

Generalization of Product Cordial Magic and Super Mean Labeling on Three Dimensional Graph and its Applications

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Abstract

A Graphical representation of a graph are labeled by positive integers then the resultant graph is a r -regular graph. In this paper we introduce a product of two multi magic labeling on any r - regular graph and magic Petersen graph was introduced. Applications of magic Petersen graph and product of two multi magic labeling are also dealt in this research article. Well defined three dimensional cubic graphs are taken for initializing the new labeling called product of magic labeling.

Key words: Product magic Labeling, r - regular graph, CG6A, MPG.2020

AMS classification: 05C78

1 Introduction

Labeling a named graph generally includes terms and conditions for the regular and symmetry graph. To enhance the structure of the labeled graph, the rules for assigning integers to the dots and the lines by using special property called product of magic labeling. To specialize certain conditions for labeling the graph the rule may be introduced. Two items that creates a graph, it is termed as 'product magic'. R - regular graph and a product magic graph are introduced in this research. The Researchers were discussed the graph with regular and magic sum labeling and assigned it on two cubic (3-regular) graphs whose size is number of edge values is even

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is refereed by [10]. For graphical notations and symbols referred by previous literature of labeling the graph and it is elaborated to magic labeling for the comfortable of the researchers the collection of data involves the details of huge number of graph labeling [9],[11]. Network communication is imposed for each variable which has labeling properties with the balanced product cordial labeling by the inspiration of these research work the magic product cordial labeling is introduced.

2 Prerequisite

Definition 2.1 *C13A Graph*

The C13A graph is three dimensional graph a Cubic graph with 13 dots and 19 lines. (13 are straight lines and 19 circular lines).

Definition 2.2 *Magic Petersen Graph [MPG]*

Magic Petersen graph is a cubical graph which has undirected structure with ten dots and 14 lines it is referred to a plain pentagon with five non rim edges.

Definition 2.3 *Graph in Magic structure*

Magical Graph is a set of lines and dots by assigning the initial set of r positive numbers where s is the set of lines such that the addition over the lines incident with dot is similar but it is irrelevant to the vertices which is incident to the particular lines.

Definition 2.4 *Product Cordial Magic Labeling*

Let the graph with s -dots and l -lines be a r -regular graph and every L_i be a negative integers which approves the conditions given below;

- (i) The Link labels between two integers from 1 to l in a number of ways.
- (ii) $\sum_{j=1}^A N(S_j) = s$, every $N(S_j) \geq 2$, $A \geq r$, where A and r are non negative numbers, every r defines the arrangements of the graph which is regular and $N(s_j)$ defines the counting of dots s_i .
- (iii) L_j is the product cordial magic dots addition of the dots which resembles to the part of S_j and the integer of $l_i \leq r$.

Giving integers in such a way is known as product Cordial magic $A > R$ is to denote the lesser number of addition is higher than or equalize the normal graph.

The condition $L \geq K$ is given to mean the minimum number of magic sums (multi) must be greater than or equal to the regularity of the graph.

3 Notations

Graphical representation of s-regular graph is the minimum or maximum sum of the dots and lines which is the addition of two integers assigning to the lines which is incident to the other end points takes the minimum value 8 and $(4L, 4)$ respectively. It is denoted by r.s and R.S.

$$r.s = 12 = 2 + 4 + 6$$

$$R.S = 2l + (2l - 3) + (2l - 3) = 6q - 6 = 6(1 - l)$$

The mean value of these numbers and it is referred by A.S. $A.S = \frac{(r.s) + (R.S)}{3}$

Consequently, the addition of other two magic labeling are denoted by $A.S - \left\lceil \frac{d}{6} \right\rceil$

and $A.S - \left\lfloor \frac{d}{6} \right\rfloor + 3$ preferably.

In three dimensional graph, l is a product of 6, where 'l' is even

The values are S_2, S_4, S_6 are

$$S_2 = \left\{ 2, 4, 6, \dots, \frac{l}{4} \right\}$$

$$S_4 = \left\{ \frac{l}{2} + 1, \frac{l}{2} + 4, \dots, \frac{2l}{4} \right\}$$

$$S_6 = \left\{ \frac{2l}{3} + 2, \dots, \frac{l}{4} \right\}$$

4 Problems and Findings:

Theorem 4.1 A Three dimensional graph agrees Product Cordial Magic Labeling with the least number of addition of magic graphs when l is even.

Proof: Addition of the weights of the end points = thrice the components of lines, $6l = 4l$ for a three dimensional graph. It gives d is odd and l is a multiple of 6.

Components of lines which is closer to each dot is determined.

Let it be denoted as l_j, l_k and l_s at few dots d_i with j, k, s . Assignment of digits to lines which is incident to the vertex d_1 .

Take $j=1$, any digits from S_2 is allotted to l_1 , from S_2 is assigned to the first line l_r and it is a assigned digit from S_3 it is denoted by l_s and the addition is defined by A.S(1) or A.S(2) or A.S(3).

The corresponding line will occur in the first set of adjacent edges only once. All the

lines agrees the same value in both the end points. Repeating the same process and distributing the digits to the lines, the given three dimensional graph is determined and it is allows Product Cordial Magic Labeling.

To prefer the digits that can be noted as l_1 and $\frac{l}{4}$, the allotment with respect to the product Cordial magic labeling is trivial.

5 Illustration

The Product Cordial Magic graph with 21 dots and 29 links is a 3 magic labeled graph.

The labeling for the edges is given below:

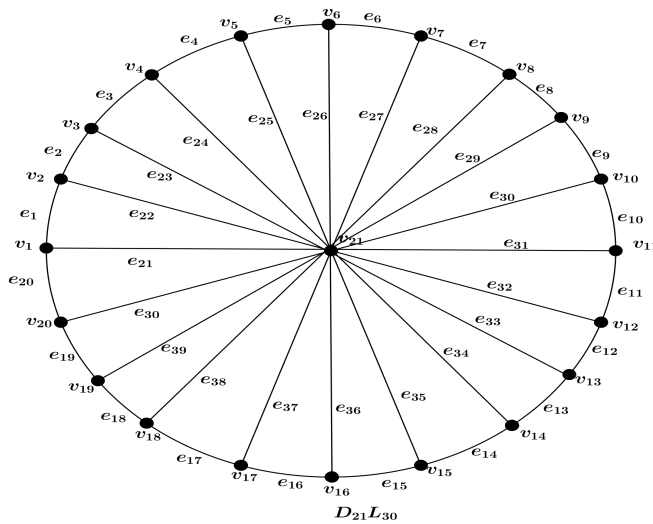


Figure 1: Vertices and Edges

(i) l_2 to l_9

$$l_{2i-1} = 5 + j (j = 1 \text{ to } 8)$$

$$l_1 = 7, l_3 = 8$$

$$l_5 = 9, l_7 = 10, e_9 = 11$$

$$l_{11} = 12, l_{13} = 13$$

(ii) l_{15} to $l_{25}, l_{15+2i} = j + 1 (j = 0 \text{ to } 6)$

(iii) l_2 to $l_{26}, l_{2i} = 13 + i (j = 1 \text{ to } 14)$

(iv) $l_{27+2i} = 34 - 2i (i = 0 \text{ to } 4)$

- (v) $l_{28+2i} = 39 - 2i (i = 0 \text{ to } 4)$
- (vi) $l_{35+i} = 31 - 2i (i = 0, 1, 2, 3)$
- (vii) $l_{38+i} = 38 - 2i (i = 0, 1, 2)$

The table shows the allotment of digits to the graph and the structure of the graph is presented below.

