{2n}]_0^t=\frac{t^n}{n!}$

$$\sum_{r=0}^{\infty}\int_{(r)0}^{t} 1dx=1+\frac{t}{1!}+\frac{t^2}{2!}+\ldots\frac{t^n}{n!}=e^x \tag{9}$$

Second Integration

$$\sum_{r=0}^{\infty}\int_{(2r)0}^{t} 1dx=1+\frac{t^2}{2!}+\frac{t^4}{4!}+\ldots=e(t^0,2)$$

Third Integration

$$\sum_{r=0}^{\infty}\int_{(3r)0}^{t} 1dx=1+\frac{t^3}{3!}+\frac{t^6}{6!}+\ldots=e(x^0,3)$$

In general, we get

$e(t^0,n) = 1+\frac{t^n}{n!}+\frac{t^{2n}}{2n!}+\ldots+\frac{t^{rn}}{(rn)!}+\ldots$

$$e(t^0,n) = \sum_{r=0}^{\infty}\int_{(rn)0}^{t} 1dx \tag{10}$$

Lemma 2.7 If $'n'$ be any positive integer. Then,

$$\sum_{k=0}^{n-1} e(t^k,n) = e^t \tag{11}$$

Proof: From the definition of generalized exponential function, we have

$$e(t^0,n) = \frac{t^0}{0!}+\frac{t^n}{n!}+\frac{t^{2n}}{2n!}\ldots \tag{12}$$

$$e(t^1,n) = \frac{t}{1!}+\frac{t^{n+1}}{n+1!}+\frac{t^{2n+1}}{2n+1}\ldots \tag{13}$$

$$e(t^{n-1},n) = \frac{t^{n-1}}{n-1!}+\frac{t^{2n-1}}{2n-1!}+\ldots \tag{14}$$

Adding $(11)+(12)+(13)$ then we get

$$1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots = e^x.$$

Example 2.8 Put $n=5$ in equ (11) then we get $\sum_{k=0}^{5-1} e(t^k,5) = e^t$

From the equation (2), for m=5,

$e(t^0,5)+e(t^1,5)+e(t^2,5)+e(t^3,5)+e(t^4,5) = e^t$,

$e(x^0,5) = \frac{t^0}{0!}+\frac{t^5}{5!}+\ldots, e(t^1,5) = \frac{t}{1!}+\frac{t^6}{6!}+\ldots, e(t^2,5) = \frac{t^2}{2!}+\frac{t^7}{7!}+\ldots,$

$e(t^3,5) = \frac{t^3}{3!}+\frac{t^8}{8!}+\ldots, e(t^4,5) = \frac{t^4}{4!}+\frac{t^9}{9!}+\ldots$

Adding all the above expansion then we get e^t.

In equation (10) we start with 1 then we start with $\frac{x}{1!}$ in (10)

$\sum_{r=0}^{\infty}\int_{(rn)0}^{t}\frac{x}{1!}dx = \int_{(0n)0}^{t}\frac{x}{1!}dx+\int_{(n)0}^{t}\frac{x}{1!}dx+\int_{(2n)0}^{t}\frac{x}{1!}dx = \frac{t}{1!}+\frac{t^{n+1}}{n+1!}+\frac{2n+1}{2n+1!}+\ldots$

$$= \sum_{r=0}^{\infty} \int_{(rn)0}^{t} \frac{x}{1!} dx = e(t^1, n)$$

In general, we derive

$$\sum_{r=0}^{\infty} \int_{(rn)0}^{t} \frac{x^k}{1!} dx = e(t^k, n) \tag{15}$$

Example 2.9 Put $n = 5$ in equation (15) then we get

$$\sum_{r=0}^{4} \int_{(r5)0}^{t} \frac{x^1}{1!} dx = e(t^1, 5)$$

Proof: $\sum\limits_{r=0}^{4} \int\limits_{(r5)0}^{t} \frac{x^1}{1!} dx = \int\limits_{(0)0}^{t} \frac{x}{1!} dx + \int\limits_{(1)0}^{t} \frac{x}{1!} dx + \int\limits_{(2)0}^{t} \frac{x}{1!} dx \int\limits_{(3)0}^{t} \frac{x}{1!} dx + \int\limits_{(4)0}^{t} \frac{x}{1!} dx$

$$\sum_{r=0}^{4} \int_{(r5)0}^{t} \frac{x}{1!} dx = e(t^1, 5)$$

Theorem 2.10 For a positive integer $'n'$ and $t > 0$

$$\sum_{k=0}^{n-1} \sum_{r=0}^{\infty} \int_{(rn)0}^{t} \frac{x^k}{k!} dx = e^t \tag{16}$$

Proof: From equation (12)

$$\sum_{r=0}^{\infty} \int_{(rn)0}^{t} \frac{x^k}{k!} = e(t^k, n) \tag{17}$$

Now equation (17) follows from lemma 2.7

Taking $\sum\limits_{k=0}^{n-1}$ on both side of equation (17), we get

$$\sum_{k=0}^{n-1} \sum_{r=0}^{\infty} \int_{(rn)0}^{t} \frac{x^k}{k!} dx = \sum_{k=0}^{n-1} e(t^k, n)$$

by lemma 2.7 we have (17)

Thus we have arrived sum of generalized exponential function interms multi integration.

3 Fuzzy Derivative Graph

In this chapter, we introduce higher order fuzzy derivative graph (upto fifth order)

Construction of fuzzy derivative graph: Applying differential on generalized exponential function $e(x^r, m)$, we see

$$\frac{d}{dx}e(x^0,m) = e(x^1,m)$$
$$\frac{d^2}{dx^2}e(x^2,m) = e(x^2,m)$$
$$\vdots$$
$$\frac{d^{m-1}}{dx^{m-1}}e(x^0,m) = e(x^{m-1},m)$$

3.1 First Order Derivative

Let $G^{(1)} = (V, E^{(1)})$ be the graph, where $V = \{e(x^r,m) | r = 0,1,2,\cdots,m-1\}$ and $E^{(1)} = \{e(x^r,m) \to \frac{d}{dx}e(x^r,m)\}$. Then $G^{(1)}$ is called first derivative graph.

Example 3.1 Applying derivative an generalized exponential function

$\frac{d}{dx}e(x^0,5) = e(x^4,5), \frac{d}{dx}e(x^1,5) = e(x^0,5), \frac{d}{dx}e(x^2,5) = e(x^1,5),$

$\frac{d}{dx}e(x^3,5) = e(x^2,5), \frac{d}{dx}e(x^4,5) = e(x^3,5)$

E $= \{e(x^0,5) \to e(x^4,5), e(x^1,5) \to e(x^0,5), e(x^2,5) \to e(x^1,5), e(x^3,5) \to e(x^2,5),$
$e(x^4,5) \to e(x^3,5)\}$

3.2 Second Order Derivative

Let $G^{(2)} = (V, E^{(2)})$ be the graph, where $V = \{e(x^r,m) | r = 0,1,2,\cdots,m-1\}$ and $E^{(2)} = \{e(x^r,m) \to \frac{d^2}{dx^2}e(x^r,m)\}$. Then $G^{(2)}$ is called second derivative graph.

Example 3.2 Applying derivative an generalized exponential function

$\frac{d^2}{dx^2}e(x^0,5) = e(x^3,5), \frac{d^2}{dx^2}e(x^1,5) = e(x^4,5), \frac{d^2}{dx^2}e(x^2,5) = e(x^0,5),$

$\frac{d^2}{dx^2}e(x^3,5) = e(x^1,5), \frac{d}{dx}e(x^4,5) = e(x^2,5)$

E $= \{e(x^0,5) \to e(x^3,5), e(x^1,5) \to e(x^4,5), e(x^2,5) \to e(x^0,5), e(x^3,5) \to e(x^1,5),$
$e(x^4,5) \to e(x^2,5)\}$

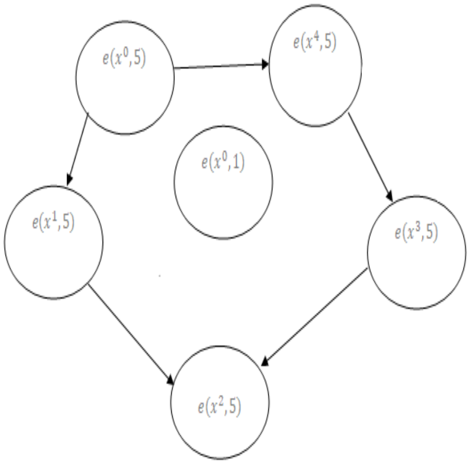

Figure 1: The graphical represention of first order derivative graph

3.3 Third Order Derivative

Let $G^{(3)} = (V, E^{(3)})$ be the graph, where $V = \{e(x^r,m) | r = 0,1,2,\cdots,m-1\}$ and $E^{(3)} = \{e(x^r,m) \rightarrow \frac{d^3}{dx^3} e(x^r,m)\}$. Then $G^{(3)}$ is called third derivative graph.

Example 3.3 Applying derivative an generalized exponential function

$$\frac{d^3}{dx^3}e(x^0,5) = e(x^2,5), \frac{d^3}{dx^3}e(x^1,5) = e(x^3,5), \frac{d^3}{dx^3}e(x^2,5) = e(x^4,5),$$

$$\frac{d^3}{dx^3}e(x^3,5) = e(x^0,5), \frac{d^3}{dx^3}e(x^4,5) = e(x^1,5)$$

$$\text{E} = \{e(x^0,5) \rightarrow e(x^2,5), e(x^1,5) \rightarrow e(x^3,5), e(x^2,5) \rightarrow e(x^4,5), e(x^3,5) \rightarrow e(x^0,5),$$
$$e(x^4,5) \rightarrow e(x^1,5)\}$$

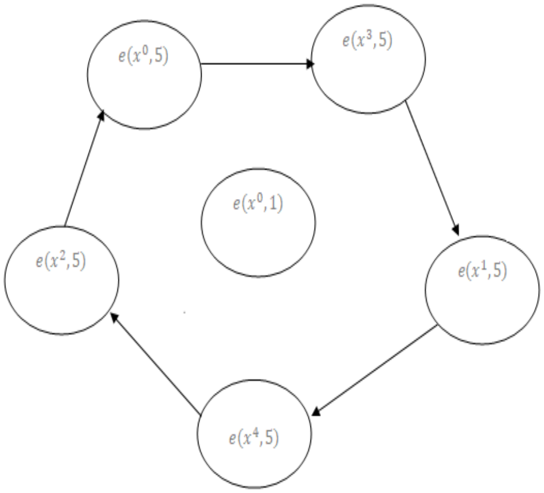

Figure 2: The graphical represention of second order derivative graph

3.4 Fourth Order Derivative

Let $G^{(4)} = (V, E^{(4)})$ be the graph, where $V = \{e(x^r, m) | r = 0, 1, 2, \cdots, m-1\}$ and $E^{(4)} = \{e(x^r, m) \to \frac{d^4}{dx^4} e(x^r, m)\}$. Then $G^{(4)}$ is called fourth derivative graph.

Example 3.4 Applying derivative an generalized exponential function

$$\frac{d^4}{dx^4} e(x^0, 5) = \frac{2x^1}{2!} + \frac{7x^6}{7!} + \ldots = \frac{x^1}{1!} + \frac{x^6}{6!} + \ldots$$

$$\frac{d^4}{dx^4} e(x^0, 5) = e(x^1, 5), \frac{d^4}{dx^4} e(x^1, 5) = e(x^2, 5), \frac{d^4}{dx^4} e(x^2, 5) = e(x^3, 5),$$

$$\frac{d^4}{dx^4} e(x^3, 5) = e(x^4, 5), \frac{d^4}{dx^4} e(x^4, 5) = e(x^0, 5)$$

E $= \{e(x^0,5) \to e(x^1,5), e(x^1,5) \to e(x^2,5), e(x^2,5) \to e(x^3,5), e(x^3,5) \to e(x^4,5),$
$e(x^4,5) \to e(x^0,5)\}$

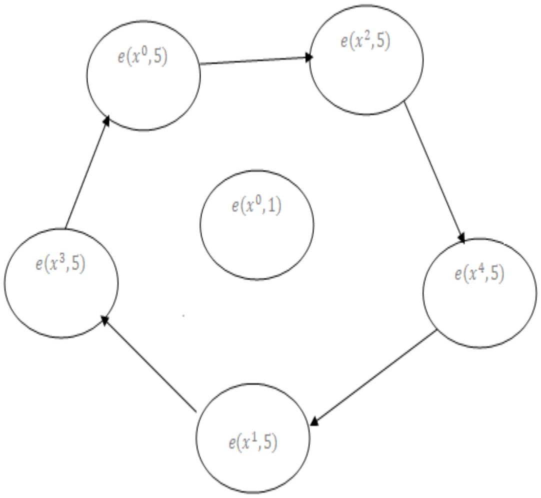

Figure 3: The graphical represention of third order derivative graph

3.5 Fifth Order Derivative

Let $G^{(5)} = (V, E^{(5)})$ be the graph, where $V = \{e(x^r,m) | r = 0,1,2,\cdots,m-1\}$ and $E^{(5)} = \{e(x^r,m) \rightarrow \frac{d^5}{dx^5} e(x^r,m)\}$. Then $G^{(5)}$ is called fifth derivative graph.

Example 3.5 Applying derivative an generalized exponential function

$$\frac{d^5}{dx^5} e(x^0,5) = \frac{1x^0}{1!} + \frac{6x^5}{6!} + \ldots = \frac{x^0}{0!} + \frac{x^5}{5!} + \ldots$$

$$\frac{d^5}{dx^5} e(x^0,5) = e(x^0,5), \frac{d^5}{dx^5} e(x^1,5) = e(x^1,5), \frac{d^5}{dx^5} e(x^2,5) = e(x^2,5),$$

$$\frac{d^5}{dx^5} e(x^3,5) = e(x^3,5), \frac{d^5}{dx^5} e(x^4,5) = e(x^4,5)$$

$$\text{E} = \{e(x^0,5) \rightarrow e(x^0,5), e(x^1,5) \rightarrow e(x^1,5), e(x^2,5) \rightarrow e(x^2,5), e(x^3,5) \rightarrow e(x^3,5)$$
$$e(x^4,5) \rightarrow e(x^4,5)\}$$

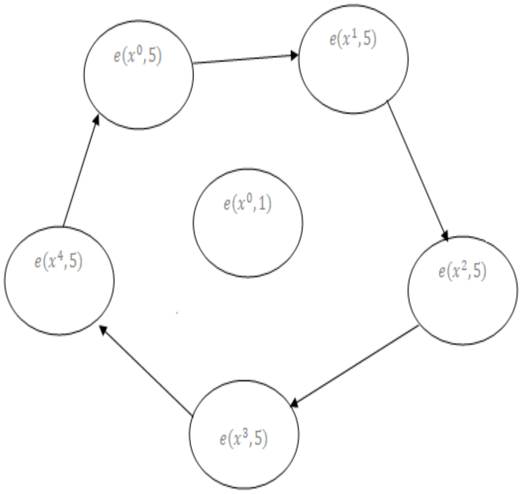

Figure 4: The graphical represention of fourth order derivative graph

For example when $x = 1/2$

Vertex value of the fuzzy derived graph

$\sigma(v_0) = \dfrac{e(1/2^0,5)}{e} = 0.3679, \sigma(v_1) = 0.1839, \sigma(v_2) = 0.0459,$

$\sigma(v_3) = 0.0076, \sigma(v_4) = 0.00095$

Edge value of the fuzzy derived graph

$\mu(v_0,v_1) = \dfrac{e(1/2^1,5)}{e} = 0.1839, \mu(v_1,v_2) = 0.0459, \mu(v_2,v_3) = 0.0076,$

$\mu(v_3,v_4) = 0.00095, \mu(v_4,v_0) = 0.00095$

Degree of Vertex

Let $\bar{G} = (\sigma,\mu)$ be a fuzzy graph on $G = (V,E)$. The degree og vertex u is $d_{\bar{G}}(u) = \sum\limits_{u \neq v \varepsilon E} \mu(u,v)$, for $u, v \varepsilon E$ and $\mu(u,v) = 0$, for (u,v) not in E, this is equivalent to $d_{\bar{G}}(u) = \sum\limits_{(u,v)\varepsilon E} \mu(u,v)$ and $d_{\bar{G}}(u) = \mu(u,v)$

$d_{\bar{G}}(v_0) = \dfrac{e(1/2^1,5)}{e} + \dfrac{e(1/2^4,5)}{e} = 0.18485, d_{\bar{G}}(v_1) = 0.2298, d_{\bar{G}}(v_2) = 0.0535,$

$d_{\bar{G}}(v_3) = 0.00855, d_{\bar{G}}(v_4) = 0.0019$

Total Degree of Vertex

Let $\bar{G} = (\sigma,\mu)$ be a fuzzy graph on $G = (V,E)$.The total degree of a vertex u is

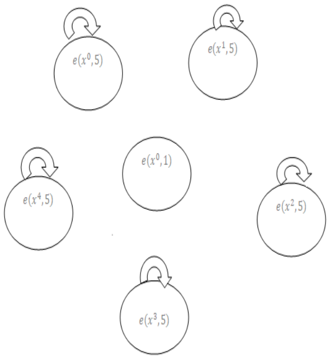

Figure 5: The graphical represention of fifth order derivative graph

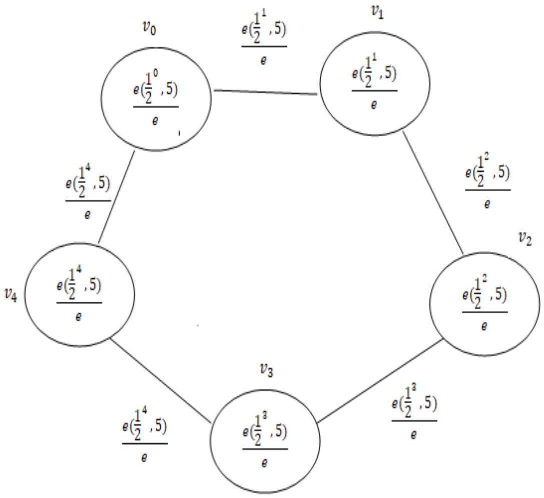

Figure 6: The graphical represention of derivative graph

defind as $td_{\bar{G}}(u) = \sum \mu(u,v) + \sigma(u) = d_{\bar{G}}(u) + \sigma(u), (u,v) \varepsilon E.$

$td_{\bar{G}}(v_0) = 0.55275, td_{\bar{G}}(v_1) = 0.4137, td_{\bar{G}}(v_2) = 0.0994,$

$td_{\bar{G}}(v_3) = 0.01615, td_{\bar{G}}(v_4) = 0.00285$

Irregular Fuzzy Graph

Let $\bar{G} = (\sigma, \mu)$ be a fuzzy graph on $G = (V, E)$. Then $\bar{G}$ is said to be irregular fuzzy graph if there exists a vertex which is adjacent to be vertices with distinct degrees.

From the graph (6) we have the following, $\sigma(v_0) = 0.3679$, $\sigma(v_1) = 0.1839$,

$\sigma(v_2) = 0.0459$, $\sigma(v_3) = 0.0076$, $\sigma(v_4) = 0.00095$ and $\mu(v_0, v_1) = 0.1839$,

$\mu(v_1, v_2) = 0.0459$, $\mu(v_2, v_3) = 0.0076$, $\mu(v_3, v_4) = 0.00095$, $\mu(v_4, v_0) = 0.00095$

such that $v_0, v_1 \varepsilon \bar{G}$ such that $d_{\bar{G}}(v_0) = 0.1839 + 0.00095 = 0.18485$ and

$d_{\bar{G}}(v_1) = 0.1839 + 0.0459 = 0.18485$ such that v_0, v_1 are adjacent to a vertices but $d_{\bar{G}}(v_0) \neq d_{\bar{G}}(v_1)$. Therefore the fuzzy graph $\bar{G}$ is said to be a irregular fuzzy graph.

Totally Irregular Fuzzy Graph

Let $\bar{G} = (\sigma, \mu)$ be a fuzzy graph on $G = (V, E)$. Then $\bar{G}$ is said to be totally irregular fuzzy graph if there exists a vertex which is adjacent to a vertices with distinct total degrees. From the graph (6) we have the following,

$\sigma(v_0) = 0.3679$, $\sigma(v_1) = 0.1839$, $\sigma(v_2) = 0.0459$, $\sigma(v_3) = 0.0076$, $\sigma(v_4) = 0.00095$ and $\mu(v_0, v_1) = 0.1839$, $\mu(v_1, v_2) = 0.0459$, $\mu(v_2, v_3) = 0.0076$, $\mu(v_3, v_4) = 0.00095$, $\mu(v_4, v_0) = 0.00095$ then $td_{\bar{G}}(v_0) = 0.55275$, $td_{\bar{G}}(v_1) = 0.4137$, $td_{\bar{G}}(v_2) = 0.0994$, $td_{\bar{G}}(v_3) = 0.01615$, $td_{\bar{G}}(v_4) = 0.00285$ such that $v_0, v_1 \varepsilon \bar{G}$ such that $td_{\bar{G}}(v_0) = 0.55275$, $td_{\bar{G}}(v_1) = 0.4137$, and v_0, v_1 are adjacent vertices $td_{\bar{G}}(v_0) \neq td_{\bar{G}}(v_1)$

Therefore the fuzzy graph $\bar{G}$ is said to be totally irregular fuzzy graph.

Strongly Irregular Fuzzy Graph

Let $\bar{G} = (\sigma, \mu)$ be a fuzzy graph on $G = (V, E)$. Then $\bar{G}$ is said to be a strongly irregular fuzzy graph if every pair of vertices have distinct degrees.

Let $\bar{G} = (\sigma, \mu)$ be a fuzzy graph on $G = (V, E)$.Then we have $d_{\bar{G}}(v_0) = 0.18485$, $d_{\bar{G}}(v_1) = 0.2298$, $d_{\bar{G}}(v_2) = 0.0535$, $d_{\bar{G}}(v_3) = 0.00855$, $d_{\bar{G}}(v_4) = 0.0019$.

From the graph (6) we have the following,

$\sigma(v_0) = 0.3679$, $\sigma(v_1) = 0.1839$, $\sigma(v_2) = 0.0459$, $\sigma(v_3) = 0.0076$, $\sigma(v_4) = 0.00095$ and $\mu(v_0, v_1) = 0.1839$, $\mu(v_1, v_2) = 0.0459$, $\mu(v_2, v_3) = 0.0076$, $\mu(v_3, v_4) = 0.00095$, $\mu(v_4, v_0) = 0.00095$.

