ISSN: 2456-8686, Volume 3, Issue 2, 2019:51-62
DOI : http://doi.org/10.26524/cm54

Journal of Computational Mathematica

Open Access

A Short note on Pairwise Fuzzy Resolvable Spaces

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Abstract

The aim of this paper is the notion of resolvability in fuzzy bitopological spaces have been studied extensively. Relativization in pairwise fuzzy resolvability and pairwise fuzzy irresolvability are also established in this paper.

Key words: Pairwise fuzzy open set, pairwise fuzzy nowhere dense set, pairwise fuzzy dense set, pairwise fuzzy resolvable space, pairwise fuzzy irresolvable space, pairwise fuzzy hyperconeected space, pairwise fuzzy nodec space, pairwise fuzzy Naire space and pairwise fuzzy submaximal space.

AMS classification: 54A40, 03E72

1. Introduction

Many mathematical concepts can be represented by the notion of set theory, which dichotomize the situation into two conditions: either "yes" or "no". Till 1965, Mathematicians were concerned only about "well-defined" things and smartly avoided any other possibilities which are more realistic in nature. In 1965, L.A.Zadeh [14] introduced the concept of fuzzy set, to accommodate real life situations by giving partial membership to each element of a situation under consideration. The usual notion of set topology was generalized with the introduction of fuzzy topology by C.L.Chang [2] in 1968, based on the concept of fuzzy sets invented by Zadeh. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of

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ISSN: 2456-8686, Volume 3, Issue 2, 2019:51-62
DOI : http://doi.org/10.26524/cm54

fuzzy topology has been developed. In 1989, A.Kandil [5] introduced the concept of fuzzy bitopological spaces and since then various notions in classical topology have been extended to fuzzy bitopological spaces.

The systematic study of resolvability in classical topology began with the works of E.Hewitt [4] and M.Katetov [6]. The concepts of resolvability and irresolvability in topological spaces were introduced and studied by E.Hewitt [4] in 1943. Since then several mathematicians found interest in the study of resolvable and irresolvable spaces. In 1993, C.Chattopadhyay and C.Bandyopadhyay [3] extended the study of resolvable and irresolvable spaces to the bitopological spaces. The concepts of resolvability and irresolvability of fuzzy bitopological spaces were introduced by G.Thangaraj [7]. In this chapter, the concepts of resolvability in fuzzy bitopological spaces have been studied extensively. Besides giving the definitions and examples, various properties of such spaces are investigated.

2. Preliminaries

Now we give some basic notions and results used in the sequel. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X, T_1, T_2), where T_1 and T_2 are two fuzzy topologies on a non-empty set X. Throughout this paper, the indices i and j take values in $\{1, 2\}$ and $i \neq j$.

Definition 2.1 [2] Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T). The closure and interior of a fuzzy set λ in a fuzzy topological space (X, T) are respectively denoted as $cl(\lambda)$ and $int(\lambda)$ are defined as

(1) $cl(\lambda) = \wedge\{\mu \mid \lambda \leq \mu, 1 - \mu \in T\}$ and
(2) $int(\lambda) = \vee\{\mu \mid \mu \leq \lambda, \mu \in T\}$.

Lemma 2.2 [1] For a fuzzy set λ of a fuzzy space X,

(a) $1 - cl(\lambda) = int(1 - \lambda)$ and
(b) $1 - int(\lambda) = cl(1 - \lambda)$.

Theorem 2.3 [13] Let X be a fuzzy topological space and λ, μ be fuzzy sets in X. Then we have

(1). λ is fuzzy closed (resp., fuzzy open) $\Leftrightarrow cl(\lambda) = \lambda$ (resp., $int(\lambda) = \lambda$);
(2). $\lambda \leq \mu \Rightarrow cl(\lambda) \leq cl(\mu) \quad (int(\lambda) \leq int(\mu))$;

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(3). $cl\ cl(\lambda) = cl(\lambda) \quad (int\ int(\lambda) = int(\lambda))$;
(4). $cl(\lambda) \vee cl(\mu) = cl(\lambda \vee \mu)$;
(5). $cl(\lambda) \wedge cl(\mu) \geq cl(\lambda \wedge \mu)$;
(6). $int(\lambda) \vee int(\mu) \leq int(\lambda \vee \mu)$;
(7). $int(\lambda) \wedge int(\mu) = int(\lambda \wedge \mu)$;

Definition 2.4 [14] A fuzzy set λ in a set X is a function from X to $[0, 1]$, that is., $\lambda : X \to [0, 1]$.

Definition 2.5 [8] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy open set if $\lambda \in T_i,\ (i = 1, 2)$. The complement of pairwise fuzzy open set in (X, T_1, T_2) is called a pairwise fuzzy closed set.

Definition 2.6 [7] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy dense set if $cl_{T_1}cl_{T_2}(\lambda) = 1$ and $cl_{T_2}cl_{T_1}(\lambda) = 1$.

Definition 2.7 [7] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nowhere dense set if $int_{T_1}cl_{T_2}(\lambda) = 0 = int_{T_2}cl_{T_1}(\lambda)$.

Definition 2.8 [11] A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy first category set if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2). Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy second category set.

Definition 2.9 [11] If λ is a pairwise fuzzy first category set in a fuzzy bitopological space (X, T_1, T_2), then $1 - \lambda$ is called a pairwise fuzzy residual set in (X, T_1, T_2).

Definition 2.10 [10] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy Baire space if $int_{T_i}(\vee_{k=1}^{\infty}(\lambda_k)) = 0$, $(i = 1, 2)$, where (λ_k)'s are pairwise fuzzy nowhere dense sets in (X, T_1, T_2).

Definition 2.11 [11] A fuzzy bitopological space (X, T_1, T_2) is said to be a pairwise fuzzy submaximal space if for each fuzzy set λ in (X, T_1, T_2) such that $cl_{T_i}(\lambda) = 1,\ (i = 1, 2)$, then λ is a pairwise fuzzy open set in (X, T_1, T_2).

Definition 2.12 [11] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy nodec space if every non-zero pairwise fuzzy nowhere dense set in (X, T_1, T_2), is a

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pairwise fuzzy closed set in (X, T_1, T_2). That is, if λ is a pairwise fuzzy nowhere dense set in a fuzzy bitopological space (X, T_1, T_2), then $1 - \lambda \in T_i$ $(i = 1, 2)$.

Definition 2.13 [9] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy hyperconnected space if λ is a pairwise fuzzy open set in (X, T_1, T_2), then $cl_{T_i}(\lambda) = 1$, $(i = 1, 2)$.

Theorem 2.14 [12] If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy resolvable space, then there exists atleast one pairwise fuzzy dense set λ in (X, T_1, T_2) such that λ is not a pairwise fuzzy open set in (X, T_1, T_2).

Theorem 2.15 [12] If λ_1 and λ_2 are any two pairwise fuzzy dense sets in a fuzzy bitopological space (X, T_1, T_2) such that $\lambda_1 \leq (1 - \lambda_2)$, then (X, T_1, T_2) is a pairwise fuzzy resolvable space.

3. Pairwise fuzzy resolvable spaces and other fuzzy bitopological spaces

Definition 3.1 [7] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy resolvable space if there exists a pairwise fuzzy dense set λ in (X, T_1, T_2) such that $1 - \lambda$ is also a pairwise fuzzy dense set in (X, T_1, T_2).

That is, (X, T_1, T_2) is called a pairwise fuzzy resolvable space if there exists a fuzzy set λ defined on X such that $cl_{T_1}cl_{T_2}(\lambda) = 1 = cl_{T_2}cl_{T_1}(\lambda)$ and $cl_{T_1}cl_{T_2}(1 - \lambda) = 1 = cl_{T_2}cl_{T_1}(1 - \lambda)$ in (X, T_1, T_2).

Definition 3.2 [7] A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy irresolvable space if for each pairwise fuzzy dense set λ in (X, T_1, T_2), $cl_{T_1}cl_{T_2}(1-\lambda) \neq 1$ and $cl_{T_2}cl_{T_1}(1 - \lambda) \neq 1$ in (X, T_1, T_2).

Theorem 3.3 [10] Let (X, T_1, T_2) be a fuzzy bitopological space. Then the following are equivalent:

(1). (X, T_1, T_2) is a pairwise fuzzy Baire space.
(2). $int_{T_i}(\lambda) = 0$, $(i = 1, 2)$, for every pairwise fuzzy first category set λ in (X, T_1, T_2).
(3). $cl_{T_i}(\mu) = 1$, $(i = 1, 2)$, for every pairwise fuzzy residual set μ in (X, T_1, T_2).

The following propositions give conditions for a pairwise fuzzy Baire space to be a pairwise fuzzy resolvable space.

Proposition 3.4 If there exists a pairwise fuzzy first category set λ such that $cl_{T_1}cl_{T_2}(\lambda) = 1 = cl_{T_2}cl_{T_1}(\lambda)$ in a pairwise fuzzy Baire space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy resolvable space.

Proof: Let λ be a pairwise fuzzy first category set in (X, T_1, T_2) such that $cl_{T_1}cl_{T_2}(\lambda) = 1 = cl_{T_2}cl_{T_1}(\lambda)$. Since (X, T_1, T_2) is a pairwise fuzzy Baire space, by theorem 3.3, $int_{T_1}(\lambda) = 0$ and $int_{T_2}(\lambda) = 0$, in (X, T_1, T_2). Then $int_{T_1}int_{T_2}(\lambda) = int_{T_1}(0) = 0$ and $int_{T_2}int_{T_1}(\lambda) = int_{T_2}(0) = 0$, in (X, T_1, T_2). That is, $int_{T_1}int_{T_2}(\lambda) = 0 = int_{T_2}int_{T_1}(\lambda)$, for the pairwise fuzzy first category set λ in (X, T_1, T_2). Then $1 - int_{T_1}int_{T_2}(\lambda) = 1 - 0 = 1$ and $1 - int_{T_2}int_{T_1}(\lambda) = 1 - 0 = 1$, in (X, T_1, T_2). This implies that $cl_{T_1}cl_{T_2}(1 - \lambda) = 1$ and $cl_{T_2}cl_{T_1}(1 - \lambda) = 1$, in (X, T_1, T_2). Hence $cl_{T_1}cl_{T_2}(\lambda) = 1 = cl_{T_2}cl_{T_1}(\lambda)$ and $cl_{T_1}cl_{T_2}(1 - \lambda) = 1 = cl_{T_2}cl_{T_1}(1 - \lambda)$ in (X, T_1, T_2). Thus, for the pairwise fuzzy dense set λ in (X, T_1, T_2), $cl_{T_1}cl_{T_2}(1 - \lambda) = 1 = cl_{T_2}cl_{T_1}(1 - \lambda)$ in (X, T_1, T_2). This implies that (X, T_1, T_2) is a pairwise fuzzy resolvable space.

Proposition 3.5 If there exists a pairwise fuzzy residual set μ such that $int_{T_1}int_{T_2}(\mu) = 0 = int_{T_2}int_{T_1}(\mu)$ in a pairwise fuzzy Baire space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy resolvable space.

Proof: Let μ be a pairwise fuzzy residual set in (X, T_1, T_2) such that $int_{T_1}int_{T_2}(\mu) = 0 = int_{T_2}int_{T_1}(\mu)$. Since μ is a pairwise fuzzy residual set in (X, T_1, T_2), $1 - \mu$ is a pairwise fuzzy first category set in (X, T_1, T_2). Now $cl_{T_1}cl_{T_2}(1 - \mu) = 1 - int_{T_1}int_{T_2}(\mu) = 1 - 0 = 1$ and $cl_{T_2}cl_{T_1}(1 - \mu) = 1 - int_{T_2}int_{T_1}(\mu) = 1 - 0 = 1$ in (X, T_1, T_2). Hence $1 - \mu$ is a pairwise fuzzy first category set in (X, T_1, T_2) such that $cl_{T_1}cl_{T_2}(1 - \mu) = 1 = cl_{T_2}cl_{T_1}(1 - \mu)$, in the pairwise fuzzy Baire space (X, T_1, T_2). Therefore, by proposition 3.4, (X, T_1, T_2) is a pairwise fuzzy resolvable space.

The following proposition shows that the pairwise fuzzy resolvability does not implies the pairwise fuzzy submaximality of fuzzy bitopological spaces.

Proposition 3.6 If a fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy resolvable space, then (X, T_1, T_2) is not a pairwise fuzzy submaximal space.

Proof: Let (X, T_1, T_2) be a pairwise fuzzy resolvable space. It has to be proved that (X, T_1, T_2) is not a pairwise fuzzy submaximal space. Assume the contrary. Suppose

ISSN: 2456-8686, Volume 3, Issue 2, 2019:51-62
DOI : http://doi.org/10.26524/cm54

that (X, T_1, T_2) is a pairwise fuzzy submaximal space. Then, each pairwise fuzzy dense set will be a pairwise fuzzy open set in (X, T_1, T_2). But, this is a contradiction to the theorem 2.14, which establishes that, in a pairwise fuzzy resolvable space (X, T_1, T_2), there exists atleast one pairwise fuzzy dense set λ in (X, T_1, T_2), which is not a pairwise fuzzy open set in (X, T_1, T_2). Hence (X, T_1, T_2) is not a pairwise fuzzy submaximal space.

The following propositions give conditions for a pairwise fuzzy hyperconnected space to be a pairwise fuzzy resolvable space.

Proposition 3.7 If there exist pairwise fuzzy open sets λ_1 and λ_2 such that $\lambda_1 \leq (1 - \lambda_2)$ in a pairwise fuzzy hyperconnected space (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy resolvable space.

Proof: Let λ_1 and λ_2 be any two pairwise fuzzy open sets in (X, T_1, T_2) such that $\lambda_1 \leq 1 - \lambda_2$. Since (X, T_1, T_2) is a pairwise fuzzy hyperconnected space, the pairwise fuzzy open sets λ_1 and λ_2 are pairwise fuzzy dense sets in (X, T_1, T_2). Hence λ_1 and λ_2 are pairwise fuzzy dense sets in (X, T_1, T_2) such that $\lambda_1 \leq 1 - \lambda_2$. Then, by theorem 2.15, (X, T_1, T_2) is a pairwise fuzzy resolvable space.

The following propositions give conditions for a pairwise fuzzy nodec and pairwise fuzzy hyperconnected space to be a pairwise fuzzy resolvable space.

Proposition 3.8 If λ_1 and λ_2 are any two pairwise fuzzy nowhere dense sets in a pairwise fuzzy nodec and pairwise fuzzy hyperconnected space (X, T_1, T_2) such that $\lambda_1 \geq (1 - \lambda_2)$, then (X, T_1, T_2) is a pairwise fuzzy resolvable space.

Proof: Suppose that λ_1 and λ_2 are pairwise fuzzy nowhere dense sets in (X, T_1, T_2) such that $\lambda_1 \geq (1 - \lambda_2)$. Since (X, T_1, T_2) is a pairwise fuzzy nodec space, the pairwise fuzzy nowhere dense sets λ_1 and λ_2 are pairwise fuzzy closed sets in (X, T_1, T_2) and hence $1 - \lambda_1$ and $1 - \lambda_2$ are pairwise fuzzy open sets in (X, T_1, T_2). Since (X, T_1, T_2) is a pairwise fuzzy hyperconnected space, the pairwise fuzzy open sets $1 - \lambda_1$ and $1 - \lambda_2$ are pairwise fuzzy dense sets in (X, T_1, T_2). Let $\mu_1 = 1 - \lambda_1$ and $\mu_2 = 1 - \lambda_2$. Then, μ_1 and μ_2 are pairwise fuzzy dense sets in (X, T_1, T_2) and hence $cl_{T_1} cl_{T_2}(\mu_1) = 1 = cl_{T_2} cl_{T_1}(\mu_1)$ and $cl_{T_1} cl_{T_2}(\mu_2) = 1 = cl_{T_2} cl_{T_1}(\mu_2)$ in (X, T_1, T_2). Now $\lambda_1 \geq (1 - \lambda_2)$ implies that $1 - \lambda_1 \leq (1 - (1 - \lambda_2))$. Then $\mu_1 \leq (1 - \mu_2)$ in (X, T_1, T_2). This implies that $cl_{T_i} cl_{T_j}(\mu_1) \leq cl_{T_i} cl_{T_j}(1 - \mu_2)$ in (X, T_1, T_2). Thus, $1 \leq cl_{T_i} cl_{T_j}(1 - \mu_2)$ in (X, T_1, T_2). That is, $cl_{T_i} cl_{T_j}(1 - \mu_2) = 1$ in (X, T_1, T_2). Thus, there exists a pairwise fuzzy dense

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set μ_2 in (X, T_1, T_2) such that $cl_{T_i}cl_{T_j}(1-\mu_2) = 1$ in (X, T_1, T_2). Therefore, (X, T_1, T_2) is a pairwise fuzzy resolvable space.

4. Relativization in pairwise fuzzy resolvability and pairwise fuzzy irresolvability

Definition 4.1 Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ defined on X is said to be pairwise fuzzy resolvable relative to X or pairwise fuzzy resolvable in X if there exists two pairwise fuzzy dense sets μ_1 and μ_2 in (X, T_1, T_2) such that $\mu_1 \wedge \lambda \neq 0$ and $\mu_2 \wedge \lambda \neq 0$, then $\mu_1 \wedge \mu_2 \wedge \lambda = 0$ in (X, T_1, T_2).

Proposition 4.2 If a fuzzy set λ is pairwise fuzzy resolvable relative to X in a fuzzy bitopological space (X, T_1, T_2), then there exists two pairwise fuzzy dense sets μ_1 and μ_2 in (X, T_1, T_2) such that $\mu_1 \wedge \lambda \neq 0$, $\mu_2 \wedge \lambda \neq 0$ and $int_{T_i}int_{T_j}(\mu_1 \wedge \mu_2) \leq 1 - \big(cl_{T_i}cl_{T_j}(\lambda)\big)$ in (X, T_1, T_2) ($i \neq j$ and $i, j = 1, 2$).

Proof: Let the fuzzy set λ in (X, T_1, T_2) be pairwise fuzzy resolvable relative to X. Then there are two pairwise fuzzy dense sets μ_1 and μ_2 in (X, T_1, T_2) with $\mu_1 \wedge \lambda \neq 0$, $\mu_2 \wedge \lambda \neq 0$ and $\mu_1 \wedge \mu_2 \wedge \lambda = 0$ in (X, T_1, T_2). Now $\mu_1 \wedge \mu_2 \wedge \lambda = 0$ in (X, T_1, T_2), implies that $\mu_1 \wedge \mu_2 \leq (1 - \lambda)$ in (X, T_1, T_2). Then $int_{T_i}int_{T_j}(\mu_1 \wedge \mu_2) \leq int_{T_i}int_{T_j}(1 - \lambda)$ and hence $int_{T_i}int_{T_j}(\mu_1 \wedge \mu_2) \leq \big(1 - cl_{T_i}cl_{T_j}(\lambda)\big)$, in (X, T_1, T_2).

Proposition 4.3 If a pairwise fuzzy dense set λ is pairwise fuzzy resolvable relative to X, then there are two pairwise fuzzy dense sets μ_1 and μ_2 in (X, T_1, T_2) such that $cl_{T_i}cl_{T_j}[(1 - \mu_1) \vee (1 - \mu_2)] = 1$, in (X, T_1, T_2) ($i \neq j$ and $i, j = 1, 2$).

Proof: Let λ be a pairwise fuzzy dense set in (X, T_1, T_2) such that λ is pairwise fuzzy resolvable relative to X. Then, by proposition 4.2, there are two pairwise fuzzy dense sets μ_1 and μ_2 in (X, T_1, T_2) such that $int_{T_i}int_{T_j}(\mu_1 \wedge \mu_2) \leq 1 - \big(cl_{T_i}cl_{T_j}(\lambda)\big)$ ($i \neq j$ and $i, j = 1, 2$) in (X, T_1, T_2). Since λ is a pairwise fuzzy dense set in (X, T_1, T_2), $cl_{T_i}cl_{T_j}(\lambda) = 1$ ($i \neq j$ and $i, j = 1, 2$) in (X, T_1, T_2) and hence $int_{T_i}int_{T_j}(\mu_1 \wedge \mu_2) \leq 1 - 1 = 0$ in (X, T_1, T_2). That is, $int_{T_i}int_{T_j}(\mu_1 \wedge \mu_2) = 0$ in (X, T_1, T_2). Then, $1 - int_{T_i}int_{T_j}(\mu_1 \wedge \mu_2) = 1 - 0 = 1$, in (X, T_1, T_2) and hence $cl_{T_i}cl_{T_j}\,(1 - (\mu_1 \wedge \mu_2)) = 1$, in (X, T_1, T_2). Therefore, $cl_{T_i}cl_{T_j}[(1 - \mu_1) \vee (1 - \mu_2)] = 1$, in (X, T_1, T_2).

Remark 4.4 If a pairwise fuzzy dense set λ in a fuzzy bitopological space (X, T_1, T_2), is pairwise fuzzy resolvable resolvable to X, then there are pairwise fuzzy dense sets μ_1 and μ_2 in (X, T_1, T_2) such that $cl_{T_i}cl_{T_j}(\lambda \vee \mu) = 1$ and $cl_{T_i}cl_{T_j}[(1 - \mu_1) \vee (1 - \mu_2)] =$

1 ($i \neq j$ and $i, j = 1, 2$), in (X, T_1, T_2). For, $cl_{T_i}cl_{T_j}(\lambda \vee \mu) = cl_{T_i}cl_{T_j}(\lambda) \vee cl_{T_i}cl_{T_j}(\mu) = 1 \vee 1 = 1$, in (X, T_1, T_2) and from proposition 4.3, $cl_{T_i}cl_{T_j}[(1-\mu_1) \vee (1-\mu_2)] = 1$, in (X, T_1, T_2).

Proposition 4.5 Let (X, T_1, T_2) be a pairwise fuzzy resolvable space. Then a fuzzy set λ in (X, T_1, T_2) is pairwise fuzzy resolvable relative to X if $int_{T_i}int_{T_j}(\lambda) \neq 0$ ($i \neq j$ and $i, j = 1, 2$).

Proof: Let (X, T_1, T_2) be a pairwise fuzzy resolvable space. Then there exists a pairwise fuzzy dense set μ in (X, T_1, T_2) such that $cl_{T_i}cl_{T_j}(1-\mu) = 1$ ($i \neq j$ and $i, j = 1, 2$), in (X, T_1, T_2). Then, $cl_{T_i}cl_{T_j}(\mu) = 1$ and $cl_{T_i}cl_{T_j}(1-\mu) = 1$ ($i \neq j$ and $i, j = 1, 2$), in (X, T_1, T_2). Let λ be a fuzzy set defined on X such that $int_{T_i}int_{T_j}(\lambda) \neq 0$ ($i \neq j$ and $i, j = 1, 2$), in (X, T_1, T_2).

Suppose that $\mu \wedge \lambda = 0$ in (X, T_1, T_2). Then $\lambda \leq (1-\mu)$ in (X, T_1, T_2) and hence $int_{T_i}int_{T_j}(\lambda) \leq int_{T_i}int_{T_j}(1-\mu)$ in (X, T_1, T_2). Then $int_{T_i}int_{T_j}(\lambda) \leq 1 - cl_{T_i}cl_{T_j}(\mu)$ and hence $int_{T_i}int_{T_j}(\lambda) \leq 1 - 1 = 0$ in (X, T_1, T_2). That is, $int_{T_i}int_{T_j}(\lambda) = 0$ in (X, T_1, T_2), a contradiction. Hence it must be that $\mu \wedge \lambda \neq 0$ in (X, T_1, T_2).

Now suppose that $(1-\mu) \wedge \lambda = 0$, in (X, T_1, T_2). Then $\lambda \leq [1-(1-\mu)]$ and hence $int_{T_i}int_{T_j}(\lambda) \leq int_{T_i}int_{T_j}[1-(1-\mu)]$ in (X, T_1, T_2). This implies that $int_{T_i}int_{T_j}(\lambda) \leq 1 - cl_{T_i}cl_{T_j}(1-\mu) = 1 - 1 = 0$ in (X, T_1, T_2). That is, $int_{T_i}int_{T_j}(\lambda) = 0$ in (X, T_1, T_2), a contradiction. Hence it must be that $(1-\mu) \wedge \lambda \neq 0$, in (X, T_1, T_2).

Now, assume that $\mu \wedge (1-\mu) \wedge \lambda \neq 0$ in (X, T_1, T_2). Then, $int_{T_i}int_{T_j}(\mu \wedge (1-\mu) \wedge \lambda) \neq 0$ and hence it will be $int_{T_i}int_{T_j}(\mu) \wedge int_{T_i}int_{T_j}(1-\mu) \wedge int_{T_i}int_{T_j}(\lambda) \neq 0$, in (X, T_1, T_2). