



## On Fuzzy Semi Regular Volterra Spaces

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### Abstract

In this paper, the notion of semi regular Volterra spaces in fuzzy setting is introduced. Some characterizations of fuzzy semi regular Volterra spaces are also studied in this paper.

**Key words:** Fuzzy semi dense set, fuzzy semi nowhere dense set, fuzzy semi  $G_\delta$ -set, fuzzy semi  $F_\sigma$ -set, fuzzy semi regular  $G_\delta$ -set, fuzzy semi regular  $F_\sigma$ -set, fuzzy semi first category set and fuzzy semi Volterra spaces.

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## 1. Introduction

In 1970, J.Mack [4] introduced the concepts of regular  $G_\delta$ -sets and regular  $F_\sigma$ -sets in classical topology. K.K.Azad [1] introduced fuzzy regular open and fuzzy regular closed sets in 1981. The concepts of regular  $G_\delta$ -sets and regular  $F_\sigma$ -sets in fuzzy setting are introduced and studied in this paper. By using fuzzy regular  $G_\delta$ -sets, the concepts of fuzzy regular Volterra and fuzzy regular weakly Volterra spaces are introduced in this paper. Several characterizations of fuzzy regular Volterra and fuzzy regular weakly Volterra spaces in terms of fuzzy regular  $F_\sigma$ -sets, fuzzy first category sets, fuzzy residual sets and fuzzy  $\sigma$ -nowhere dense sets are also established in this paper.

## 2. Preliminaries

In 1965, L.A.Zadeh [9] introduced the concept of fuzzy set  $\lambda$  on a base set  $X$  as a function from  $X$  into the unit interval  $I = [0, 1]$ . This function is also called a membership function. A membership function is a generalization of a characteristic function.

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**Definition 2.1** [3] Let  $\lambda$  and  $\mu$  be fuzzy sets in  $X$ . Then for all  $x \in X$ ,

1.  $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$ ,
2.  $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$ ,
3.  $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = \max\{\lambda(x), \mu(x)\}$ ,
4.  $\delta = \lambda \wedge \mu \Leftrightarrow \delta(x) = \min\{\lambda(x), \mu(x)\}$ ,
5.  $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$ .

For a family  $\{\lambda_i/i \in I\}$  of fuzzy sets in  $X$ , the union  $\psi = \vee_i \lambda_i$  and intersection  $\delta = \wedge_i \lambda_i$  are defined by  $\psi(x) = \sup_i\{\lambda_i(x), x \in X\}$ , and  $\delta(x) = \inf_i\{\lambda_i(x), x \in X\}$ .

The fuzzy set  $0_X$  is defined as  $0_X(x) = 0$ , for all  $x \in X$  and the fuzzy set  $1_X$  defined as  $1_X(x) = 1$ , for all  $x \in X$ .

**Definition 2.2** [3] A fuzzy topology is a family ‘ $T$ ’ of fuzzy sets in  $X$  which satisfies the following conditions:

- (1)  $\Phi, X \in T$ ,
- (2) If  $A, B \in T$ , then  $A \cap B \in T$ ,
- (3) If  $A_i \in T$ , for each  $i \in I$ , then  $\cup_{i \in I} A_i \in T$ .

$T$  is called a fuzzy topology for  $X$  and the pair  $(X, T)$  is a fuzzy topological space or fts in short. Every member of  $T$  is called a  $T$ -open fuzzy set. A fuzzy set is  $T$ -closed if and only if its complement is  $T$ -open. When no confusion is likely to arise, we shall call a  $T$ -open ( $T$ -closed) fuzzy set simply an open (closed) fuzzy set.

**Lemma 2.3** [1] Let  $(X, T)$  be any fuzzy topological space and  $\lambda$  be any fuzzy set in  $(X, T)$ . We define the fuzzy semi-closure and the fuzzy semi-interior of  $\lambda$  as follows:

- (i).  $scl(\lambda) = \wedge\{\mu/\lambda \leq \mu, \mu \text{ is fuzzy semi-closed set of } X\}$
- (ii).  $sint(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \text{ is fuzzy semi-open set of } X\}$ .

**Lemma 2.4** [1] For a fuzzy set  $\lambda$  of a fuzzy space  $X$ ,

- (a)  $1 - scl(\lambda) = sint(1 - \lambda)$  and
- (b)  $1 - sint(\lambda) = scl(1 - \lambda)$ .

