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Faber polynomial coefficient estimates of bi-univalent functions connected with bounded boundary rotation by using Ruscheweyh derivative

Murugan A[1], Prathviraj Sharma[2] and Sivasubramanian S[3]

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Abstract

In this article, Utilizing the Ruscheweyh derivative operator in the complex domain, we propose and examine a new class of analytic and bi-univalent functions with bounded boundary rotation situated in the open unit disk. Through the application of Faber polynomial expansions, we determine upper bounds for the general coefficients of these functions, which are subject to a gap series condition, as well as for their first two coefficients.

Key words: Analytic; Bi-univalent functions; Ruscheweyh derivative; Bounded boundary rotation; Coefficient estimates.

AMS classification: Primary 30C45, 33C50; Secondary 30C80.

1 Introduction

Indicate by $\mathcal{P}_m(\eta)$ [26] for $m \geq 2$ and $0 \leq \eta < 1$, which represents a class of analytic functions χ that are normalized to ensure $\chi(0) = 1$. Furthermore, these functions must comply with the condition that

$$\int_0^{2\pi} \left| \frac{\Re(\chi(\varsigma)) - \eta}{1-\eta} \right| dt \leq m\pi,$$

where $\varsigma = e^{it} \in \mathbb{U}$, which is defined as $\{\varsigma \in \mathbb{C} : |\varsigma| < 1\}$. When $\eta = 0$, we represent $\mathcal{P}_m$ as $\mathcal{P}_m(0)$. Thus, the class $\mathcal{P}_m$ relates to functions χ that are analytic within $\mathbb{U}$,

[1]Department of Mathematics, College of Engineering Guindy, Anna University, Chennai 600025, Tamil Nadu, India. Email:murumaths2020@gmail.com
[2]Department of Mathematics, University College of Engineering Tindivanam, Anna University, Tindivanam 604001, Tamil Nadu, India. Email:sirprathvi99@gmail.com
[3]Department of Mathematics, University College of Engineering Tindivanam, Anna University, Tindivanam 604001, Tamil Nadu, India. Email:sivasaisastha@rediffmail.com

normalized to satisfy $\chi(0) = 1$, and can be expressed as

$$\chi(\varsigma) = \int_0^{2\pi} \frac{1 + \varsigma e^{-it}}{1 - \varsigma e^{-it}} d\delta(t),$$

where δ is a real-valued function with bounded variation, which ensures

$$\int_0^{2\pi} d\delta(t) = 2\pi \quad and \int_0^{2\pi} |d\delta(t)| \leq m\pi, \quad m \geq 2.$$

Evidently, $\mathcal{P}_2 := \mathcal{P}$ represents the well-known class of Carath'eodory functions, which are defined as normalized functions possessing a positive real part within the open unit disc $\mathbb{U}$.

Define $\mathcal{A}$ as the set of analytic functions given by

$$f(\varsigma) = \varsigma + \sum_{k=2}^{\infty} a_k \varsigma^k, \tag{1}$$

where $\varsigma \in \mathbb{U}$. In addition, let $\mathcal{S}$ be the subset of $\mathcal{A}$ that comprises univalent functions within the domain $\mathbb{U}$. The two subclasses of $\mathcal{S}$, specifically the starlike and convex function classes, will be indicated as $\mathcal{S}^*$ and $\mathcal{C}$, respectively. For formal definitions of the different subclasses of $\mathcal{S}$, refer to sources [9, 27]. According to the Koebe one-quarter theorem, the transformation of the unit disk $\mathbb{U}$ by any univalent function f from the class $\mathcal{S}$ will yield a disk centered at the origin with a radius of $1/4$. Let us examine the fact that f^{-1} denotes the inverse of the function f belonging to set $\mathcal{A}$, which can be expressed through the equations

$$\varsigma = f^{-1}(f(\varsigma)), \quad \varsigma \in \mathbb{U}$$

and

$$f(f^{-1}(w)) = w, \quad |w| < \rho(f) \geq \frac{1}{4}.$$

Here,

$$h(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (a_4 - 5a_2 a_3 + 5a_2^3)w^4 + \cdots. \tag{2}$$

If both $f(\varsigma)$ and $f^{-1}(\varsigma)$ are univalent in the domain $\mathbb{U}$, the function is classified as bi-univalent in $\mathbb{U}$. The complete collection of bi-univalent functions in $\mathbb{U}$ is symbolized by Σ. For a concise history and notable examples within the class Σ, refer to [5]. The concepts of bi-starlike functions of order η (where $0 < \eta \leq 1$), referred to as $\mathcal{S}^*_\Sigma(\eta)$, and bi-convex functions of the same order, designated as $\mathcal{S}_\Sigma(\eta)$, were first presented by Brannan and Taha in [5]. However, a significant challenge persists regarding the Taylor-Maclaurin coefficients $|a_k|$ for $k \in \mathbb{N} \setminus \{1, 2\}$. Following an examination of several intriguing subclasses of Σ, many researchers (see [4, 7, 21, 24, 15, 29, 36, 37, 38, 39, 40, 43, 44, 45, 46, 48, 49]) have determined that the approximations for the first two Taylor-Maclaurin coefficients, $|a_2|$ and $|a_3|$, are not precise. It is clear that the family Σ is not empty. Examples of functions in this family Σ are

$$\frac{\varsigma}{1-\varsigma}, \quad \frac{1}{2}\log\left(\frac{1+\varsigma}{1-\varsigma}\right) \quad \textit{and} \quad -\log(1-\varsigma)$$

along with their corresponding inverse functions:

$$\frac{w}{1+w}, \quad \frac{1-e^{-2w}}{1+e^{-2w}} \quad \textit{and} \quad 1-e^{-w}.$$

Furthermore, other common functions that are not part of this family Σ include

$$\varsigma - \frac{\varsigma^2}{2} \quad \textit{and} \quad \frac{\varsigma}{1-\varsigma^2}.$$

Fekete and Szegö (see [10]) disproof of the Littlewood-Paley conjecture that the coefficients of odd univalent functions are bounded by unity is the basis of the Fekete-Szego problem $|a_3 - \Psi d_2^2|$ for $f \in \mathcal{S}$, which has a long history in the field of Geometric Function Theory. Fekete-Szego inequalities for various function families were obtained by many different authors. This topic has become of considerable interest among researchers in Geometric Function Theory (see, for example, [50, 20, 35, 42, 34, 6, 16, 30, 32, 23, 22, 31]).

Utilizing the Faber polynomial expansion [1, 12] for functions $h \in \Sigma$ allows for the expression of the coefficients of the inverse map $g = h^{-1}$, as follows:

$$\begin{aligned} g(w) = h^{-1}(w) &= w + \sum_{k=2}^{\infty} A_k w^k \\ &= w + \sum_{k=2}^{\infty} \frac{1}{k} \mathcal{O}_{k-1}^{-k}(a_2, a_3, \ldots, a_k) w^k, \end{aligned}$$

where

$$\begin{aligned}\mathcal{O}_{k-1}^{-k} &= \frac{(-k)!}{(1-2k)!(k-1)!}a_2^{k-1} + \frac{(-k)!}{(2(1-k))!(-3+k)!}a_2^{k-3}a_3 \\ &+ \frac{(-k)!}{(3-2k)!(k-4)!}a_2^{k-4}a_4 \\ &+ \frac{(-k)!}{(2(2-k))!(k-5)!}a_2^{k-5}\left[a_5+(2-k)a_3^2\right] \\ &+ \frac{(-k)!}{(5-2k)!(k-6)!}a_2^{k-6}\left[a_6+(5-2k)a_2a_4\right] + \sum_{r=1}^{\infty} a^{n-r}\mathcal{Z}_r,\end{aligned}$$

and $\mathcal{Z}_r$ for $j \in [7, k]$ in a homogeneous polynomial in the variable $a_2, a_3, \ldots, a_k$. The first three terms of $\mathcal{O}_{k-1}^{-k}$ are,

$$\mathcal{O}_1^{-2} = -2a_2,$$

$$\mathcal{O}_2^{-3} = 3(2a_2^2 - a_3)$$

and

$$\mathcal{O}_3^{-4} = -4(5a_2^3 - 5a_2a_3 + a_4).$$

In most cases, the expansion for $\mathcal{O}_k^{\tau}$ is provided by

$$\mathcal{O}_k^{\tau} = \tau a_k + \frac{\tau(\tau-1)}{2}\mathcal{V}_k^2 + \frac{\tau!}{(\tau-3)!3!}\mathcal{V}_k^3 + \ldots + \frac{\tau!}{(\tau-k)!k!}\mathcal{V}_k^k$$

where $\mathcal{V}_k^{\tau} = B_k^{\tau}(a_1, a_2, \ldots, a_k)$ and

$$B_k^q(a_1, a_2, \ldots, a_k) = \sum_{n=1}^{\infty} \frac{(n)!(a_1)^{\gamma_1}(a_2)^{\gamma_2}\ldots(a_k)^{\gamma_k}}{\gamma_1!\gamma_2!\ldots\gamma_k!}.$$

where $a_1 = 1$ and the sum is taken over all non-negative integers $\gamma_1!, \gamma_2!, \ldots, \gamma_k!$ such that

$$\begin{aligned}\gamma_1 + \gamma_2 + \ldots + \gamma_k &= n \\ \gamma_1 + 2\gamma_2 + \ldots + k\gamma_k &= k.\end{aligned}$$

For a function f defined on the set $\mathcal{A}$ as specified in (1), the Ruscheweyh derivative operator $\mathcal{R}^{\lambda}:\mathcal{A}\longrightarrow\mathcal{A}$ (refer to [28]) is defined in the following manner:

$$\mathcal{R}^{\lambda}f(\varsigma)=\varsigma+\sum_{k=2}^{\infty}\frac{\Gamma(\lambda+k)}{\Gamma(k)\Gamma(\lambda+1)}a_k\varsigma^k,$$

where $\lambda\in\mathbb{N}_0:=\mathbb{N}\cup\{0\}=\{0,1,2,3,\dots\}$.

Remark 1.1 (i) Choosing the parameter $\lambda=0$ the operator $\mathcal{R}^{\lambda}f(\varsigma)$ simplifies to $f(\varsigma)$.

(ii) Choosing the parameter $\lambda=1$ the operator $\mathcal{R}^{\lambda}f(\varsigma)$ simplifies to $\mathcal{D}f(\varsigma)=\varsigma f'(\varsigma)$.

(iii) Choosing the parameter $\lambda=2$ the operator $\mathcal{R}^{\lambda}f(\varsigma)$ simplifies to $\mathcal{L}f(\varsigma)=\dfrac{f(\varsigma)+\varsigma f'(\varsigma)}{2}$ as defined by Livingston [19].

To establish our main results, we need the subsequent lemma.

Lemma 1.2 [3] If a function $\chi\in\mathcal{P}_m(\eta)$ and is represented as

$$\chi(\varsigma)=1+\chi_1\varsigma+\chi_2\varsigma^2+\chi_3\varsigma^3+\cdots$$

then for all $k\geq 1$, the following inequality is satisfied:

$$|\chi_k|\leq m(1-\eta).$$

Utilizing the Ruscheweyh derivative operator in the complex domain, we propose and examine a new class of analytic and bi-univalent functions with bounded boundary rotation situated in the open unit disk. Through the application of Faber polynomial expansions, we determine upper bounds for the general coefficients of these functions, which are subject to a gap series condition, as well as for their first two coefficients.

2 Coefficient estimates for the function class $\mathcal{G}_{\Sigma}^{\beta,\lambda}(m,\eta)$.

Definition 2.1 For the parameters $\lambda \in \mathbb{N}_0$, $0 \leq \beta \leq 1$, $0 \leq \eta < 1$ and $2 \leq m \leq 4$. A function $f \in \Sigma$ as described in (1) is considered to belong to the $\mathcal{G}_{\Sigma}^{\beta,\lambda}(m,\eta)$ class if it fulfills the outlined conditions

$$(1-\beta)\frac{\mathcal{R}^{\lambda} f(\varsigma)}{\varsigma} + \beta(\mathcal{R}^{\lambda} f)'(\varsigma) \in \mathcal{P}_m(\eta)$$

and

$$(1-\beta)\frac{\mathcal{R}^{\lambda} h(w)}{w} + \beta(\mathcal{R}^{\lambda} h)'(w) \in \mathcal{P}_m(\eta),$$

where the function h, defined as the inverse of f, is represented by equation (2).

Remark 2.2 It should be remarked that the family $\mathcal{G}_{\Sigma}^{\beta,\lambda}(m,\eta)$ is a generalization of well-known families consider earlier. For the choice of parameters in Definition 2.1, we have the following cases:
(i) If $m = 2$, then $\mathcal{G}_{\Sigma}^{\beta,\lambda}(m,\eta) \equiv \mathcal{G}_{\Sigma}^{\beta,\lambda}(\eta)$ is the class of functions hold the following conditions:

$$\Re\left((1-\beta)\frac{\mathcal{R}^{\lambda} f(\varsigma)}{\varsigma} + \beta(\mathcal{R}^{\lambda} f)'(\varsigma)\right) > \eta$$

and

$$\Re\left((1-\beta)\frac{\mathcal{R}^{\lambda} h(w)}{w} + \beta(\mathcal{R}^{\lambda} h)'(w)\right) > \eta.$$

(ii) If $\lambda = 0$, then $\mathcal{G}_{\Sigma}^{\beta,\lambda}(m,\eta) \equiv \mathcal{K}_{\Sigma}[m,\eta]$ is the class of functions hold the following conditions:

$$(1-\beta)\frac{f(\varsigma)}{\varsigma} + \beta f'(\varsigma) \in \mathcal{P}_m(\eta)$$

and

$$(1-\beta)\frac{h(w)}{w} + \beta h'(w) \in \mathcal{P}_m(\eta).$$

The class $\mathcal{K}_{\Sigma}[m,\eta]$ was examined by Sharma et al. [33].

(iii) If $\beta = 1$ and $\lambda = 0$, then $\mathcal{G}_{\Sigma}^{\beta,\lambda}(m,\eta) \equiv \mathcal{H}_{\Sigma}[m,\eta]$ is the class of functions hold the following conditions:

$$f'(\varsigma) \in \mathcal{P}_m(\eta)$$

and

$$h'(w) \in \mathcal{P}_m(\eta).$$

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The class $\mathcal{H}_\Sigma[m,\eta]$ was examined by Sharma et al. [33].

(iv) If $\beta = 1$, $m = 2$ and $\lambda = 0$, then $\mathcal{G}_\Sigma^{\beta,\lambda}(m,\eta) \equiv \mathcal{H}_\Sigma(\eta)$ is the class of functions hold the following conditions:

$$\Re(f'(\varsigma)) > \eta$$

and

$$\Re(h'(w)) > \eta.$$

The class $\mathcal{H}_\Sigma(\eta)$ was examined by Srivastava et al. [41].
(v) If $\lambda = 2$, then $\mathcal{G}_\Sigma^{\beta,\lambda}(m,\eta) \equiv \mathcal{L}\Sigma[m,\eta]$ is the class of functions hold the following conditions:

$$(1-\beta)\frac{\mathcal{L}f(\varsigma)}{\varsigma} + \beta(\mathcal{L}f)'(\varsigma) \in \mathcal{P}_m(\eta)$$

and

$$(1-\beta)\frac{\mathcal{I}h(w)}{w} + \beta(\mathcal{I}h)'(w) \in \mathcal{P}_m(\eta).$$

(vi) If $\lambda = 0$ and $m = 2$, then $\mathcal{G}_\Sigma^{\beta,\lambda}(m,\eta) \equiv \mathcal{B}_\Sigma(\eta,\beta)$ is the class of functions hold the following conditions:

$$\Re\left((1-\beta)\frac{f(\varsigma)}{\varsigma} + \beta f'(\varsigma)\right) > \eta$$

and

$$\Re\left((1-\beta)\frac{h(w)}{w} + \beta h'(w)\right) > \eta.$$

The class $\mathcal{B}_\Sigma(\eta,\beta)$ was examined by Frasin and Aouf [11].

Theorem 2.3 For the parameters $0 \le \beta \le 1$, $0 \le \eta < 1$ and $2 \le m \le 4$. Let $f \in \mathcal{G}_\Sigma^{\beta,\lambda}(m,\eta)$ as defined by (1). If $a_2 = \cdots = a_{k-1} = 0$, then

$$|a_k| \le \frac{m(1-\eta)\Gamma(k)\Gamma(\lambda+1)}{[1+\beta(k-1)]\Gamma(\lambda+k)}, \quad \forall \quad k \ge 3. \tag{3}$$

Proof: Given that $f \in \mathcal{G}_\Sigma^{\beta,\lambda}(m,\eta)$, according to Definition 2.1, there exist functions $y(\varsigma)$ and $v(w)$ belongs to the class $\mathcal{P}_m(\eta)$ such that

$$(1-\beta)\frac{\mathcal{R}^\lambda f(\varsigma)}{\varsigma} + \beta(\mathcal{R}^\lambda f)'(\varsigma) = y(\varsigma) \tag{4}$$

and

$$(1-\beta)\frac{\mathcal{R}^{\lambda}g(w)}{w}+\beta(\mathcal{R}^{\lambda}g)'(w)=v(w), \tag{5}$$

where

$$y(\varsigma)=1+y_1\varsigma+y_2\varsigma^2+y_3\varsigma^3+\cdots \tag{6}$$

and

$$v(w)=1+v_1w+v_2w^2+v_3w^3+\cdots. \tag{7}$$

As the function f presented in the form (1), it follows that

$$(1-\beta)\frac{\mathcal{R}^{\lambda}f(\varsigma)}{\varsigma}+\beta(\mathcal{R}^{\lambda}f)'(\varsigma)=1+\sum_{k=2}^{\infty}[1+\beta(k-1)]\frac{\Gamma(\lambda+k)}{\Gamma(k)\Gamma(\lambda+1)}a_k\varsigma^k. \tag{8}$$

In a similar manner, for the inverse function h presented in the form (2) and utilizing the Faber polynomial, we obtain

$$(1-\beta)\frac{\mathcal{R}^{\lambda}g(w)}{w}+\beta(\mathcal{R}^{\lambda}g)'(w)=1+\sum_{k=2}^{\infty}[1+\beta(k-1)]\frac{\Gamma(\lambda+k)}{\Gamma(k)\Gamma(\lambda+1)}A_kw^k, \tag{9}$$

where

$$A_k=\frac{1}{k}\mathcal{O}_{k-1}^{-k}(a_2,a_3,\ldots,a_k).$$

Alternatively, utilizing the Faber polynomial in (4) and (5) yields

$$(1-\beta)\frac{\mathcal{R}^{\lambda}f(\varsigma)}{\varsigma}+\beta(\mathcal{R}^{\lambda}f)'(\varsigma)=1+\sum_{k=1}^{\infty}\mathcal{C}_k^1(y_1,y_2,\ldots,y_k)\varsigma^k \tag{10}$$

and

$$(1-\beta)\frac{\mathcal{R}^{\lambda}g(w)}{w}+\beta(\mathcal{R}^{\lambda}g)'(w)=1+\sum_{k=1}^{\infty}\mathcal{C}_k^1(v_1,v_2,\ldots,v_k)w^k. \tag{11}$$

Based on the equations presented in (8), (9), (10) and (11), we can conclude that

$$[1+\beta(k-1)]\frac{\Gamma(\lambda+k)}{\Gamma(k)\Gamma(\lambda+1)}a_k=\mathcal{C}_k^1(y_1,y_2,\ldots,y_k) \tag{12}$$

and

$$[1+\beta(k-1)]\frac{\Gamma(\lambda+k)}{\Gamma(k)\Gamma(\lambda+1)}\times\frac{1}{k}\mathcal{O}_{k-1}^{-k}(a_2,a_3,\ldots,a_k)=\mathcal{C}_k^1(v_1,v_2,\ldots,v_k). \tag{13}$$

In the case of $a_m = 0$, with the condition that $2 \leq m \leq n-1$, we can conclude that $A_n = -a_n$, and

$$[1+\beta(k-1)]\frac{\Gamma(\lambda+k)}{\Gamma(k)\Gamma(\lambda+1)}a_k = y_{k-1} \tag{14}$$

and

$$-[1+\beta(k-1)]\frac{\Gamma(\lambda+k)}{\Gamma(k)\Gamma(\lambda+1)}a_k = v_{k-1}. \tag{15}$$

An application of Lemma 1.2, to either equation (14) or equation (15) yields the required result as stated in equation (3). This concludes the proof of Theorem 2.3.

By assigning the values $\lambda = 0$, in Theorem 2.3, we obtain the subsequent corollary.

Corollary 2.4 For the parameters $0 \leq \eta < 1$ and $2 \leq m \leq 4$. Let $f \in \mathcal{K}_\Sigma[m,\eta]$ as defined by (1). If $a_2 = \cdots = a_{k-1} = 0$, then

$$|a_k| \leq \frac{m(1-\eta)}{1+\beta(k-1)}, \quad \forall \quad k \geq 3.$$

By assigning the values $\lambda = 0$ and $\beta = 1$, in Theorem 2.3, we obtain the subsequent corollary.

Corollary 2.5 For the parameters $0 \leq \eta < 1$ and $2 \leq m \leq 4$. Let $f \in \mathcal{H}_\Sigma(m,\eta)$ as defined by (1). If $a_2 = \cdots = a_{k-1} = 0$, then

$$|a_k| \leq \frac{m(1-\eta)}{k}, \quad \forall \quad k \geq 3.$$

By assigning the values $\lambda = 0$, $m = 2$ and $\beta = 1$, in Theorem 2.3, we obtain the subsequent corollary.

Corollary 2.6 For the parameters $0 \leq \eta < 1$. Let $f \in \mathcal{H}_\Sigma(\eta)$ as defined by (1). If $a_2 = \cdots = a_{k-1} = 0$, then

$$|a_k| \leq \frac{2(1-\eta)}{k}, \quad \forall \quad k \geq 3.$$

Remark 2.7 (i) Corollary 2.4 and Corollary 2.5, verifies the results obtained by Hou Tang et. al. [47].

(ii) Corollary 2.6, verifies the results obtained by Hamidi and Jahangiri [13].

Theorem 2.8 For the parameters $0 \leq \beta \leq 1$, $0 \leq \eta < 1$ and $2 \leq m \leq 4$. Let $f \in \mathcal{G}_{\Sigma}^{\beta,\lambda}(m,\eta)$ as defined by (1), then

$$|a_2| \leq \sqrt{\frac{2m(1-\eta)}{(1+2\beta)(\lambda+1)(\lambda+2)}}, \tag{16}$$

$$|a_3| \leq \frac{2m(1-\eta)}{(1+2\beta)(\lambda+1)(\lambda+2)} \tag{17}$$

and

$$|a_3 - \Psi a_2^2| \leq \begin{cases} \dfrac{2m(1-\eta)(1-\Psi)}{(1+2\beta)(\lambda+1)(\lambda+2)} & \text{if } \Psi < 0, \\ \dfrac{2m(1-\eta)}{(1+2\beta)(\lambda+1)(\lambda+2)} & \text{if } 0 \leq \Psi \leq 2, \\ \dfrac{2m(1-\eta)(\Psi-1)}{(1+2\beta)(\lambda+1)(\lambda+2)} & \text{if } \Psi > 2, \end{cases} \tag{18}$$

where Ψ is any real number.
Proof: By substituting $k = 3$ in equations (12) and (13), we obtain

$$(1+2\alpha)(\lambda+1)(\lambda+2)\frac{a_3}{2} = y_2 \tag{19}$$

and

$$(1+2\alpha)\frac{(\lambda+1)(\lambda+2)}{2}(2a_2^2 - a_3) = v_2. \tag{20}$$

By analyzing (19) and (20), we can conclude the following

$$(1+2\alpha)(\lambda+1)(\lambda+2)a_2^2 = y_2 + v_2. \tag{21}$$

An application of Lemma 1.2, in (21), we get

$$|a_2|^2 \leq \frac{2m(1-\eta)}{(1+2\alpha)(\lambda+1)(\lambda+2)}. \tag{22}$$

Equation (22) establishes the bound of $|a_2|$ as indicated in (16). Likewise, by applying

Lemma 1.2, in equation (19), we derive the bounds for $|a_3|$ as presented in (17). Consequently, for any real number Ψ, and from equations (19) and (21), we have

$$a_3 - \Psi a_2^2 = \frac{(2-\Psi)y_2 - \Psi v_2}{(1+2\alpha)(\lambda+1)(\lambda+2)}. \tag{23}$$

An application of Lemma 1.2, in (23), we get

$$|a_3 - \Psi a_2^2| \leq \frac{m(1-\eta)[|2-\Psi| + |\Psi|]}{(1+2\alpha)(\lambda+1)(\lambda+2)}. \tag{24}$$

Equation (24) gives the bounds of $|a_3 - \Psi a_2^2|$ given in (18). This completes the proof of Theorem 2.8.

By assigning the values $\lambda = 0$, in Theorem 2.8, we obtain the subsequent corollary.

Corollary 2.9 For the parameters $0 \leq \eta < 1$ and $2 \leq m \leq 4$. Let $f \in \mathcal{K}_\Sigma[m, \eta]$ as defined by (1), then

$$|a_2| \leq \sqrt{\frac{m(1-\eta)}{1+2\beta}},$$

$$|a_3| \leq \frac{m(1-\eta)}{1+2\beta}$$

and

$$|a_3 - \Psi a_2^2| \leq \begin{cases} \dfrac{m(1-\eta)(1-\Psi)}{1+2\beta} & \text{if } \Psi < 0, \\ \dfrac{m(1-\eta)}{1+2\beta} & \text{if } 0 \leq \Psi \leq 2, \\ \dfrac{m(1-\eta)(\Psi-1)}{1+2\beta} & \text{if } \Psi > 2, \end{cases}$$

where Ψ is any real number.

By assigning the values $\lambda = 0$ and $\beta = 1$, in Theorem 2.8, we obtain the subsequent corollary.

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Corollary 2.10 For the parameters $0 \leq \eta < 1$ and $2 \leq m \leq 4$. Let $f \in \mathcal{H}_{\Sigma}(m, \eta)$ as defined by (1), then

$$|a_2| \leq \sqrt{\frac{m(1-\eta)}{3}},$$

$$|a_3| \leq \frac{m(1-\eta)}{3}$$

and

$$|a_3 - \Psi a_2^2| \leq \begin{cases} \dfrac{m(1-\eta)(1-\Psi)}{3} & \text{if } \Psi < 0, \\ \dfrac{m(1-\eta)}{3} & \text{if } 0 \leq \Psi \leq 2, \\ \dfrac{m(1-\eta)(\Psi-1)}{3} & \text{if } \Psi > 2, \end{cases}$$

where Ψ is any real number.