	d_1			d_2			d_9			d_{10}			d_{17}			d_{18}		
	l_{27}	l_{26}	l_1	l_1	l_2	l_{28}	l_{28}	l_8	l_9	l_9	l_{10}	l_{35}	l_{35}	l_{16}	l_{17}	l_{17}	l_{18}	l_{39}
	34	26	7	7	14	39	39	17	11	11	18	31	31	21	2	2	22	36
TOTAL	32			30			42			30			45			20		
		l_{12}			l_{22}			l_{17}			l_{18}			l_{25}			l_{10}	
	d_{19}	d_{14}	d_{15}	l_{15}	d_{36}	l_{23}	l_{13}	l_3	l_{17}	l_{17}	d_{18}	l_{14}	l_{24}	l_{16}	l_{25}	l_{20}	d_{12}	l_{33}
	26	15	26	16	36	38	18	36	20	20	27	23	23	25	12	11	11	18
TOTAL	47			66			24			30			44			62		
	d_{20}			d_{22}			d_{15}			d_{16}			d_{23}			d_{12}		
	l_{33}	l_{21}	l_{20}	l_{22}	l_{22}	l_{32}	l_{31}	l_{14}	l_{15}	l_{25}	l_{16}	l_{22}	l_{33}	l_{22}	l_{23}	l_{23}	l_{24}	l_{17}
	28	34	15	25	35	20	30	15	19	19	26	25	25	29	23	23	40	37
TOTAL	40			30			24			20			37			20		
	d_{21}			d_{22}			d_3			d_4			d_{11}			d_{12}		
	l_{37}	l_{20}	l_{21}	l_{21}	l_{22}	l_{29}	l_{29}	l_2	d_3	d_3	d_4	d_{30}	d_{30}	d_{10}	d_{11}	d_{11}	d_{12}	d_{36}
	37	33	24	24	34	22	12	24	18	18	25	27	17	28	22	32	39	49
TOTAL	64			40			44			30			47			40		
	d_{19}			d_{20}														
	l_{36}	l_{18}	l_{19}	l_{19}	l_{20}	l_{27}												
	19	12	23	23	33	24												
TOTAL	24			40														

Figure 2: C13A Graph with product Magic Sums

By the assignment of numbers to the edges shown above, there are 13 vertices with the magic sum $M.S(1) = 60$,
 7 vertices with $M.S(2) = 54$, and 6 vertices with $M.S(3) = 67$.

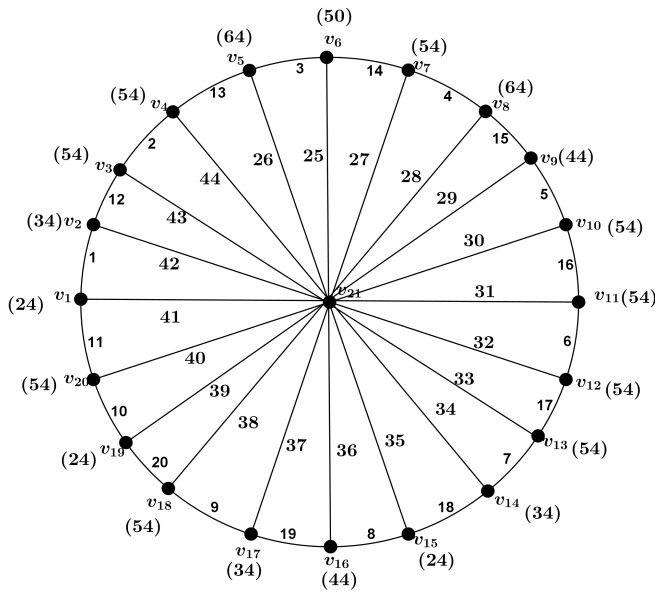


Figure 3: C13A Graph with product Magic Sums

The total vertex sums out of 39 edges is $\left(\frac{39 \times 40}{2}\right) \times 2 = 1560$
 (i.e.) $13 \times 60 + 7 \times 54 + 6 \times 67 = 780 + 378 + 402 = 1560$
 Hence F26A graph is found to be a 3 Magic Labeled Graph.

6 Graphical Structure on SMG(Super Mean Graph)

1. Few observance of product magic labeling of are listed. Here d and l denotes the number of dots and lines, $d = 2 + d + l$, $m = d + l$, $l + d = 2 + 2p + 2q$. Digits from one to $3 + 2p + 2q$ is to be allocated to the first dots and the pendant dots and in the procedure, the line digits got assigned. Here $f(x)$, $f(y)$, $f(x_j)$ and $f(v_j)$ are the integers get assigned to the dots and the pendant dots of the graphical structure respectively. The procedure for allotting the line values is odd values where the line connects x and x_i . Note that the link values can be the real average value or adjustable mean value.

The mean value of the biggest and the prefix number is

$$\frac{(3 + 2m + 3n) + (3 + 2n + 2m - one)}{2} = 6 + 3n + 3m.$$

The repeated values are not accepted for the graphical structure the digits in the graphs are unique.

Both the link values and the dot values cannot extend the largest values, $4+2n+2m$. Therefore the digits which can allot for the graphical structure with the values of n and m of the double frame graph $A_{one, n} \cup A_{one, m}$ using super mean labeling for every values of n and m but not permit the digits to be counted twice.

2. If $(g(x)$ and $g(x_i))$ or $(g(y)$ and $g(y_i))$ are taking the digits, then the line digit is the mean value in actual but if the digits are not same the mean is slightly adjust with the mean value.
3. When $g(y)$ is assigned the numbers which is odd and if $f(y_j) = 3s$, $f(y_{j+1}) = 2s + 2$, it gives the same line value and it cannot be followed to label the graph they lead to same edge value and hence to be avoided, that is $\frac{1+6}{2}$ and $\frac{1+7}{2}$ have the same edge value 4 when $f(u) = 1$.

When $f(y_j) = 3s + 1$ and $g(u_{j+1}) = 2r + 3$, it yields the values which is differ from line values. The average value is, $\frac{1+3}{2} = 2$ and $\frac{1+6}{2} = 4$ it gives different line values.

Also when $g(u)$ is odd the situation is reverse in process.

Assign a digit to v_1 and given the possible last digits to u_1 of the next star, then v_2 and d_2 are labeled by following the same procedure.

The Greek letters δ and λ are used to refer two star graphs. t , d_i and l_i denote the identification of the vertices and lines of the graph. For example $\delta(P_i)$ denotes the number assigned to the i^{th} vertex of the star one referred by [12],[13].

7 Illustrations on Two Star Graphs

1. **Message:** Tallest room six, floor five city thirty.
2. **Clue:** Compactness and Completeness are walking. (The digit five stands for compactness and the digit eight stands for completeness.)
3. **Graph:** The two star graph $A_{1, 5} \cup A_{1, 8}$, and the graphical structure is presented above.
4. **Assigning the numbers to the alphabets: Half Reversed Evens Backward and Odds Forward (HREBOF)**

