ISSN: 2456-8686, Volume 2, Issue 2, 2018:80-88
DOI : http://doi.org/10.26524/cm41

Journal of Computational Mathematica

Open Access

Some Applications of Higher Order Generalized $\alpha-$ Difference Operator

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Abstract

In this paper, we derive the discrete version of the Bernoulli's formula according to the generalized α- difference operator for negative ℓ, and to find the sum of several type of arithmetic series in the field of Numerical Methods. Suitable example are provided to illustrate the main results.

Key words: Generalized α- Difference, Operator, Polynomial, Polynomial factorial.
AMS classification: 39A10, 39A11, 39A13.

1.Introduction

The theory of difference equations is based on the operator Δ defined as

$$\Delta u(k) = u(k+1) - u(k), k \in \mathbb{N}(0) = \{0, 1, 2, ...\}. \tag{1}$$

Eventhough many authors [1], [11], [14] have suggested the definition of Δ as

$$\Delta u(k) = u(k+\ell) - u(k), k \in [0, \infty),\ \ell \in (0, \infty), \tag{2}$$

and no significant progress took place in the field of numerical methods, they took up the definition of Δ as given in (2), and developed the theory of difference equations in a different direction and many interesting results were obtained in number theory. For convenience, they labelled the operator Δ defined by (2) as Δ_ℓ and its inverse by Δ_ℓ^{-1}. When Δ_ℓ is operated on a complex function $u(k)$ and considering ℓ to be real, some new qualitative properties like rotatory, expanding, shrinking, spiral and weblike were noticed. The results obtained can be found in [15].

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Jerzy Popenda, et al.,[18], while discussing the behavior of solutions of a particular type of difference equation, defined Δ_α as $\Delta_\alpha u(k) = u(k+1) - \alpha u(k)$. This definition of Δ_α is being ignored for a long time. In [5], we have generalized the definition of Δ_α by $\Delta_{\alpha(\ell)}$defined as $\Delta_{\alpha(\ell)} u(k) = u(k+\ell) - \alpha u(k)$ for the real valued function $u(k)$ and $\ell \in (0, \infty)$ and also obtained the solutions of certain types of generalized α-difference equations, in particular, the generalized Clairaut's α-difference equation, generalized Euler α-difference equation and the generalized α-Bernoulli polynomial $B_{\alpha(n)}(k, \ell)$, which is a solution of the α-difference equation $u(k+\ell) - \alpha u(k) = nk^{n-1}$, for $n \in \mathbb{N}(1)$ [19], [4].

Recently, G.B.A.Xavier, et.al. extended from the definition of generalized α-difference operator of n^{th} kind and to obtain the formula for sum of partial sums of various types of arithmetic-geometric progression in the field of Numerical Analysis [17].

With this background, in this paper we derive the generalized discrete $\alpha-$Bernoulli's formula and to obtain the formula for sum of several types of arithmetic and geometric series using the stirling numbers of first and second kind respectively.

Throughout this paper, we make use of the following notations:

1. $[\frac{k}{\ell}]$ means integer part of $\frac{k}{\ell}$,
2. $N_\ell(j) = \{j, \ell + j, 2\ell + j, \cdots\}$,
3. $\Delta^{-m}_{\alpha(-\ell)} u(k)||^{k}_{(m-1)\ell+j} =$
   $\left[\Delta^{-1}_{\alpha(-\ell)} u(k)|^{k}_{j}\right] \left[\Delta^{-1}_{\alpha(-\ell)} u(k)|^{k}_{\ell+j}\right] \cdots \left[\Delta^{-1}_{\alpha(-\ell)} u(k)|^{k}_{(m-1)\ell+j}\right]$,
4. $(m-1)^{(m-1)} = m!$.

2. Preliminaries

In this section, we present some basic definitions and preliminary results which will be useful for further subsequent discussions.