Since $\bar{G}$ has two pair v_0, v_1, v_2, v_4 vertices having degrees then the graph $\bar{G}$ is strongly irregular fuzzy graph.

Highly Irregular Fuzzy Graph

Let $\bar{G} = (\sigma, \mu)$ be a fuzzy graph on $G = (V, E)$. Then $\bar{G}$ is said to be highly irregular fuzzy graph if every vertex in $\bar{G}$ is adjacent to the vertices having distinct degrees.

From the graph (6) we have the following,

$\sigma(v_0) = 0.3679$, $\sigma(v_1) = 0.1839$,

$\sigma(v_2) = 0.0459$, $\sigma(v_3) = 0.0076$, $\sigma(v_4) = 0.00095$ and $\mu(v_0, v_1) = 0.1839$,

$\mu(v_1, v_2) = 0.0459$, $\mu(v_2, v_3) = 0.0076$, $\mu(v_3, v_4) = 0.00095$, $\mu(v_4, v_0) = 0.00095$

$d_{\bar{G}}(v_0) = 0.18485$, $d_{\bar{G}}(v_1) = 0.2298$, $d_{\bar{G}}(v_2) = 0.0535$, $d_{\bar{G}}(v_3) = 0.00855$,

$d_{\bar{G}}(v_4) = 0.0019$ here every vertex in $\bar{G}$ is adjacent to the vertices having distinct degrees then $\bar{G}$ is highly irregular fuzzy graph.

Neighbourly irregular fuzzy graph

Let $\bar{G} = (\sigma, \mu)$ be a fuzzy graph on $G = (V, E)$. Then $\bar{G}$ is said to be a neighbourly irregular fuzzy graph if every two adjacent vertices of a fuzzy graph $\bar{G}$ have distinct degree. From the graph (6) every two adjacent vertices of a fuzzy graph $\bar{G}$ have distinct degree. Then the fuzzy graph $\bar{G}$ neighbourly irregular fuzzy graph.

Degree of an edge

$\bar{G} = (\sigma, \mu)$ be a fuzzy graph. The degree of an edge (u, v) is defined as
$d_{\bar{G}}(u, v) = d_{\bar{G}}(u) + d_{\bar{G}}(v) - 2\mu(u, v)$.
$d_{\bar{G}}(v_0, v_1) = 0.0468, d_{\bar{G}}(v_1, v_2) = 0.1915, d_{\bar{G}}(v_2, v_3) = 0.0468, d_{\bar{G}}(v_3, v_4) = 0.0084,$

$d_{\bar{G}}(v_4, v_0) = 0.18485$

Minimum degree of an edge

The minimum degree of an edge is $\delta_E(\bar{G}) = \wedge\{d_{\bar{G}}(u, v) : (u, v)\varepsilon E\}$.

$$\delta_E(\bar{G}) = min\{d_{\bar{G}}(v_0, v_1), d_{\bar{G}}(v_1, v_2), d_{\bar{G}}(v_2, v_3), d_{\bar{G}}(v_3, v_4), d_{\bar{G}}(v_4, v_0)\}$$

$$= min\{0.0468, 0.1915, 0.0468, 0.0084, 0.18485\} = 0.0084$$

Maximum degree of an edge

The maximum degree of an edge is $\Delta_E(\bar{G}) = \vee\{d_{\bar{G}}(u, v) : (u, v)\varepsilon E\}$.

$$\Delta_E(\bar{G}) = max\{d_{\bar{G}}(v_0, v_1), d_{\bar{G}}(v_1, v_2), d_{\bar{G}}(v_2, v_3), d_{\bar{G}}(v_3, v_4), d_{\bar{G}}(v_4, v_0)\}$$

$$= max\{0.0468, 0.1915, 0.0468, 0.0084, 0.18485\} = 0.1915$$

Total degree of an edge

Let $\bar{G} = (\sigma, \mu)$ be a fuzzy graph. The total degree of an edge (u, v) is defined as $td_{\bar{G}}(u, v) = d_{\bar{G}}(u) + d_{\bar{G}}(v) - \mu(u, v)$
$td_{\bar{G}}(v_0, v_1) = 0.23075, td_{\bar{G}}(v_1, v_2) = 0.2374, td_{\bar{G}}(v_2, v_3) = 0.054450,$

$td_{\bar{G}}(v_3, v_4) = 0.0094, td_{\bar{G}}(v_4, v_0) = 0.1857$

Minimum total degree of an edge

The minimum degree of an edge is $\delta_{tE}(\bar{G}) = \wedge\{td_{\bar{G}}(u,v) : (u,v)\varepsilon E\}$.

$$\delta_{tE}(\bar{G}) = min\{td_{\bar{G}}(v_0,v_1), td_{\bar{G}}(v_1,v_2), td_{\bar{G}}(v_2,v_3), td_{\bar{G}}(v_3,v_4), td_{\bar{G}}(v_4,v_0)\}$$
$$= min\{0.23075, 0.2374, 0.054450, 0.0094, 0.1857\} = 0.0094$$

Maximum total degree of an edge

The maximum degree of an edge is $\Delta_{tE}(\bar{G}) = \vee\{td_{\bar{G}}(u,v) : (u,v)\varepsilon E\}$.

$$\delta_{tE}(\bar{G}) = max\{td_{\bar{G}}(v_0,v_1), td_{\bar{G}}(v_1,v_2), td_{\bar{G}}(v_2,v_3), td_{\bar{G}}(v_3,v_4), td_{\bar{G}}(v_4,v_0)\}$$
$$= max\{0.23075, 0.2374, 0.054450, 0.0094, 0.1857\} = 0.2374$$

Neighbourly edge irregular fuzzy graph

Let $\bar{G} = (\sigma,\mu)$ be a connected fuzzy graph on $G = (V,E)$. Then $\bar{G}$ is said to be neighbourly edge irregular fuzzy graph if every adjacent edges having distinct degree.

From the graph (6) every adjacent edges having distinct degree.

Then $\bar{G}$ neighbourly edge irregular fuzzy graph.

Neighbourly edge totally irregular fuzzy graph

Let $\bar{G} = (\sigma,\mu)$ be a connected fuzzy graph on $G = (V,E)$. Then $\bar{G}$ is said to be a neighbourly edge totally irregular fuzzy graph if every adjacent edges having distinct total degree. From the graph (6) we have the following,

$d_{\bar{G}}(v_1,v_2) = 0.1915$, $d_{\bar{G}}(v_2,v_3) = 0.0468$, $d_{\bar{G}}(v_3,v_4) = 0.0084$, $d_{\bar{G}}(v_4,v_0) = 0.18485$, $td_{\bar{G}}(v_i,v_j) \neq td_{\bar{G}}(v_k,v_l)$ for all i,j,k,l Then $\bar{G}$ neighbourly edge totally irregular fuzzy graph.

Complete Fuzzy Graph

A fuzzy graph $\bar{G} = (\sigma,\mu)$ is said to be complete if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all $u,v\varepsilon V$ and is denoted by K_σ From the graph (6) we have the following,

$\sigma(v_0) = 0.3679$, $\sigma(v_1) = 0.1839$, $\sigma(v_2) = 0.0459$, $\sigma(v_3) = 0.0076$, $\sigma(v_4) = 0.00095$, $\mu(v_0,v_1) = 0.1839$, $\mu(v_1,v_2) = 0.0459$, $\mu(v_2,v_3) = 0.0076$, $\mu(v_3,v_4) = 0.00095$, $\mu(v_4,v_0) = 0.00095$ $\mu(v_0,v_1) = 0.1839, \mu(v_1,v_2) = 0.0459, \mu(v_2,v_3) = 0.0076,$

$\mu(v_3,v_4) = 0.00095, \mu(v_0,v_1) = 0.00095$

Then the fuzzy graph $\bar{G} = (\sigma,\mu)$ is called complete fuzzy graph.