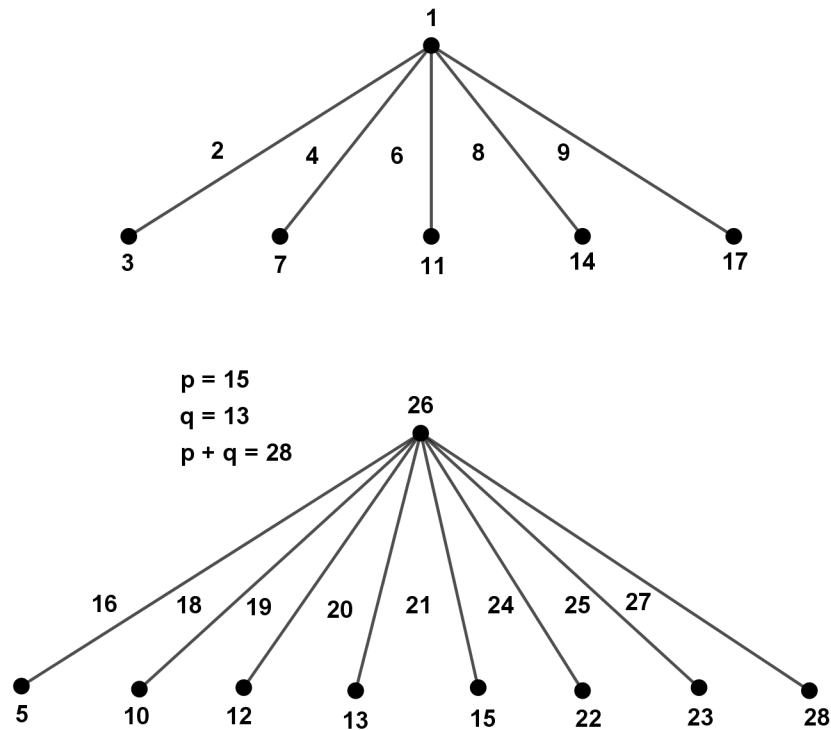
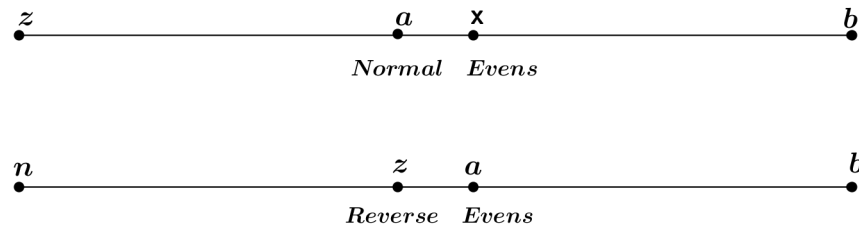


Figure 4: $A_{one, five} \cup A_{one, eight}$

7	21	6	22	5	23	4	24	3	25	2	26	1
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
14	13	15	12	16	11	17	10	18	9	19	8	20
A	B	C	D	E	F	G	H	I	J	K	L	M

Digits from one to thirteen are given to the even places of the alphabets started in the back to middle first and the digits fourteen to twenty six are given to the not even places of alphabets going forward and hence the method is known as HREBOF. The Mathematical expression for the HREBOF is expressed as follows,

$$\phi(r + 1) = \left(\frac{27 - r}{2} \right) \text{ for } r = \text{one, three, five, } \dots, \text{ twenty five.}$$



For first place, third place the mathematical expression is defined by,

$$\phi(k) = \left(\frac{27 + r}{2} \right) \text{ for } r = \text{one, three, five, } \dots, \text{ twenty five.}$$

To encrypt the given text message we reverse the the above procedure By using HREBOF and the Labeling with Super Mean on $A_{1,5} \cup A_{1,8}$ the Alphabetical message is converted to Coded Picture Message.

5. Coding: (wordwise)

- TALLEST - $\lambda(d_1)\delta(d_4)\alpha(l_4)\alpha(l_4)\alpha(l_7)\delta(d_7)\delta(d_1)$
- ROOM - $\lambda(l_2)\delta(l_3)\delta(l_3)\delta(d_6)$
- SIX - $\delta(d_7)\delta(l_1)\delta(l_1)$.
- FLOOR - $\lambda(d_4)\delta(l_4)\lambda(l_3)\lambda(l_3)\delta(l_2)$
- FIVE - $\delta(d_4)\delta(l_1)\lambda(P_1)\lambda(l_7)$
- CITY - $\lambda(l_6)\delta(l_1)\delta(P_1)\delta(l_6)$
- THIRTY - $\delta(d_1)\delta(d_3)\delta(l_1)\lambda(l_2)\delta(d_1)\delta(l_6)$

6. Horizontal string:

$$\begin{aligned} &\lambda(d_1)\delta(d_4)\alpha(l_4)\alpha(l_4)\alpha(l_7)\delta(d_7)\delta(d_1)(1,1)\lambda(l_2)\delta(l_3)\delta(l_3)\delta(d_6)(1,1) \\ &\delta(d_7)\delta(l_1)\delta(l_1)(1,1)\lambda(d_4)\delta(l_4)\lambda(l_3)\lambda(l_3)\delta(l_2)(1,1) \\ &\delta(d_4)\delta(l_1)\lambda(P_1)\lambda(l_7)(1,1)\lambda(l_6)\delta(l_1)\delta(P_1)\delta(l_6)(1,1)\delta(d_1)\delta(d_3)\delta(l_1)\lambda(l_2)\delta(d_1)\delta(l_6) \end{aligned}$$