By assigning the values $\lambda = 0$, $m = 2$ and $\beta = 1$, in Theorem 2.8, we obtain the subsequent corollary.

Corollary 2.11 For the parameters $0 \leq \eta < 1$. Let $f \in \mathcal{H}_{\Sigma}(\eta)$ as defined by (1), then

$$|a_2| \leq \sqrt{\frac{2(1-\eta)}{3}},$$

$$|a_3| \leq \frac{2(1-\eta)}{3}$$

and

$$|a_3 - \Psi a_2^2| \leq \begin{cases} \dfrac{2(1-\eta)(1-\Psi)}{3} & \text{if } \Psi < 0, \\ \dfrac{2(1-\eta)}{3} & \text{if } 0 \leq \Psi \leq 2, \\ \dfrac{2(1-\eta)(\Psi-1)}{3} & \text{if } \Psi > 2, \end{cases}$$

where Ψ is any real number.

By assigning the values $\lambda = 0$ and $m = 2$, in Theorem 2.8, we obtain the subsequent corollary.

Corollary 2.12 For the parameters $0 \leq \eta < 1$. Let $f \in \mathcal{B}_\Sigma(\eta, \beta)$ as defined by (1), then

$$|a_2| \leq \sqrt{\frac{2(1-\eta)}{1+2\beta}},$$

$$|a_3| \leq \frac{2(1-\eta)}{1+2\beta}$$

and

$$|a_3 - \Psi a_2^2| \leq \begin{cases} \dfrac{2(1-\eta)(1-\Psi)}{1+2\beta} & \text{if } \Psi < 0, \\ \dfrac{2(1-\eta)}{1+2\beta} & \text{if } 0 \leq \Psi \leq 2, \\ \dfrac{2(1-\eta)(\Psi-1)}{1+2\beta} & \text{if } \Psi > 2, \end{cases}$$

where Ψ is any real number.

Remark 2.13 (i) Corollary 2.9 and Corollary 2.10, verifies the results obtained by Sharma et. al. [33].
(ii) Corollary 2.11 and Corollary 2.12, verifies the bounds of $|a_2|$ and improves the bounds of $|a_3|$ obtained by Srivastava et. al. [41] and Frasin and Aouf [11].

3 Coefficient estimates for the function class $\mathcal{S}_\Sigma^{*,\lambda}(m, \eta)$.

Definition 3.1 For the parameters $0 \leq \eta < 1$ and $2 \leq m \leq 4$. A function $f \in \Sigma$ as described in (1) is considered to belong to the $\mathcal{S}_\Sigma^{*,\lambda}(m, \eta)$ class if it fulfills the outlined conditions

$$\frac{\varsigma(\mathcal{R}^\lambda f)'(\varsigma)}{\mathcal{R}^\lambda f(\varsigma)} \in \mathcal{P}_m(\eta)$$

and

$$\frac{w(\mathcal{R}^\lambda h)'(w)}{\mathcal{R}^\lambda h(w)} \in \mathcal{P}_m(\eta),$$

where the function h, defined as the inverse of f, is represented by equation (2).

Remark 3.2 It should be remarked that the family $\mathcal{S}_\Sigma^{*,\lambda}(m, \eta)$ is a generalization of well-known families consider earlier. For the choice of parameters in Definition 3.1, we have the following cases:
(i) If $m = 2$, then $\mathcal{S}_\Sigma^{*,\lambda}(m, \eta) \equiv \mathcal{S}_\Sigma^{*,\lambda}(\eta)$ is the class of functions hold the following

conditions:

$$\Re\left(\frac{\varsigma(\mathcal{R}^\lambda f)'(\varsigma)}{\mathcal{R}^\lambda f(\varsigma)}\right) > \eta$$

and

$$\Re\left(\frac{w(\mathcal{R}^\lambda h)'(w)}{\mathcal{R}^\lambda h(w)}\right) > \eta.$$

(ii) If $\lambda = 0$, then $\mathcal{S}_\Sigma^{*,\lambda}(m,\eta) \equiv \mathcal{S}_\Sigma^{*}(m,\eta)$ is the class of functions hold the following conditions:

$$\frac{\varsigma f'(\varsigma)}{f(\varsigma)} \in \mathcal{P}_m(\eta)$$

and

$$\frac{wh'(w)}{h(w)} \in \mathcal{P}_m(\eta).$$

The class $\mathcal{S}_\Sigma^{*}(m,\eta)$ was examined by Li et al. [18].

(iii) If $\lambda = 0$ and $m = 2$, then $\mathcal{S}_\Sigma^{*,\lambda}(m,\eta) \equiv \mathcal{S}_\Sigma^{*}(\eta)$ is the class of functions hold the following conditions:

$$\Re\left(\frac{\varsigma f'(\varsigma)}{f(\varsigma)}\right) > \eta$$

and

$$\Re\left(\frac{wh'(w)}{h(w)}\right) > \eta.$$

The class $\mathcal{S}_\Sigma^{*}(\eta)$ was examined by Brannan and Taha [5].

Theorem 3.3 For the parameters $0 \leq \beta \leq 1$, $0 \leq \eta < 1$ and $2 \leq m \leq 4$. Let $f \in \mathcal{S}_\Sigma^{*,\lambda}(m,\eta)$ as defined by (1). If $a_2 = \cdots = a_{k-1} = 0$, then

$$|a_k| \leq \frac{m(1-\eta)\Gamma(k)\Gamma(\lambda+1)}{(k-1)\Gamma(\lambda+k)}, \quad \forall \quad k \geq 3. \tag{25}$$

Proof: Given that $f \in \mathcal{S}_\Sigma^{*,\lambda}(m,\eta)$, according to Definition 3.1, there exist functions $y(\varsigma)$ and $v(w)$ belongs to the class $\mathcal{P}_m(\eta)$ such that

$$\frac{\varsigma(\mathcal{R}^\lambda f)'(\varsigma)}{\mathcal{R}^\lambda f(\varsigma)} = y(\varsigma) \tag{26}$$

and

$$\frac{w(\mathcal{R}^\lambda h)'(w)}{\mathcal{R}^\lambda h(w)} = v(w), \tag{27}$$

where $y(\varsigma)$ and $v(w)$ are presented in the form (6) and (7), respectively. As the function f presented in the form (1), it follows that

$$\frac{\varsigma(\mathcal{R}^\lambda f)'(\varsigma)}{\mathcal{R}^\lambda f(\varsigma)} = 1 - \sum_{k=2}^{\infty} \mathcal{Q}_{k-1}^{k}(\bar{a}_2, \bar{a}_3, \ldots, \bar{a}_k)\varsigma^k, \tag{28}$$

where

$$\bar{a}_k = \frac{\Gamma(\lambda+k)}{\Gamma(k)\Gamma(\lambda+1)} a_k.$$

In a similar manner, for the inverse function h presented in the form (2) and utilizing the Faber polynomial, we obtain

$$\frac{w(\mathcal{R}^\lambda h)'(w)}{\mathcal{R}^\lambda h(w)} = 1 - \sum_{k=2}^{\infty} \mathcal{Q}_{k-1}^{k}(\bar{A}_2, \bar{A}_3, \ldots, \bar{A}_k)w^k, , \tag{29}$$

where

$$\bar{A}_k = \frac{1}{k}\mathcal{O}_{k-1}^{-k}(\bar{a}_2, \bar{a}_3, \ldots, \bar{a}_k).$$

Alternatively, utilizing the Faber polynomial in (26) and (27) yields

$$\frac{\varsigma(\mathcal{R}^\lambda f)'(\varsigma)}{\mathcal{R}^\lambda f(\varsigma)} = 1 + \sum_{k=1}^{\infty} \mathcal{C}_k^1(y_1, y_2, \ldots, y_k)\varsigma^k \tag{30}$$

and

$$\frac{w(\mathcal{R}^\lambda h)'(w)}{\mathcal{R}^\lambda h(w)} = 1 + \sum_{k=1}^{\infty} \mathcal{C}_k^1(v_1, v_2, \ldots, v_k)w^k. \tag{31}$$

Based on the equations presented in (28), (29), (30) and (31), we can conclude that

$$\mathcal{Q}_{k-1}^{k}(\bar{a}_2, \bar{a}_3, \ldots, \bar{a}_k) = \mathcal{C}_k^1(y_1, y_2, \ldots, y_k) \tag{32}$$

and

$$\mathcal{Q}_{k-1}^{k}(\bar{A}_2, \bar{A}_3, \ldots, \bar{A}_k) = \mathcal{C}_k^1(v_1, v_2, \ldots, v_k). \tag{33}$$

In the case of $a_m = 0$, with the condition that $2 \leq m \leq k-1$, we can conclude that

$\bar{A}_k = -\bar{a}_k$, and

$$(k-1)\frac{\Gamma(\lambda+k)}{\Gamma(k)\Gamma(\lambda+1)}a_k = y_{k-1} \tag{34}$$

and

$$-(k-1)\frac{\Gamma(\lambda+k)}{\Gamma(k)\Gamma(\lambda+1)}a_k = v_{k-1}. \tag{35}$$

An application of Lemma 1.2, to either equation (34) or equation (35) yields the required result as stated in equation (25). This concludes the proof of Theorem 3.3.

By assigning the values $\lambda = 0$, in Theorem 3.3, we obtain the subsequent corollary.

Corollary 3.4 For the parameters $0 \leq \eta < 1$ and $2 \leq m \leq 4$. Let $f \in \mathcal{S}^*_\Sigma(m, \eta)$ as defined by (1). If $a_2 = \cdots = a_{k-1} = 0$, then

$$|a_k| \leq \frac{m(1-\eta)}{k-1}, \quad \forall \quad k \geq 3.$$

By assigning the values $\lambda = 0$ and $m = 2$, in Theorem 3.3, we obtain the subsequent corollary.

Corollary 3.5 For the parameters $0 \leq \eta < 1$. Let $f \in \mathcal{S}^*_\Sigma(\eta)$ as defined by (1). If $a_2 = \cdots = a_{k-1} = 0$, then

$$|a_k| \leq \frac{2(1-\eta)}{k-1}, \quad \forall \quad k \geq 3.$$

Remark 3.6 (i) Corollary 3.4 verifies the results obtained by Hou Tang et. al. [47].
(ii) Corollary 3.5, verifies the results obtained by Hamidi and Jahangiri [13].