Strongly edge irregular fuzzy graph

Let $\bar{G} = (\sigma,\mu)$ be a fuzzy graph on $\bar{G} = (V,E)$ then $\bar{G}$ is said to be strongly edge irregular fuzzy graph if every pair of edges having distinct degrees.

From the graph (6) we have every pair of edges distinct degrees.

Strongly edge totally irregular fuzzy graph

Let $\bar{G} = (\sigma, \mu)$ be a fuzzy graph on $\bar{G} = (V, E)$ then fuzzy graph $\bar{G}$ is said to be strongly edge totally irregular fuzzy graph if every pair of edges having distinct total degrees. From the graph (6) we have every pair of edges distinct total degrees.

Regular fuzzy graph

Let $\bar{G} = (\sigma, \mu)$ be a fuzzy graph on $\bar{G} = (V, E)$ if $d_G(v) = k$ for all $v \varepsilon V$

That is if each vertex has the same degrees k, then $\bar{G}$ is said to be regular fuzzy graph of degree k. Let $\bar{G} = (V, E)$ be the fuzzy graph. Therefore the vertex of degrees calculated as $d_{\bar{G}}(v_0) = 0.18485$, $d_{\bar{G}}(v_1) = 0.2298$, $d_{\bar{G}}(v_2) = 0.0535$, $d_{\bar{G}}(v_3) = 0.00855$, $d_{\bar{G}}(v_4) = 0.0019$

Each vertex have distinct degree, Hence the fuzzy graph $\bar{G}$ is not regular fuzzy graphs.

Thus we have discusses irregular and regular graphs.

4. Product Graph of Fuzzy Graph

Let G_1 and G_2 be the two graphs. where $G_1 = \{u_0, u_1, u_2, u_3, u_4\}$ and $G_2 = \{v_0, v_1, v_2\}$.

Then the product of $G_1 \times G_2$ is defined below

$$G_1 \times G_2 = \{(u_0v_0), (u_0v_1), (u_0v_2), (u_1v_0), (u_1v_1), (u_1v_2), (u_2v_0), (u_2v_1), (u_2v_2), (u_3v_0), (u_3v_1), (u_3v_2), (u_4v_0), (u_4v_1), (u_4v_2)\}$$

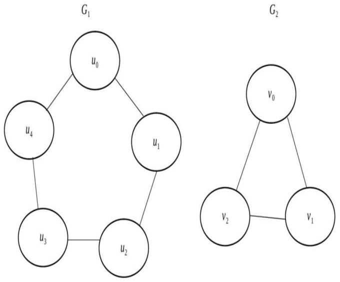

Figure 7: The graphical represention of Product Graphs

σ_1-values ,μ_1 - values of G_1 is given below,

σ_1	u_0	u_1	u_2	u_3	u_4
	$\frac{e(x^{(0)},5)}{e}$	$\frac{e(x^{(1)},5)}{e}$	$\frac{e(x^{(2)},5)}{e}$	$\frac{e(x^{(3)},5)}{e}$	$\frac{e(x^{(4)},5)}{e}$
μ_1	e_1	e_2	e_3	e_4	e_5
	$\frac{e(x^{(1)},5)}{e}$	$\frac{e(x^{(2)},5)}{e}$	$\frac{e(x^{(3)},5)}{e}$	$\frac{e(x^{(4)},5)}{e}$	$\frac{e(x^{(4)},5)}{e}$

σ_2-values ,μ_2 - values of G_2 is given below,

σ_2	u_0	u_1	u_2
	$\frac{e(x^{(0)},5)}{e}$	$\frac{e(x^{(1)},5)}{e}$	$\frac{e(x^{(2)},5)}{e}$
μ_1	e_1	e_2	e_3
	$\frac{e(x^{(1)},5)}{e}$	$\frac{e(x^{(2)},5)}{e}$	$\frac{e(x^{(3)},5)}{e}$

$$\sigma_{12}(u_0v_0) = \sigma_1(u_0).\sigma_2(v_0)$$

The fuzzy vertex values calculated as, $\sigma_{12}(u_0v_0) = \sigma_1(u_0).\sigma_2(v_0) = 0.1381$
$\sigma_{12}(u_0v_1) = 0.0680, \sigma_{12}(u_0v_2) = 0.01695, \sigma_{12}(u_1v_0) = 0.0690, \sigma_{12}(u_1v_1) = 0.0340,$
$\sigma_{12}(u_1v_2) = 0.0084, \sigma_{12}(u_2v_0) = 0.0172, \sigma_{12}(u_2v_1) = 0.0084, \sigma_{12}(u_2v_2) = 0.0021,$
$\sigma_{12}(u_3v_0) = 0.0028, \sigma_{12}(u_3v_1) = 0.0014, \sigma_{12}(u_3v_2) = 0.00035, \sigma_{12}(u_4v_0) = 0.00035,$
$\sigma_{12}(u_4v_1) = 0.00017, \sigma_{12}(u_4v_2) = 0.00017$

The Degree of vertex:

Let $\bar{G} = (\sigma,\mu)$ be a fuzzy graph on $G = (V,E)$. The degree of vertex u is $d_{\bar{G}}(u) = \sum\limits_{u\neq v\varepsilon E} \mu(u,v)$, for $u, v\varepsilon E$ and $\mu(u,v) = 0$, for (u,v) not in E,
this is equivalent to $d_{\tilde{G}}(u,v) = \sum\limits_{(u,v)\in E} \mu(u,v)$
$d_{12}(u_0v_0) = 0.1543, d_{12}(u_1v_0) = 0.1286, d_{12}(u_2v_0) = 0.0305,$
$d_{12}(u_3v_0) = 0.0049, d_{12}(u_4v_0) = 0.00091, d_{12}(u_0v_2) = 0.0339, d_{12}(u_0v_1) = 0.0933,$
$d_{12}(u_1v_2) = 0.0252, d_{12}(u_2v_1) = 0.0189, d_{12}(u_2v_2) = 0.0056, d_{12}(u_3v_1) = 0.00315,$
$d_{12}(u_3v_2) = 0.00087, d_{12}(u_4v_1) = 0.00038, d_{12}(u_4v_2) = 0.000129.$

Total degree of a vertex:

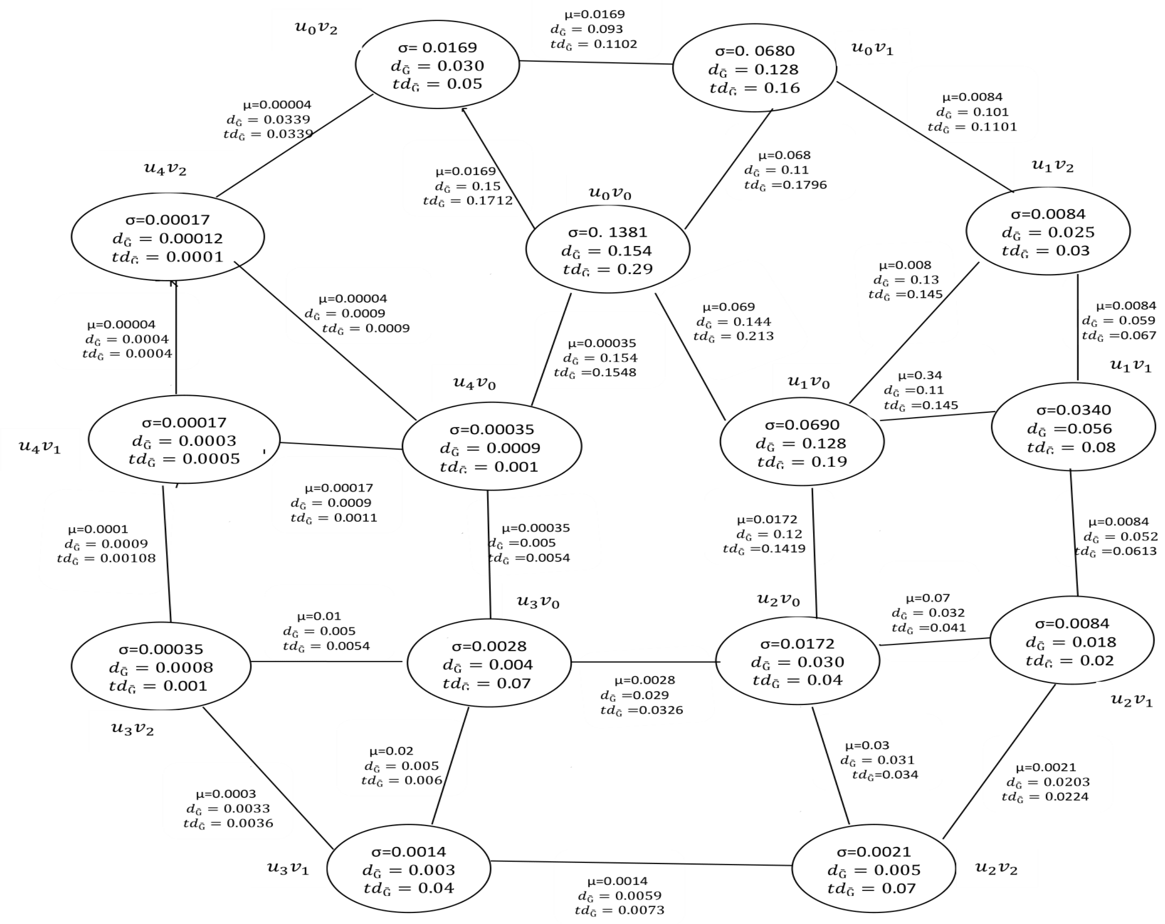

Figure 8: The graphical represention of product of fuzzy derived graph

Let $\bar{G} = (\sigma, \mu)$ be a fuzzy graph on $G = (V, E)$.The total degree of a vertex u is defind as $td_{\widetilde{G}}(u) = d_{\widetilde{G}}(u) + \sigma(u)$
$td_{12}(u_0v_0) = 0.2924, td_{12}(u_1v_0) = 0.1976, td_{12}(u_2v_0) = 0.0477, td_{12}(u_3v_0) = 0.0077,$
$td_{12}(u_4v_0) = 0.0012, td_{12}(u_0v_2) = 0.0508, td_{12}(u_1v_1) = 0.0848, td_{12}(u_2v_1) = 0.0273,$
$td_{12}(u_2v_2) = 0.0077, td_{12}(u_3v_1) = 0.00455, td_{12}(u_3v_2) = 0.00122, td_{12}(u_4v_1) = 0.00055$

$td_{12}(u_4v_2) = 0.00017$.

Irregular Fuzzy Graph

Let $\bar{G_{12}} = (\sigma, \mu)$ be a fuzzy graph on $G = (V,E)$. Then $\bar{G_{12}}$ is said to be irregular fuzzy graph if there exists a vertex which is adjacent to be vertices with distinct degrees.

From the graph 8 we have the following, $u_0, u_1 \in \bar{G_1}$ and $v_0, v_1 \in \bar{G_2}$ such that $d_{12}(u_0v_0) = 0.15428, d_{12}(u_1v_0) = 0.12860$ such that u_0v_0, u_1v_0 are adjacent to vertices but $d_{12}(u_0v_0) \neq d_{12}(u_1v_0)$. Therefore the fuzzy product graph G_{12} is said to be a irregular fuzzy graph.

Totally Irregular Fuzzy Graph

Let $\bar{G_{12}} = (\sigma, \mu)$ be a fuzzy graph on $G = (V,E)$. Then $\bar{G_{12}}$ is said to be totally irregular fuzzy graph if there exists a vertex which is adjacent to a vertices with distinct total degrees.

From the graph 8 we have the following, $u_0, u_1 \in \bar{G_1}$ and $v_0, v_1 \in \bar{G_2}$ such that $td_{12}(u_0v_0) = 0.2924, td_{12}(u_1v_0) = 0.1976$ such that u_0v_0, u_1v_0 are adjacent to vertices but $td_{12}(u_0v_0) \neq td_{12}(u_1v_0)$. Therefore the fuzzy product graph G_{12} is said to be a totally irregular fuzzy graph.

Strongly Irregular Fuzzy Graph

Let $\bar{G_{12}} = (\sigma, \mu)$ be a fuzzy graph on $G = (V,E)$. Then $\bar{G_{12}}$ is said to be a strongly irregular fuzzy graph if every pair of vertices have distinct degrees.

$\bar{G_{12}}$ has two pair $\{u_0v_0, u_1, v_0\}, \{u_0v_0, u_0, v_1\}$ vertices having distinct degrees then $\bar{G_{12}}$ is strongly irregular fuzzy graph.

From the fuzzy product graph 8 having two pair vertices distinct degrees.

Highly Irregular Fuzzy Graph

Let $\bar{G_{12}} = (\sigma, \mu)$ be a fuzzy graph on $G = (V,E)$. Then $\bar{G_{12}}$ is said to be a highly irregular fuzzy graph if every vertex in $\bar{G_{12}}$ is adjacent to the vertices having distinct degrees. From the fuzzy product graph 8 we have the following, if u_0v_0, u_1v_0 are adjacent but having distinct degress then $\bar{G_{12}}$ is highly irregular fuzzy graph.

Neighbourly irregular fuzzy graph

Let $\bar{G_{12}} = (\sigma, \mu)$ be a fuzzy graph on $G = (V,E)$. Then $\bar{G_{12}}$ is said to be a neighbourly irregular fuzzy graph if every two adjacent vertices of a fuzzy graph $\bar{G_{12}}$ have distinct degree. From the fuzzy product graph 8. If every two adjacent vretices have distinct degrees. Therefore $\bar{G_{12}}$ is Neighbourly irregular fuzzy graph.

Edge value of product fuzzy graph

$\mu_{12}(u_0v_0,u_1v_0) = 0.0690, \mu_{12}(u_1v_0,u_2v_0) = 0.0172, \mu_{12}(u_2v_0,u_3v_0) = 0.0028,$

$\mu_{12}(u_3v_0,u_4v_0) = 0.00035, \mu_{12}(u_4v_0,u_0v_0) = 0.00035, \mu_{12}(u_0v_0,u_0v_2) = 0.01695,$

$\mu_{12}(u_0v_0,u_0v_1) = 0.0680, \mu_{12}(u_1v_0,u_1v_2) = 0.0084, \mu_{12}(u_1v_0,u_1v_1) = 0.0340,$

$\mu_{12}(u_2v_0,u_2v_1) = 0.0084, \mu_{12}(u_2v_0,u_2v_2) = 0.0021, \mu_{12}(u_3v_0,u_3v_1) = 0.0014,$

$\mu_{12}(u_3v_0,u_3v_2) = 0.00035, \mu_{12}(u_4v_0,u_4v_1) = 0.00017, \mu_{12}(u_4v_0,u_4v_2) = 0.000043,$

$\mu_{12}(u_0v_2,u_0v_1) = 0.01695, \mu_{12}(u_0v_1,u_1v_2) = 0.0084, \mu_{12}(u_1v_2,u_1v_1) = 0.0084,$

$\mu_{12}(u_1v_1,u_2v_1) = 0.0084, \mu_{12}(u_2v_1,u_2v_2) = 0.0021, \mu_{12}(u_2v_2,u_3v_1) = 0.0014,$

$\mu_{12}(u_3v_1,u_3v_2) = 0.00035, \mu_{12}(u_3v_2,u_4v_1) = 0.00017,$

$\mu_{12}(u_4v_1,u_4v_2) = 0.000043, \mu_{12}(u_4v_2,u_0v_2) = 0.000043$

The Degree of an Edge:

$\bar{G} = (\sigma,\mu)$ be a fuzzy graph. The degree of an edge (u,v) is defined as

$d_{\widetilde{G}}(u,v) = d_{\widetilde{G}}(u) + d_{\widetilde{G}}(v) - 2\mu(u,v)$

$d_{\widetilde{G}}(u_0v_0,u_1v_0) = 0.1449, d_{\widetilde{G}}(u_1v_0,u_2v_0)0.1247, d_{\widetilde{G}}(u_2v_0,u_3v_0)0.0298,$

$d_{\widetilde{G}}(u_3v_0,u_4v_0) = 0.0051, d_{\widetilde{G}}(u_4v_0,u_0v_0) = 0.1545, d_{\widetilde{G}}(u_0v_0,u_0v_2) = 0.1543,$

$d_{\widetilde{G}}(u_0v_0,u_0v_1) = 0.1116, d_{\widetilde{G}}(u_1v_0,u_1v_2) = 0.1370, d_{\widetilde{G}}(u_1v_0,u_1v_1) = 0.1114,$

$d_{\widetilde{G}}(u_2v_0,u_2v_1) = 0.0326, d_{\widetilde{G}}(u_2v_0,u_2v_2) = 0.0319, d_{\widetilde{G}}(u_3v_0,u_3v_1) = 0.00525,$

$d_{\widetilde{G}}(u_3v_0,u_3v_2) = 0.00507, d_{\widetilde{G}}(u_4v_0,u_4v_1) = 0.00095, d_{\widetilde{G}}(u_0v_2,u_0v_1) = 0.0933,$

$d_{\widetilde{G}}(u_0v_1,u_1v_2) = 0.1017, d_{\widetilde{G}}(u_1v_2,u_1v_1) = 0.0592, d_{\widetilde{G}}(u_1v_1,u_2v_1) = 0.0529,$

$d_{\widetilde{G}}(u_2v_1,u_2v_2) = 0.0203, d_{\widetilde{G}}(u_2v_2,u_3v_1) = 0.00595, d_{\widetilde{G}}(u_3v_1,u_3v_2) = 0.0033,$

$d_{\widetilde{G}}(u_3v_2,u_4v_1) = 0.00091, d_{\widetilde{G}}(u_4v_1,u_4v_2) = 0.00042, d_{\widetilde{G}}(u_4v_2,u_0v_2) = 0.0339$

The Minimum degree of an edge is

$$\delta_E(G_1G_2) = \wedge\{d_{G_1G_2}(u,v) : (u,v) \in E\} = 0.00042$$

The Maximum degree of an edge is

$$\Delta_E(G_1G_2) = \vee\{d_{G_1G_2}(u,v) : (u,v) \in E\} = 0.1545$$

Total Degree of an Edge:

Let $\bar{G} = (\sigma, \mu)$ be a fuzzy graph. The total degree of an edge (u, v) is defined as $td_{\widetilde{G}}(u,v) = d_{\widetilde{G}}(u) + d_{\widetilde{G}}(v) - \mu(u,v)$

$td_{12}(u_0v_0, u_1v_0) = 0.2139, td_{12}(u_1v_0, u_2v_0) = 0.1419, td_{12}(u_2v_0, u_3v_0) = 0.0326,$

$td_{12}(u_3v_0, u_4v_0) = 0.005463, td_{12}(u_4v_0, u_0v_0) = 0.1548, td_{12}(u_0v_0, u_0v_2) = 0.17125,$

$td_{12}(u_0v_0, u_0v_1) = 0.1796, td_{12}(u_1v_0, u_1v_2) = 0.1454, td_{12}(u_1v_0, u_1v_1) = 0.1454,$

$td_{12}(u_2v_0, u_2v_1) = 0.0410, td_{12}(u_2v_0, u_2v_2) = 0.0340, td_{12}(u_3v_0, u_3v_1) = 0.0066,$

$td_{12}(u_3v_0, u_3v_2) = 0.00542, td_{12}(u_4v_0, u_4v_1) = 0.0011, td_{12}(u_4v_0, u_4v_2) = 0.000999,$

$td_{12}(u_0v_2, u_0v_1) = 0.11025, td_{12}(u_0v_1, u_1v_2) = 0.1101, td_{12}(u_1v_2, u_1v_1) = 0.0676,$

$td_{12}(u_1v_1, u_2v_1) = 0.0613, td_{12}(u_2v_1, u_2v_2) = 0.0224, td_{12}(u_2v_2, u_3v_1) = 0.00735,$

$td_{12}(u_3v_1, u_3v_2) = 0.00367, td_{12}(u_3v_2, u_4v_1) = 0.00108, td_{12}(u_4v_1, u_4v_2) = 0.000466,$

$td_{12}(u_4v_2, u_0v_2) = 0.0339.$

The Minimum total degree of an edge is

$$\delta_{tE}(G_1G_2) = \wedge\{td_{G_1G_2}(u,v) : (u,v) \in E\} = 0.000466$$

The Maximum total degree of an edge is

$$\Delta_{tE}(G_1G_2) = \vee\{td_{G_1G_2}(u,v) : (u,v) \in E\} = 0.2139$$

Neighbourly edge irregular fuzzy graph

Let $\bar{G_{12}} = (\sigma, \mu)$ be a connected fuzzy graph on $G = (V, E)$. Then $\bar{G_{12}}$ is said to be a neighbourly edge irregular fuzzy graph if every adjacent edges having distinct degree.

From the product graph 8 we have every adjacent edges having distinct degree.

Then the fuzzy product graph $\bar{G_{12}}$ neighbourly edge irregular fuzzy graph.

Neighbourly edge totally irregular fuzzy graph

Let $\bar{G_{12}} = (\sigma, \mu)$ be a connected fuzzy graph on $G = (V, E)$. Then $\bar{G_{12}}$ is said to be a neighbourly edge totally irregular fuzzy graph if every adjacent edges having distinct total degree. From the product graph 8 we have every adjacent edges having distinct total degree. Then the fuzzy product graph $\bar{G_{12}}$ neighbourly edge totally irregular fuzzy graph.

Complete Fuzzy Graph

A fuzzy graph $\bar{G_{12}} = (\sigma, \mu)$ is said to be complete if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all

$u, v \varepsilon V$ and is denoted by K_σ.

Then the fuzzy product graph $\bar{G_{12}} = (\sigma, \mu)$ is called complete fuzzy graph.

Strongly edge irregular fuzzy graph

Let $\bar{G_{12}} = (\sigma, \mu)$ be a fuzzy graph on $\bar{G} = (V, E)$ then $\bar{G_{12}}$ is said to be strongly edge irregular fuzzy graph if every pair of edges having distinct degrees. From the product graph 8 we have every pair of edges having distinct degrees.

Then the fuzzy product graph $\bar{G_{12}} = (\sigma, \mu)$ is called strongly edge irregular fuzzy graph.

Strongly edge totally irregular fuzzy graph

Let $\bar{G_{12}} = (\sigma, \mu)$ be a fuzzy graph on $\bar{G} = (V, E)$ then $\bar{G_{12}}$ is said to be strongly edge totally irregular fuzzy graph if every pair of edges having distinct total degrees. From the product graph 8 we have every pair of edges having distinct total degrees. Then the fuzzy product graph $\bar{G_{12}} = (\sigma, \mu)$ is called storngly edge total irregualr fuzzy graph.

Regular fuzzy graph

Let $\bar{G_{12}} = (\sigma, \mu)$ be a fuzzy graph on $\bar{G} = (V, E)$ if $d_{G_{12}}(v) = k$ for all $v \in V$

That is if each vertex has the same degrees k, then $\bar{G_{12}}$ is said to be regular fuzzy graph of degree k. Here each vertex have distinct degree, Hence the fuzzy poduct grpah $\bar{G_{12}}$ is not regular fuzzy graphs.

Totally regular fuzzy graph

If each vertex $\bar{G_{12}}$ have the distinct total degree, Hence the fuzzy product graph $\bar{G_{12}}$ is not totally regular fuzzy graph.

d_2^{th} degree of a vertex:

$d_2(u_0v_2) = 0.0084 + 0.01695 + 0.00035 + 0.000043 = 0.02574, d_2(u_0v_1) = 0.07679,$

$d_2(u_1v_2) = 0.0336, d_2(u_1v_1) = 0.0617, d_2(u_2v_1) = 0.0210, d_2(u_2v_2) = 0.00665,$

$d_2(u_2v_0) = 0.0986, d_2(u_0v_0) = 0.06016, d_2(u_4v_0) = 0.0021, d_2(u_3v_0) = 0.00826,$

$d_2(u_3v_1) = 0.00332, d_2(u_3v_2) = 0.00109, d_2(u_4v_1) = 0.00055, d_2(u_4v_2) = 0.000215$

d_3^{th} degree of a vertex:

$d_3(u_0v_2) = 0.03429, d_3(u_0v_1) = 0.02612, d_3(u_1v_2) = 0.00529, d_3(u_1v_1) = 0.0215,$

$d_3(u_2v_1) = 0.0175, d_3(u_2v_2) = 0.00472, d_3(u_2v_0) = 0.0343, d_3(u_1v_0) = 0.001963,$

$d_3(u_0v_0) = 0.0112, d_3(u_4v_0) = 0.0014, d_3(u_3v_0) = 0.0063, d_3(u_3v_1) = 0.00319,$

$d_3(u_3v_2) = 0.00109, d_3(u_4v_1) = 0.00068, d_3(u_4v_2) = 0.00017$

d_4^{th} degree of a vertex:

$d_4(u_0v_2) = 0.01085, d_4(u_0v_1) = 0.0028, d_4(u_1v_2) = 0.00192, d_4(u_1v_1) = 0.00074,$

$d_4(u_2v_1) = 0.00874, d_4(u_2v_2) = 0.00424, d_4(u_3v_1) = 0.0021, d_4(u_3v_2) = 0.00105,$

$d_4(u_4v_1) = 0.00051, d_4(u_4v_2) = 0.00012$

Thus we have explained fuzzy product graph with suitable examples by finding irregular and regular graphs.

5 Conclusion

The study on fuzzy derived graphs offers a detailed exploration of their properties, such as fuzzy edge degrees, regularity, and irregularity. It distinguishes between various types of fuzzy graphs, including neighbourly edge irregular and strongly edge irregular graphs, providing a clear framework to understand their complexities. Through examples, the research emphasizes the importance of fuzzy graphs in mathematical modeling and their potential applications in different fields.

The introduction of generalized exponential functions adds depth to the discussion, opening up new areas for future research. In conclusion, the study improves the theoretical understanding of fuzzy derived graphs, making it a valuable contribution to graph theory. It encourages further investigation into fuzzy graph properties and their implications in real-world scenarios.

References

[1] Ahmad U and Batool T, Domination in rough fuzzy digraphs with application. Soft Computing, 27(5), pp.2425-2442(2023).

[2] Ajay D, Selvachandran G, Aldring J, Thong PH, Son LH and Cuong BC, Einstein exponential operation laws of spherical fuzzy sets and aggregation operators in decision making. Multimedia Tools and Applications, 82(27), pp.41767-41790(2023).

[3] Al Mutab, Huda Mutab. "Fuzzy Graphs." Journal of Advances in Mathematics 17 : 232-247(2019).

[4] Alspach B, and Parsons TD, Isomorphism of circulant graphs and digraphs. Discrete Mathematics, 25(2): 97-108(1979).

[5] Atpinar S and Yazlik Y, Qualitative behavior of exponential type of fuzzy difference equations system. Journal of Applied Mathematics and Computing, 69(6), pp.4135-4162(2023).

[6] Beaini D, Passaro S, Létourneau V, Hamilton W, Corso G and Lio P, Directional graph networks. In International Conference on Machine Learning (pp. 748-758)2021.

[7] Behmanesh M, Krahn M and Ovsjanikov M, TIDE: Time derivative diffusion for deep learning on graphs. In International Conference on Machine Learning (pp. 2015-2030). PMLR,2023, July..

[8] Gani A Nagoor and Latha SR, "On irregular fuzzy graphs", Applied Mathematical Sciences 6.11 : 517-523(2012).

[9] Garai T, Chakraborty D and Roy TK, Expected value of exponential fuzzy number and its application to multi-item deterministic inventory model for deteriorating items. Journal of Uncertainty Analysis and Applications, 5, pp.1-20(2017).

[10] Haciomeroglu ES, Aspinwall L and Presmeg NC, Contrasting cases of calculus students understanding of derivative graphs, Mathematical thinking and learning, 12(2), pp.152-176(2010).

[11] Harary F and Schwenk AJ, The communication problem on graphs and digraphs(1974).

[12] Kóczy L, Fuzzy graphs in the evaluation and optimization of networks. Fuzzy sets and systems, 46(3), pp.307-319(1992).

[13] Krivitsky PN, Exponential-family random graph models for valued networks. Electronic journal of statistics, 6, p.1100(2012).

[14] Krishnaveni J and Maheswari NS, Support neighbourly irregular fuzzy graphs. International Journal of advanced research in engineering and technology, 11(1), pp.147-154(2020).

[15] Nešetřil J, Homomorphisms of derivative graphs. Discrete Mathematics, 1(3), pp.257-268(1971).

[16] Pjanıć K, Lıđan E and Kurtanovıć A, Visualization of relationship between a function and its derivative. Eğitim Bilimleri Araştırmaları Dergisi, 5(1), pp.205-213(2015).

[17] Radha K and Sri Harini S, Characterizations of the Direct Sum of Two Difference-Mean Fuzzy Graphs. Indian Journal of Science and Technology, 17(20), pp.2043-2049(2024).

[18] Sabidussi G, Graph derivatives. Mathematische Zeitschrift, 76(1), pp.385-401(1961).

[19] Santhimaheswari NR and Sekar C, "On strongly edge irregular fuzzy graphs." Kragujevac Journal of Mathematics 40.1: 125-135(2016).

[20] Santhimaheswari NR and Sekar C, On neighbourly edge irregular fuzzy graphs, International Journal of Mathematical Archive, 6(10): 224-231(2015).

[21] Santhimaheswari NR and Sekar C, "On (r, m, k)-Regular Fuzzy Graphs." Mathematical Combinatorics 1: 18-26(2016).

[22] Ubuz B, Interpreting a graph and constructing its derivative graph: Stability and change in students' conceptions. International Journal of Mathematical Education in Science and Technology, 38(5), pp.609-637 (2007).