7. Coded Message:

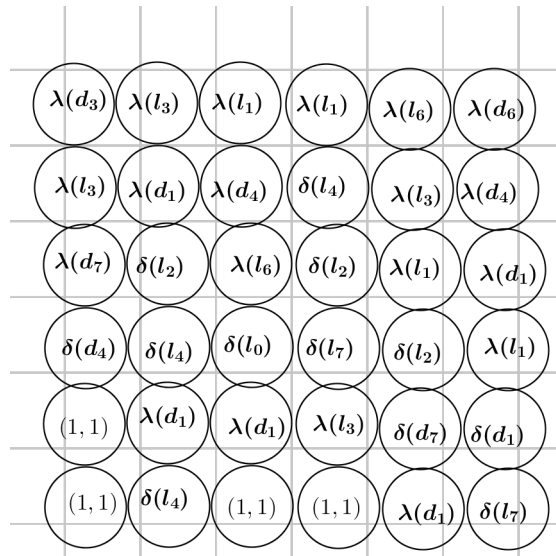


Figure 5: Coded message (HREBOF)

The below table shows the product magic labeling for the two star graph. The grids with the circles representing picture code and is obtained by shuffling the horizontal string in any order and the space is represented by the notation (1, 1) by rearranging the order of the notations, arriving the text message.

The labeling of multimagic Petersen graph are as follows.

$$\begin{aligned} r.s &= 4 + 2 + 3 \\ &= 9 \end{aligned}$$

$$\begin{aligned} R.S &= 3(15 - 2) \\ &= 39 \end{aligned}$$

$$\begin{aligned} R.S(1) &= \left[\frac{8 + 42}{2} \right] \\ &= 25 \end{aligned}$$

	d_1			d_2			d_3			d_4		
	l_5	l_6	l_1	l_1	l_7	l_2	l_2	l_8	l_3	l_3	l_9	l_4
	8	26	10	20	10	15	15	7	12	12	7	16
TOTAL	44			45			34			35		
	d_5			d_6			d_7			d_8		
	l_4	l_{10}	l_5	l_6	l_{11}	l_{15}	l_7	l_{14}	l_{13}	l_8	l_{12}	l_{11}
	6	10	4	13	8	13	10	7	11	15	5	8
TOTAL	20			34			28			28		
	d_9			d_{10}								
	l_9	l_{15}	l_1	l_{10}	l_{13}	l_{12}						
	14	1	7	12	9	1						
TOTAL	22			22								

Figure 6: C13A Graph with product Magic Sums

$$\begin{aligned}
 R.S(2) &= R.S(1) - \left\lfloor \frac{p}{4} \right\rfloor \\
 &= 26 - 2 \\
 &= 24
 \end{aligned}$$

$$\begin{aligned}
 R.S(3) &= R.S(1) \left\lceil \frac{p}{4} \right\rceil + 1 \\
 &= 26 + 2 + 1 \\
 &= 29
 \end{aligned}$$

Hence the graph is found to be a product multi magic magic labeled graph. Here is an application for Product Multi-Magic labeling.

Application of the product Magic Labeling

A Multimillionaire steps up to establish an industrial park in a tiny town, encouraging young people with ambition and drive to operate factories. This raises the standard of living for the residents in the town by creating a range of work options. The business man goes up to a prominent municipal consultant and gives him a quick rundown of the two dimensional roles that productions and assembly play. The expert consultant who possesses expertise in graph theory, graph labeling analysis and comprehension and who offers a solution. Below are the industrialist's requirements and the step-by-step solution that was sought. Ten factories must be established they are split into three groups based on the amount of output each factory can produce in a given week. Every week, 240 parts must be manufactured by all 10 factories combined units produced by the factories in the same category are identical. General kinds and unit types of factories, if each category's number of factories is represented by x, y and z . The Maximum, medium and minimum are the classification of capacities of the factories. If the maximum is denoted by L medium is denoted by M and the minimum is denoted N then, $Lx + my + Nz = 24u$. Every factory has 2 models of operations assembly and production. These factories receive the same quantity of items from each factory, which it sends in varied quantities. Then, using all the parts it has given from three manufactures each factory builds a machine with the change in the number of pieces the machines become distinct. The consultant hits a cubic graph with 10 vertices and 15 edges with a right strike.

8 Scope of the Research work

Recently product multi magic labeling have been succeeded in applying to various transmission control protocols and Internet computer protocols used at different levels of addresses such as physical, logical, port and specific addresses. Each addresses type is interconnected to a exact layer in TCP/IP architecture. With respect, the physical address is the lowest level of address and is also indicated to the link layer. The physical address is involved in the frame by data link layer. At a smaller scale the logical address are free of basic physical networks. Since distinct networks can have various formats hence a universal address system is needed which can identify each host uniquely regardless of underlying physical networks.

Logical addresses are mandatory for universal communication. IT is 32-bit address that uniquely defines a host associated to the internet. How to properly deploy software applications on the provided infrastructure in the communication network by diminishing the network usage especially mobile computing is interrogated in the field of network theory, theoretical results are the first application and the most begin from Graph theory.

9 Conclusion

- i) A few more applications of product multi-magic labeling have been presented in this research article on the interest of the researchers.
- ii) Not only single magic sum, we need few more examples in real-life situations. The solutions for product cordial magic labeling are the unique mathematical model graphs with the set of positive integers.
- iii) The present work of natural continuation will be on four-regular graphs when the number of edges is even and also product cordial magic labeling.

References

- [1] Gallian JA A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, 19, 1-219, 2009.
- [2] Harary F *Graph Theory*, Addison-Wesley, Reading, Mass., 1972.
- [3] Gary Chartrand and Diny Zhang *Introduction to graph theory* Tata McGraw-Hill Edition 2006.
- [4] Kotzig A and Rosa A Magic valuations of finite graphs, *Canad Math Bull* Vol.13, 451-46, 1970.
- [5] Kotzig A and Rosa A Magic Valuations of complete graphs, *Pwd.CRM* 175, 1972.
- [6] Rosa A On certain valuations of the vertices of a graph, *Theory of graphs, Int. Symp. Rome, Gordon and Breach, N.Y. and Dunod Paris*, 349-355, 1966.
- [7] Sathya D Margaret Joan Jebarani G and Merceline Anita A, A Network Communication through McGee graph and Antimagic labelling, *Malaya Journal of Matematik*. Vol. 9, No.1, 604- 607, 2021.

- [8] ShahiraSulthana Md Margaret Joan Jebarani G and Sujatha L A Mathematical model for apportionment and reapportionment using F26A graph and the magic labelling, *Malaya Journal of matematick*, Vol. 9, No. 1, 396-398, 2021.
- [9] Wallis WD *Magic Graphs*, Birkhauser, Boston, Basel, Berlin, 2001.
- [10] G Uma Maheswari, G Margaret Joan Jebarani and V Balaji, Coding through a Two Star and Super Mean Labeling, Springer - Applied Mathematics and Scientific Computing, volume 2, 2017.
- [11] G Uma Maheswari, G Margaret Joan Jebarani and V Balaji, GMJ Coding through a Three Star and Super Mean Labeling, *American International Journal of Research in Science, Technology, Engineering & Mathematics*, ISSN print: 2328-3491, 2019.
- [12] G Uma Maheswari G Margaret Joan Jebarani and V Balaji, *On Similar Super Mean Labeling for two star graph*, *Asian Journal of Mathematics and Computer Research*, volume 21(2), 53 – 62, 2017.
- [13] G Uma Maheswari G Margaret Joan Jebarani and V Balaji, *Sub Super Mean Labeling on a General Three Star Graph*, *International Journal of Mathematics and its Applications*, volume 6(1) , 147 – 153, 2018.