Definition 2.1 [5] If $u(k)$ is real valued function then the generalized α-difference operator is defined by

$$\Delta_{\alpha(\pm\ell)} u(k) = u(k \pm \ell) - \alpha u(k),\ \alpha > 0, \ell \in (0, \infty) \tag{3}$$

and inverse is defined by

$$\Delta^{-1}_{\alpha(\pm\ell)} u(k) = v(k) - \alpha^{\pm[\frac{k}{\ell}]} v(j), \tag{4}$$

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where $v(j)$ is constant for all $k \in N_\ell(j)$.

Lemma 2.2 [17] If $u(k)$ is real valued function then

$$\Delta^{-m}_{\alpha(\pm\ell)} u(k) = \sum_{r=m}^{[\frac{k}{\ell}]} \frac{(r-1)^{(m-1)}}{(m-1)^{(m-1)}} (\pm 1)^m (\alpha)^{\pm r} u(k \mp (m-r)\ell), k \in [m\ell, \infty). \qquad (5)$$

Theorem 2.3 [17] For $k \in [m\ell, \infty)$,

$$\Delta^{-m}_{\alpha(-\ell)} u(k) ||^{k}_{(m-1)\ell+j} = \sum_{r=m}^{[\frac{k}{\ell}]} \frac{(r-1)^{(m-1)}}{(m-1)^{(m-1)}} (-1)^m \left(\frac{1}{\alpha}\right)^r u(k + (m-r)\ell). \qquad (6)$$

Lemma 2.4 Let $u(k)$ and $v(k)$ be any two real valued functions. Then,

$$\Delta_{\alpha(-\ell)}[u(k)v(k)] = u(k-\ell)\Delta_{\alpha(-\ell)} v(k) + \alpha v(k) \Delta_{-\ell} u(k). \qquad (7)$$

and

$$\Delta^{-1}_{\alpha(-\ell)}[u(k)v(k)] = u(k+\ell)\Delta^{-1}_{\alpha(-\ell)} v(k) - \alpha \Delta^{-1}_{\alpha(-\ell)}[\Delta^{-1}_{\alpha(-\ell)} v(k) \Delta_{-\ell} u(k+\ell)]. \qquad (8)$$

3. Generalized Discrete $\alpha-$ Bernoulli's Formula

In this section, for negative index $-\ell$, we derive the discrete $\alpha-$ Bernoulli's formula and establish as the sum of general partial sums of products of polynomials and polynomial factorials using the inverse of generalized α- difference operator and stirling numbers of first kind and second kind respectively.

Lemma 3.1 If $u_i(k), i = 1, 2, \cdots, w$ are real valued function then

$$\Delta^{-m}_{\alpha(-\ell)} \left[\prod_{i=1}^{w} u_i(k)\right] \Bigg|\Bigg|^{k}_{((m-1)\ell+j)} = \left\{ \Delta^{-(m-1)}_{\alpha(-\ell)} \left[u_w(k+\ell) \Delta^{-1}_{\alpha(-\ell)} \prod_{i=1}^{w-1} u_i(k)\right] + \right.$$

$$\sum_{r=0}^{w-1} \alpha^{r+1} \sum_{i_1=1}^{w} n_w C_{i_1} \ell^{i_1} \sum_{t=0}^{i_1-1} i_1 C_t r_w^t \left[\prod_{a=2}^{r+1} \sum_{i_a=1}^{w-\sum_{b=1}^{a-1} i_b} (w - \sum_{b=1}^{a-1} i_b) C_{i_a} \ell^{i_a}\right]$$

$$\Delta^{-(m-2)}_{\alpha(-\ell)} \left[u_{w-\sum_{p=1}^{r+1} i_p} (k+m\ell)^{w-\sum_{p=1}^{r+1} i_p} \Delta^{-(m+1)}_{\alpha(-\ell)} \prod_{i=1}^{w-1} u_i(k) + \sum_{q=0}^{m} (-\alpha)^{q+1} \right.$$

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$$\sum_{r_1=1}^{m-q}(m-\sum_{p=1}^{r+1} i_p)C_{r_1}\ell^{r_1}\prod_{b=2}^{q+1}\sum_{r_b=1}^{m-\sum_{p=1}^{r+1} i_p-(q-1)}\Bigg[m-\sum_{p=1}^{r+1} i_p-\sum_{a=1}^{b-1} r_a\Bigg]C_{r_b}\ell^{r_b}$$

$$u_{w-\sum_{p=1}^{r+1} i_p}(k+m\ell)^{m-\sum_{p=1}^{r+1} i_p-q-r_{q+1}}\Bigg]\Delta^{-4}_{\alpha(-\ell)}\prod_{i=1}^{w-1}u_i(k)\Bigg\}\Bigg|_{(m-1)\ell+j}^{k}.$$

Proof: From Lemma 2.4, we have

$$\Delta^{-1}_{\alpha(-\ell)}\Bigg[\prod_{i=1}^{w}u_i(k)\Bigg]\Big|\Big|_j^k = u_w(k+\ell)\Delta^{-1}_{\alpha(-\ell)}\Bigg[\prod_{i=1}^{w}u_i(k)\Bigg]+\Delta^{-2}_{\alpha(-\ell)}\Bigg[\prod_{i=1}^{w-1}u_i(k)\Bigg]$$

$$\Bigg[\sum_{r=0}^{w-1}\alpha^{r+1}\sum_{i_1=1}^{w-r} wC_{i_1}\ell^{i_1}\Bigg[\prod_{a=2}^{r+1}\sum_{i_a=1}^{w-\sum_{b=1}^{a-1}i_b}(w-\sum_{b=1}^{a-1}i_b)C_{i_a}\ell^{i_a}\Bigg]u_i(k+\ell)^{w-\sum_{p=1}^{r+1}i_p}\Bigg]\Bigg|_j^k$$

where $\Delta^{-1}_{\alpha(-\ell)}\left[\prod_{i=1}^{w}u_i(k)\right]$ is a function of k and $\Delta^{-1}_{\alpha(-\ell)}\left[\prod_{i=1}^{m}u_i(j)\right]$ is constant. Again taking $\Delta^{-1}_{\alpha(-\ell)}$ and applying the limit from $\ell+j$ to k, we obtain

$$\Delta^{-2}_{\alpha(-\ell)}\Bigg[\prod_{i=1}^{w}u_i(k)\Bigg]\Big|\Big|_{\ell+j}^k = \Delta^{-1}_{\alpha(-\ell)}\Bigg[u_w(k+\ell)\Delta^{-1}_{\alpha(-\ell)}\prod_{i=1}^{w-1}u_i(k)\Bigg]+$$

$$\sum_{r=0}^{w-1}\alpha^{r+1}\sum_{i_1=1}^{w} n_wC_{i_1}\ell^{i_1}\sum_{t=0}^{i_1-1} i_1C_t r_w^t\Bigg[\prod_{a=2}^{r+1}\sum_{i_a=1}^{w-\sum_{b=1}^{a-1}i_b}(w-\sum_{b=1}^{a-1}i_b)C_{i_a}\ell^{i_a}\Bigg]$$

$$\Delta^{-(m-2)}_{\alpha(-\ell)}\Bigg[u_{w-\sum_{p=1}^{r+1}i_p}(k+2\ell)^{w-\sum_{p=1}^{r+1}i_p}\Delta^{-3}_{\alpha(-\ell)}\prod_{i=1}^{w-1}u_i(k)+\sum_{q=0}^{m}(-\alpha)^{q+1}$$

$$\sum_{r_1=1}^{2-q}(2-\sum_{p=1}^{r+1}i_p)C_{r_1}\ell^{r_1}\prod_{b=2}^{q+1}\sum_{r_b=1}^{2-\sum_{p=1}^{r+1}i_p-(q-1)}\Bigg[2-\sum_{p=1}^{r+1}i_p-\sum_{a=1}^{b-1}r_a\Bigg]C_{r_b}\ell^{r_b}$$

$$u_{w-\sum_{p=1}^{r+1}i_p}(k+2\ell)^{2-\sum_{p=1}^{r+1}i_p-q-r_{q+1}}\Bigg]\Delta^{-4}_{\alpha(-\ell)}\prod_{i=1}^{w-1}u_i(k)\Big|_{\ell+j}^k.$$

Similarly again operating $\Delta^{-1}_{\alpha(\ell)}$ on both sides and applying the limit from $2\ell + j$ to k and which can be expressed as

$$\Delta^{-3}_{\alpha(-\ell)}\left[\prod_{i=1}^{w} u_i(k)\right]\Big\|_{2\ell+j}^{k} = \Delta^{-2}_{\alpha(-\ell)}\left[u_w(k+2\ell)\Delta^{-2}_{\alpha(-\ell)}\prod_{i=1}^{w-1} u_i(k)\right] +$$

$$\sum_{r=0}^{w-1}\alpha^{r+1}\sum_{i_1=1}^{w} n_w C_{i_1}\ell^{i_1}\sum_{t=0}^{i_1-1} i_1 C_t r_w^t\left[\prod_{a=2}^{r+1}\sum_{i_a=1}^{w-\sum\limits_{b=1}^{a-1} i_b}(w-\sum_{b=1}^{a-1} i_b)C_{i_a}\ell^{i_a}\right]$$

$$\Delta^{-(m-2)}_{\alpha(-\ell)}\Bigg[u_{w-\sum\limits_{p=1}^{r+1} i_p}(k+3\ell)^{w-\sum\limits_{p=1}^{r+1} i_p}\Delta^{-4}_{\alpha(-\ell)}\prod_{i=1}^{w-1} u_i(k)+\sum_{q=0}^{m}(-\alpha)^{q+1}$$

$$\sum_{r_1=1}^{2-q}(3-\sum_{p=1}^{r+1} i_p)C_{r_1}\ell^{r_1}\prod_{b=2}^{q+1}\sum_{r_b=1}^{3-\sum\limits_{p=1}^{r+1} i_p-(q-1)}\left[3-\sum_{p=1}^{r+1} i_p-\sum_{a=1}^{b-1} r_a\right]C_{r_b}\ell^{r_b}$$

$$u_{w-\sum\limits_{p=1}^{r+1} i_p}(k+3\ell)^{3-\sum\limits_{p=1}^{r+1} i_p-q-r_{q+1}}\Bigg]\Delta^{-4}_{\alpha(-\ell)}\prod_{i=1}^{w-1} u_i(k)\Big|_{2\ell+j}^{k}.$$

The proof completes by continuing this process.

Corollary 3.2 Let k^n be the generalized polynomial then

$$\Delta^{-m}_{\alpha(-\ell)}\left[\prod_{i=1}^{w}(k+r_i\ell)^{n_i}\right]\Big\|_{((m-1)\ell+j)}^{k} = \Delta^{-(m-1)}_{\alpha(-\ell)}\left[(k+\ell+r_w\ell)^{n_w}\Delta^{-1}_{\alpha(-\ell)}\prod_{i=1}^{w-1}(k+r_i\ell)^{n_i}\right]$$

$$+\sum_{r=0}^{n_{w-1}}\alpha^{r+1}\sum_{i_1=1}^{n_{w-r}} n_w C_{i_1}\ell^{i_1}\sum_{t=0}^{i_1-1} i_1 C_t r_w^t\left[\prod_{a=2}^{r+1}\sum_{i_a=1}^{n_w-\sum\limits_{b=1}^{a-1} i_b}(n_w-\sum_{b=1}^{a-1} i_b)C_{i_a}\ell^{i_a}\right]$$

$$\Delta^{-(m-2)}_{\alpha(-\ell)}\Bigg[(k+m\ell)^{n_w-\sum\limits_{p=1}^{r+1} i_p}\Delta^{-(m+1)}_{\alpha(-\ell)}\prod_{i=1}^{w-1}(k+r_i\ell)^{n_i}+\sum_{q=0}^{n_m}(-\alpha)^{q+1}$$

$$\sum_{r_1=1}^{n_m-q}(n_m-\sum_{p=1}^{r+1} i_p)C_{r_1}\ell^{r_1}\prod_{b=2}^{q+1}\sum_{r_b=1}^{n_m-\sum\limits_{p=1}^{r+1} i_p-(q-1)}\left[n_m-\sum_{p=1}^{r+1} i_p-\sum_{a=1}^{b-1} r_a\right]C_{r_b}\ell^{r_b}$$

$$(k+(m-1)\ell)^{n_m-\sum\limits_{p=1}^{r+1} i_p-q-r_{q+1}}\Bigg]\Delta^{-4}_{\alpha(-\ell)}\prod_{i=1}^{w-1}(k+r_i\ell)^{n_i}\Big|_{(m-1)\ell+j}^{k}.$$

Proof: The proof follows by taking $u_i(k) = (k + r_i\ell)^n$ for $i = 1, 2, \cdots, w$ in (3.1)

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Example 3.3 Let $k \in [0, \infty)$ and $j = k - \left[\frac{k}{\ell}\right]\ell$. Then, Substituting $w = 3, m = 2, n_1 = 2, n_2 = 3, n_3 = 4$ in (3.2), we get

$$\sum_{r=2}^{[\frac{k}{\ell}]}(r-1)(\frac{1}{\alpha})^r(k+(r_1+2-r)\ell)^2(k+(r_2+2-r)\ell)^3(k+(r_3+2-r)\ell)^4$$

$$= (k+\ell)^{4-i_1}\Delta^{-3}_{\alpha(-\ell)}(k+r_1\ell)^2(k+r_2\ell)^3 + \sum_{r=0}^{3}\alpha^{r+1}\sum_{r_1=1}^{4-r}(4-i_1)C_{r_1}\ell^{r_1}$$

$$\prod_{t=2}^{r+1}\sum_{r_t=1}^{4-r_1-r+1}\left[4-i_1-(\sum_{a=1}^{t-1}r_a)\right]C_{r_t}\ell^{r_t}(k+\ell)^{4-i_1-r-r_{r+1}} \tag{9}$$

In particular, $k = 55$, $\ell = 6$, $j = 1$, $r_1 = 7$,$r_2 = 8$, $r_3 = 9$
and $\alpha = 0.5$ in (9), we arrive

Corollary 3.4 Let $k_\ell^{(n)} = k(k-\ell)\cdots(k-(n-1)\ell)$ be the generalized polynomial factorial. Then,

$$\sum_{r=m}^{[\frac{k}{\ell}]}(-1)^m\frac{(r-1)^{(m-1)}}{(m-1)^{(m-1)}}(\frac{1}{\alpha})^r\left[\prod_{i=1}^{w}(k+r_i\ell)_\ell^{(n_i)}\right]$$

$$= \Delta^{-(m-1)}_{\alpha(-\ell)}\left[(k+\ell+r_w\ell)_\ell^{(n_w)}\Delta^{-1}_{\alpha(-\ell)}\prod_{i=1}^{w-1}(k+r_i\ell)_\ell^{(n_w)}\right]$$

$$+ \sum_{r=0}^{n_{w-1}}\alpha^{r+1}\sum_{i_1=1}^{n_{w-r}}n_wC_{i_1}\ell^{i_1}\sum_{t=0}^{i_1-1}i_1C_tr_w^t\left[\prod_{a=2}^{r+1}\sum_{i_a=1}^{n_w-\sum_{b=1}^{a-1}i_b}(n_w-\sum_{b=1}^{a-1}i_b)C_{i_a}\ell^{i_a}\right]$$

$$\Delta^{-(m-2)}_{\alpha(-\ell)}\left[(k+m\ell)_\ell^{\left(n_w-\sum_{p=1}^{r+1}i_p\right)}\Delta^{-(m+1)}_{\alpha(-\ell)}\prod_{i=1}^{w-1}(k+r_i\ell)_\ell^{(n_i)} + \sum_{q=0}^{n_m}(-\alpha)^{q+1}\right.$$

$$\sum_{r_1=1}^{n_m-q}(n_m-\sum_{p=1}^{r+1}i_p)C_{r_1}\ell^{r_1}\prod_{b=2}^{q+1}\sum_{r_b=1}^{n_m-\sum_{p=1}^{r+1}i_p-(q-1)}\left[n_m-\sum_{p=1}^{r+1}i_p-\sum_{a=1}^{b-1}r_a\right]C_{r_b}\ell^{r_b}$$

$$\left.(k+(m-1)\ell)_\ell^{\left(n_m-\sum_{p=1}^{r+1}i_p-q-r_{q+1}\right)}\right]\Delta^{-4}_{\alpha(-\ell)}\prod_{i=1}^{w-1}(k+r_i\ell)_\ell^{(n_i)}\Big|_{(n-1)\ell+j}^{k}. \tag{10}$$

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Proof: Substituting $u_i(k)=\prod_{i=1}^{w}(k+r_i\ell)_{\ell}^{(n_i)}$ in (3.1), we get

$$\Delta_{\alpha(-\ell)}^{-n}\left[\prod_{i=1}^{w}(k+r_i\ell)^{n_i}\right]\Big|\Big|_{(n-1)\ell+j}^{k}=\sum_{r=m}^{[\frac{k}{\ell}]}(-1)^m\frac{(r-1)^{(m-1)}}{(m-1)^{(m-1)}}(\frac{1}{\alpha})^r\left[\prod_{i=1}^{w}(k+r_i\ell)^{n_i}\right] \tag{11}$$

The proof follows by equating (10) and (11).

Corollary 3.5 Let $k\in[0,\infty)$ and $j=k-\left[\frac{k}{\ell}\right]\ell$. Then,

$$\sum_{r=2}^{[\frac{k}{\ell}]}(r-1)(\frac{1}{\alpha})^r(k+(r_1+2-r)\ell)_{\ell}^{(2)}(k+(r_2+2-r)\ell)_{\ell}^{(3)}(k+(r_3+2-r)\ell)_{\ell}^{(4)}$$

$$=(k+\ell)^{4-i_1}\Delta_{\alpha(-\ell)}^{-3}(k+r_1\ell)^2(k+r_2\ell)^3+\sum_{r=0}^{3}\alpha^{r+1}\sum_{r_1=1}^{4-r}(4-i_1)C_{r_1}\ell^{r_1}$$
$$\prod_{t=2}^{r+1}\sum_{r_t=1}^{4-r_1-r+1}\left[4-i_1-(\sum_{a=1}^{t-1}r_a)\right]C_{r_t}\ell^{r_t}(k+\ell)^{4-i_1-r-r_{r+1}} \tag{12}$$

Proof: Substituting $n=3,m=2,n_1=2,n_2=3,n_3=4$ in (10), we get (12).

Example 3.6 Taking $k=45$, $\ell=4$, $j=1$, $r_1=5$, $r_2=6$, $r_3=7$ and $\alpha=2$ in (12), we arrive

$$\sum_{r=2}^{11}(r-1)\frac{1}{2^r}(45+(7-r)4)_4^{(2)}(45+(8-r)4)_4^{(3)}(45+(9-r)4)_4^{(4)}=1.034768252\times10^{16}$$

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