Theorem 3.7 For the parameters $0 \leq \beta \leq 1$, $0 \leq \eta < 1$ and $2 \leq m \leq 4$. Let $f \in \mathcal{S}^{*,\lambda}_\Sigma(m, \eta)$ as defined by (1), then

$$|a_2| \leq \sqrt{\frac{m(1-\eta)}{\lambda+1}}, \tag{36}$$

$$|a_3| \leq \frac{m(1-\eta)}{\lambda+1} \tag{37}$$

and

$$|a_3 - \Psi a_2^2| \leq \begin{cases} \dfrac{m(1-\eta)(1-\Psi)}{\lambda+1} & \text{if } \Psi < \dfrac{\lambda+1}{\lambda+2}, \\ \dfrac{m(1-\eta)}{(\lambda+1)(\lambda+2)} & \text{if } \dfrac{\lambda+1}{\lambda+2} \leq \Psi \leq \dfrac{\lambda+3}{\lambda+2}, \\ \dfrac{m(1-\eta)(\Psi-1)}{\lambda+1} & \text{if } \Psi > \dfrac{\lambda+3}{\lambda+2}, \end{cases} \tag{38}$$

where Ψ is any real number.

Proof: By substituting $k = 3$ in equations (32) and (33), we obtain

$$(\lambda+1)(\lambda+2)a_3 - (\lambda+1)^2 a_2^2 = y_2 \tag{39}$$

and

$$(\lambda+1)(\lambda+3)a_2^2 - (\lambda+1)(\lambda+2)a_3 = v_2. \tag{40}$$

By analyzing (39) and (40), we can conclude the following

$$2(\lambda+1)a_2^2 = y_2 + v_2. \tag{41}$$

An application of Lemma 1.2, in (41), we get

$$|a_2|^2 \leq \frac{m(1-\eta)}{\lambda+1}. \tag{42}$$

Equation (42) establishes the bound of $|a_2|$ as indicated in (36). Likewise, by analyzing (39), (40) and (41), we can conclude the following

$$a_3 = \frac{(\lambda+3)y_2 + (\lambda+1)v_2}{2(\lambda+1)(\lambda+2)}. \tag{43}$$

By applying Lemma 1.2, in equation (43), we derive the bounds for $|a_3|$ as presented in (37). Consequently, for any real number Ψ, and from equations (41) and (43), we have

$$a_3 - \Psi a_2^2 = \frac{[(\lambda+3) - (\lambda+2)\Psi]y_2 + [(\lambda+1) - (\lambda+2)\Psi]v_2}{2(\lambda+1)(\lambda+2)}. \tag{44}$$

An application of Lemma 1.2, in (44), we get

$$|a_3 - \Psi a_2^2| \leq \frac{m(1-\eta)[|(\lambda+3) - (\lambda+2)\Psi| + |(\lambda+1) - (\lambda+2)\Psi|]}{2(\lambda+1)(\lambda+2)}. \tag{45}$$

Equation (45) gives the bounds of $|a_3 - \Psi a_2^2|$ given in (38). This completes the proof of Theorem 3.7.

By assigning the values $\lambda = 0$, in Theorem 3.7, we obtain the subsequent corollary.

Corollary 3.8 For the parameters $0 \leq \eta < 1$ and $2 \leq m \leq 4$. Let $f \in \mathcal{S}_{\Sigma}^{*}(m,\eta)$ as defined by (1), then

$$|a_2| \leq \sqrt{m(1-\eta)},$$

$$|a_3| \leq m(1-\eta)$$

and

$$|a_3 - \Psi a_2^2| \leq \begin{cases} m(1-\eta)(1-\Psi) & \text{if } \Psi < \dfrac{1}{2}, \\ \dfrac{m(1-\eta)}{2} & \text{if } \dfrac{1}{2} \leq \Psi \leq \dfrac{3}{2}, \\ m(1-\eta)(\Psi-1) & \text{if } \Psi > \dfrac{3}{2}, \end{cases}$$

where Ψ is any real number.

By assigning the values $\lambda = 0$ and $m = 2$, in Theorem 3.7, we obtain the subsequent corollary.

Corollary 3.9 For the parameters $0 \leq \eta < 1$. Let $f \in \mathcal{S}_{\Sigma}^{*}(\eta)$ as defined by (1), then

$$|a_2| \leq \sqrt{2(1-\eta)},$$

$$|a_3| \leq 2(1-\eta)$$

and

$$|a_3 - \Psi a_2^2| \leq \begin{cases} 2(1-\eta)(1-\Psi) & \text{if } \Psi < \dfrac{1}{2}, \\ \dfrac{2(1-\eta)}{2} & \text{if } \dfrac{1}{2} \leq \Psi \leq \dfrac{3}{2}, \\ 2(1-\eta)(\Psi-1) & \text{if } \Psi > \dfrac{3}{2}, \end{cases}$$

where Ψ is any real number.

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Remark 3.10 (i) Corollary 3.8 verifies the bounds of $|a_2|$ and improves the bounds of $|a_3|$ obtained by Li et. al. [18] and verifies the bounds of $|a_2|$ and $|a_3|$ obtained by Sharma et. al. [33].
(ii) Corollary 3.9, verifies the results obtained by Brannan and Taha [5].

4 Concluding remarks and observations

In this article, we introduce two new subclasses of bi-univalent functions associated with bounded boundary rotation, utilizing Ruscheweyh derivative operators. Using Faber polynomial expansions, we have identified general coefficient bounds for well-known classes of bi-univalent functions characterized by bounded boundary rotation. We have also obtained the first two initial non-sharp Taylor–Maclaurin coefficient bounds for these newly defined function classes. Moreover, the esteemed Fekete–Szegö inequality is derived for these new function classes. Compared to previously published results, several improved findings are presented. Given the multitude of differential operators documented in the literature, alternative operators can also be applied to the classes considered here or their related subclasses in $\mathcal{S}$.

Additionally, more corollaries can be proposed regarding the selection of parameters in Ruscheweyh derivative operators, which include a variety of fractional derivatives and integral operators.

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