**Lemma 2.5** [1] For a family  $\mathcal{A} = \{\lambda_\alpha\}$  of fuzzy sets of a fuzzy space  $X$ . Then,

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$\bigvee cl \lambda_\alpha \leq cl(\bigvee \lambda_\alpha)$ . In case  $\mathcal{A}$  is a finite set,  $\bigvee cl \lambda_\alpha = cl(\bigvee \lambda_\alpha)$ . Also  $\bigvee int \lambda_\alpha \leq int(\bigvee \lambda_\alpha)$ .

**Definition 2.6** [2] A fuzzy set  $\lambda$  in a fuzzy topological space  $X$  is called fuzzy semi-open if  $\lambda \leq clint(\lambda)$  and fuzzy semi-closed if  $intcl(\lambda) \leq \lambda$ .

**Definition 2.7** [8] A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called a fuzzy semi  $G_\delta$ -set in  $(X, T)$  if  $\lambda = \bigwedge_{i=1}^{\infty} (\lambda_i)$ , where  $(\lambda_i)$ 's are fuzzy semi open sets in  $(X, T)$ .

**Definition 2.8** [8] A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called a fuzzy semi  $F_\sigma$ -set in  $(X, T)$  if  $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$ , where  $(\lambda_i)$ 's are fuzzy semi closed sets in  $(X, T)$ .

**Definition 2.9** [7] A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called fuzzy semi dense if there exists no fuzzy semi closed set  $\mu$  in  $(X, T)$  such that  $\lambda < \mu < 1$ . That is,  $scl(\lambda) = 1$ .

**Definition 2.10** [6] Let  $(X, T)$  be a fuzzy topological space. A fuzzy set  $\lambda$  in  $(X, T)$  is called a fuzzy semi nowhere dense set if there exists no non-zero fuzzy semi open set  $\mu$  in  $(X, T)$  such that  $\mu < scl(\lambda)$ . That is,  $sintscl(\lambda) = 0$ .

**Definition 2.11** [6] Let  $(X, T)$  be a fuzzy topological space. A fuzzy set  $\lambda$  in  $(X, T)$  is called fuzzy semi first category if  $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$ , where  $(\lambda_i)$ 's are fuzzy semi nowhere dense sets in  $(X, T)$ . Any other fuzzy set in  $(X, T)$  is said to be of fuzzy semi second category.

**Definition 2.12** [6] If  $\lambda$  is a fuzzy semi first category set in a fuzzy topological space  $(X, T)$ , then  $1 - \lambda$  is called a fuzzy semi residual set in  $(X, T)$ .

**Definition 2.13** [5] A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called a fuzzy semi  $\sigma$ -nowhere dense set if  $\lambda$  is fuzzy semi  $F_\sigma$ -set in  $X, T$  such that  $sint(\lambda) = 0$ .

**Definition 2.14** [5] A fuzzy topological space  $(X, T)$  is called a fuzzy semi Volterra space if  $scl(\bigwedge_{i=1}^N (\lambda_i)) = 1$ , where  $(\lambda_i)$ 's are fuzzy semi dense and fuzzy semi  $G_\delta$ -sets in  $(X, T)$ .

**Theorem 2.15** [1] In a fuzzy topological space  $(X, T)$ ,

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- (a). The closure of a fuzzy open set is a fuzzy regular closed set
- (b). The interior of a fuzzy closed set is a fuzzy regular open set.

### 3. Fuzzy semi regular $G_\delta$ -sets

**Definition 3.1** A fuzzy set  $\lambda$  in a fuzzy topological space  $(X, T)$  is called a fuzzy semi regular  $G_\delta$ -set if  $\lambda = \bigwedge_{i=1}^{\infty} (sint(\lambda_i))$ , where  $1 - \lambda_i \in T$ .

**Definition 3.2** A fuzzy set  $\mu$  in a fuzzy topological space  $(X, T)$  is called a fuzzy semi regular  $F_\sigma$ -set if  $\mu = \bigvee_{i=1}^{\infty} (scl(\mu_i))$ , where  $\mu_i \in T$ .

**Proposition 3.3** If  $\lambda$  is a fuzzy semi regular  $G_\delta$ -set in a fuzzy topological space  $(X, T)$  if and only if  $1 - \lambda$  is a fuzzy semi regular  $F_\sigma$ -set in  $(X, T)$ .

Proof: Let  $\lambda$  be a fuzzy semi regular  $G_\delta$ -set in  $(X, T)$ . Then  $\lambda = \bigwedge_{i=1}^{\infty} (sint(\lambda_i))$ , where  $1 - \lambda_i \in T$ . Now  $1 - \lambda = 1 - \bigwedge_{i=1}^{\infty} (sint(\lambda_i)) = \bigvee_{i=1}^{\infty} (1 - sint(\lambda_i)) = \bigvee_{i=1}^{\infty} (scl(1 - \lambda_i))$ . Let  $\mu_i = 1 - \lambda_i$ . Then  $\mu_i \in T$ . Hence  $1 - \lambda = \bigvee_{i=1}^{\infty} (scl(\mu_i))$ ,  $\mu_i \in T$ . Therefore  $1 - \lambda$  is a fuzzy semi regular  $F_\sigma$ -set in  $(X, T)$ .

Conversely, let  $\lambda$  be a fuzzy semi regular  $F_\sigma$ -set in  $(X, T)$ . Then  $\lambda = \bigvee_{i=1}^{\infty} (scl(\mu_i))$ , where  $\mu_i \in T$ . Now  $1 - \lambda = 1 - \bigvee_{i=1}^{\infty} (scl(\mu_i)) = \bigwedge_{i=1}^{\infty} (1 - scl(\mu_i)) = \bigwedge_{i=1}^{\infty} (sint(1 - \mu_i))$ . Let  $1 - \mu_i = \lambda_i$ . Then implies that  $\mu_i = 1 - \lambda_i$  and  $1 - \lambda_i \in T$ . Hence  $1 - \lambda = \bigwedge_{i=1}^{\infty} (sint(\lambda_i))$ , where  $1 - \lambda_i \in T$ . Therefore  $1 - \lambda$  is a fuzzy semi regular  $G_\delta$ -set in  $(X, T)$ .

**Proposition 3.4** Let  $(X, T)$  be a fuzzy topological space.

- (1). If  $\lambda$  is a fuzzy semi regular  $G_\delta$ -set in  $(X, T)$ , then  $\lambda = \bigwedge_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy semi regular open sets in  $(X, T)$ .
- (2). If  $\lambda$  is a fuzzy semi regular  $F_\sigma$ -set in  $(X, T)$ , then  $\lambda = \bigvee_{i=1}^{\infty} (\mu_i)$ , where  $(\mu_i)$ 's are fuzzy semi regular closed sets in  $(X, T)$ .

Proof: (1). Let  $\lambda$  be a fuzzy semi regular  $G_\delta$ -set in  $(X, T)$ . Then  $\lambda = \bigwedge_{i=1}^{\infty} (sint(\lambda_i))$ , where  $1 - \lambda_i \in T$ . Now  $1 - \lambda_i \in T$  implies that  $\lambda_i$  is a fuzzy semi-closed set in  $(X, T)$ . By theorem 2.15,  $sint(\lambda_i)$  is a fuzzy semi regular open set in  $(X, T)$ . Let  $\delta_i = sint(\lambda_i)$ . Then  $\lambda = \bigwedge_{i=1}^{\infty} (\delta_i)$ , where  $(\delta_i)$ 's are fuzzy semi regular open sets in  $(X, T)$ .

(2). Let  $\lambda$  be a fuzzy semi regular  $F_\sigma$ -set in  $(X, T)$ . Then  $\lambda = \bigvee_{i=1}^{\infty} (scl(\mu_i))$ , where  $\mu_i \in T$ . Now  $\mu_i \in T$ . By theorem 2.15,  $scl(\mu_i)$  is a fuzzy semi regular closed set

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in  $(X, T)$ . Let  $\eta_i = scl(\mu_i)$ . Then  $\lambda = \bigvee_{i=1}^{\infty}(\eta_i)$ , where  $(\eta_i)$ 's are fuzzy semi regular closed sets in  $(X, T)$ .

**Proposition 3.5** If  $\lambda$  is a fuzzy semi regular  $G_\delta$ -set in a fuzzy topological space  $(X, T)$ , then  $\lambda$  is a fuzzy semi  $G_\delta$ -set in  $(X, T)$ .

Proof: Let  $\lambda$  be a fuzzy semi regular  $G_\delta$ -set in  $(X, T)$ . Then by proposition 3.4,  $\lambda = \bigwedge_{i=1}^{\infty}(\delta_i)$ , where  $(\delta_i)$ 's are fuzzy semi regular open sets in  $(X, T)$ . Since every fuzzy semi regular open set is a fuzzy semi-open set in  $(X, T)$ ,  $(\delta_i)$ 's are fuzzy semi-open sets in  $(X, T)$ . Hence  $\lambda = \bigwedge_{i=1}^{\infty}(\delta_i)$ , where  $\delta_i \in T$ . Therefore  $\lambda$  is a fuzzy semi  $G_\delta$ -set in  $(X, T)$ .

**Proposition 3.6** If  $\lambda$  is a fuzzy semi regular  $F_\sigma$ -set in a fuzzy topological space  $(X, T)$ , then  $\lambda$  is a fuzzy semi  $F_\sigma$ -set in  $(X, T)$ .

Proof: Let  $\lambda$  be a fuzzy semi regular  $F_\sigma$ -set in  $(X, T)$ . Then by proposition 3.4,  $\lambda = \bigvee_{i=1}^{\infty}(\eta_i)$ , where  $(\eta_i)$ 's are fuzzy semi regular closed sets in  $(X, T)$ . Since every fuzzy semi regular closed set is a fuzzy semi-closed set in  $(X, T)$ ,  $(\eta_i)$ 's are fuzzy semi-closed sets in  $(X, T)$ . Hence  $\lambda = \bigvee_{i=1}^{\infty}(\eta_i)$ , where  $1 - \eta_i \in T$ . Therefore  $\lambda$  is a fuzzy semi  $F_\sigma$ -set in  $(X, T)$ .

#### 4. Fuzzy semi regular Volterra spaces

**Definition 4.1** A fuzzy topological space  $(X, T)$  is called a fuzzy semi regular Volterra space if  $scl(\bigwedge_{i=1}^N(\lambda_i)) = 1$ , where  $(\lambda_i)$ 's are fuzzy semi dense and fuzzy semi regular  $G_\delta$ -sets in  $(X, T)$ .

**Proposition 4.2** If  $sint(\bigvee_{i=1}^N(\mu_i)) = 0$  where  $(\mu_i)$ 's are fuzzy semi regular  $F_\sigma$ -sets with  $sint(\mu_i) = 0$  in a fuzzy topological space  $(X, T)$ , then  $(X, T)$  is a fuzzy semi regular Volterra space.

Proof: Suppose that  $sint(\bigvee_{i=1}^N(\mu_i)) = 0$ , where  $(\mu_i)$ 's are fuzzy semi regular  $F_\sigma$ -sets with  $sint(\mu_i) = 0$ . Now  $1 - sint(\bigvee_{i=1}^N(\mu_i)) = 1$ . Then,  $scl(1 - \bigvee_{i=1}^N(\mu_i)) = 1$ . This implies that  $scl(\bigwedge_{i=1}^N(1 - \mu_i)) = 1$ . Since  $(\mu_i)$ 's are fuzzy semi regular  $F_\sigma$ -sets in  $(X, T)$ , by proposition 3.3,  $(1 - \mu_i)$ 's are fuzzy semi regular  $G_\delta$ -sets in  $(X, T)$ . Also,  $sint(\mu_i) = 0$  implies that  $1 - sint(\mu_i) = 1$ . Then,  $scl(1 - \mu_i) = 1$ . Let  $\lambda_i = 1 - \mu_i$ . Then  $(\lambda_i)$ 's are fuzzy semi dense and fuzzy semi regular  $G_\delta$ -sets in  $(X, T)$ . Hence,

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$scl(\bigwedge_{i=1}^N (\lambda_i)) = 1$ , where  $(\lambda_i)$ 's are fuzzy semi dense and fuzzy semi regular  $G_\delta$ -sets in  $(X, T)$ . Therefore  $(X, T)$  is a fuzzy semi regular Volterra space.

**Proposition 4.3** If a fuzzy topological space  $(X, T)$  is a fuzzy semi Volterra space, then  $(X, T)$  is a fuzzy semi regular Volterra space.

Proof: Let  $\lambda = scl(\bigwedge_{i=1}^N (\lambda_i)) \dots (1)$ , where  $(\lambda_i)$ 's are fuzzy semi dense and fuzzy semi regular  $G_\delta$ -sets in  $(X, T)$ . By proposition 3.5, the fuzzy semi regular  $G_\delta$ -sets  $(\lambda_i)$ 's are fuzzy semi  $G_\delta$ -sets in  $(X, T)$ . Since  $(X, T)$  is a fuzzy semi Volterra space,  $scl(\bigwedge_{i=1}^N (\lambda_i)) = 1 \dots (2)$ , where  $(\lambda_i)$ 's are fuzzy semi dense and fuzzy semi  $G_\delta$ -sets in  $(X, T)$ . Hence, from (1) and (2),  $\lambda = 1$ . Therefore  $(X, T)$  is a fuzzy semi regular Volterra space.

**Proposition 4.4** If a fuzzy topological space  $(X, T)$  is a fuzzy semi regular Volterra space, then  $sint(\bigvee_{i=1}^N (\mu_i)) = 0$ , where  $(\mu_i)$ 's are fuzzy semi  $\sigma$ -nowhere dense sets in  $(X, T)$ .

Proof: Let  $(X, T)$  be a fuzzy semi regular Volterra space. Then  $scl(\bigwedge_{i=1}^N (\lambda_i)) = 1$ , where  $(\lambda_i)$ 's are fuzzy semi dense and fuzzy semi regular  $G_\delta$ -sets in  $(X, T)$ . Now  $1 - scl(\bigwedge_{i=1}^N (\lambda_i)) = 0$  implies that  $sint(\bigvee_{i=1}^N (1 - \lambda_i)) = 0$ . Since  $(\lambda_i)$ 's are fuzzy semi regular  $G_\delta$ -sets, by proposition 3.3,  $(1 - \lambda_i)$ 's are fuzzy semi regular  $F_\sigma$ -sets in  $(X, T)$ . By proposition 3.6,  $(1 - \lambda_i)$ 's are fuzzy semi  $F_\sigma$ -sets in  $(X, T)$ . Also,  $scl(\lambda_i) = 1$  implies that  $1 - scl(\lambda_i) = 0$  and hence  $sint(1 - \lambda_i) = 0$ . Let  $\mu_i = 1 - \lambda_i$ . Then  $(\mu_i)$ 's are fuzzy semi  $F_\sigma$ -sets with  $sint(\mu_i) = 0$ . Then, by the definition of fuzzy semi  $\sigma$ -nowhere dense sets,  $(\mu_i)$ 's are fuzzy semi  $\sigma$ -nowhere dense sets in  $(X, T)$ . Hence  $sint(\bigvee_{i=1}^N (\mu_i)) = 0$ , where  $(\mu_i)$ 's are fuzzy semi  $\sigma$ -nowhere dense sets in  $(X, T)$ .

**Proposition 4.5** If  $sint(\lambda) = 0$  for a fuzzy semi regular  $F_\sigma$ -set  $\lambda$  in a fuzzy topological space  $(X, T)$ , then  $\lambda$  is a fuzzy semi first category set in  $(X, T)$ .

Proof: Let  $\lambda$  be a fuzzy semi regular  $F_\sigma$ -set in  $(X, T)$ . Then  $\lambda = \bigvee_{i=1}^\infty (scl(\mu_i))$ , where  $\mu_i \in T$ . Now  $sint(\lambda) = 0$  implies that  $sint(\bigvee_{i=1}^\infty (scl(\mu_i))) = 0$ . But  $\bigvee_{i=1}^\infty (sint(scl(\mu_i))) \leq sint(\bigvee_{i=1}^\infty (scl(\mu_i))) = 0$ . Then,  $\bigvee_{i=1}^\infty (sint(scl(\mu_i))) = 0$ . This implies that  $sint(scl(\mu_i)) = 0$ . Hence  $\mu_i$  is a fuzzy semi nowhere dense set in  $(X, T)$ . Also  $sint(scl(scl(\mu_i))) = sint(scl(\mu_i)) = 0$  implies that  $scl(\mu_i)$  is a fuzzy semi nowhere dense set in  $(X, T)$ . Hence  $\lambda = \bigvee_{i=1}^\infty (scl(\mu_i))$ , where  $(scl(\mu_i))$ 's are

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fuzzy semi nowhere dense sets in  $(X, T)$ . Therefore  $\lambda$  is a fuzzy semi first category set in  $(X, T)$ .

**Proposition 4.6** If a fuzzy semi regular  $G_\delta$ -set  $\lambda$  is a fuzzy semi dense set in a fuzzy topological space  $(X, T)$ , then  $\lambda$  is a fuzzy semi residual set in  $(X, T)$ .

Proof: Let  $\lambda$  be a fuzzy semi regular  $G_\delta$ -set with  $scl(\lambda) = 1$ . Then  $1 - \lambda$  is a fuzzy semi regular  $F_\sigma$ -set with  $1 - scl(\lambda) = 0$ . That is,  $1 - \lambda$  is a fuzzy semi regular  $F_\sigma$ -set with  $sint(1 - \lambda) = 0$ . Then by proposition 4.5,  $1 - \lambda$  is a fuzzy semi first category set in  $(X, T)$ . Therefore  $\lambda$  is a fuzzy semi residual set in  $(X, T)$ .

**Proposition 4.7** If a fuzzy topological space  $(X, T)$  is a fuzzy semi regular Volterra space, then  $scl(\bigwedge_{i=1}^N (\lambda_i)) = 1$  where  $(\lambda_i)$ 's are fuzzy semi residual sets in  $(X, T)$ .

Proof: Let  $(X, T)$  be a fuzzy semi regular Volterra space. Then  $scl(\bigwedge_{i=1}^N (\lambda_i)) = 1$ , where  $(\lambda_i)$ 's are fuzzy semi dense and fuzzy semi regular  $G_\delta$ -sets in  $(X, T)$ . By proposition 4.6,  $(\lambda_i)$ 's are fuzzy semi residual sets in  $(X, T)$ . Hence  $scl(\bigwedge_{i=1}^N (\lambda_i)) = 1$ , where  $(\lambda_i)$ 's are fuzzy semi residual sets in  $(X, T)$ .

**Proposition 4.8** If a fuzzy topological space  $(X, T)$  is a fuzzy semi regular Volterra space, then  $sint(\bigvee_{i=1}^N (\mu_i)) = 0$ , where  $(\mu_i)$ 's are fuzzy semi first category sets in  $(X, T)$ .

Proof: Let  $(X, T)$  be a fuzzy semi regular Volterra space. Then  $scl(\bigwedge_{i=1}^N (\lambda_i)) = 1$ , where  $(\lambda_i)$ 's are fuzzy semi dense and fuzzy semi regular  $G_\delta$ -sets in  $(X, T)$ . Now  $1 - scl(\bigwedge_{i=1}^N (\lambda_i)) = 0$  implies that  $sint(1 - \bigwedge_{i=1}^N (\lambda_i)) = 0$ . Then,  $sint(\bigvee_{i=1}^N (1 - \lambda_i)) = 0$ . Now  $(\lambda_i)$ 's are fuzzy semi regular  $G_\delta$ -sets in  $(X, T)$  implies that  $(1 - \lambda_i)$ 's are fuzzy semi regular  $F_\sigma$ -sets in  $(X, T)$ . Also  $scl(\lambda_i) = 1$  implies that  $1 - scl(\lambda_i) = 0$ . Then  $sint(1 - \lambda_i) = 0$ . Hence,  $(1 - \lambda_i)$ 's are fuzzy semi regular  $F_\sigma$ -sets with  $sint(1 - \lambda_i) = 0$ . Therefore by proposition 4.5,  $(1 - \lambda_i)$ 's are fuzzy semi first category sets in  $(X, T)$ . Let  $\mu_i = 1 - \lambda_i$ . Hence if  $(X, T)$  is a fuzzy semi regular Volterra space, then  $sint(\bigvee_{i=1}^N (\mu_i)) = 0$ , where  $(\mu_i)$ 's are fuzzy semi first category sets in  $(X, T)$ .

## 5. Conclusion

In this paper, the concept of fuzzy semi regular Volterra spaces have introduced and also some of their characteristics have investigated and studied.

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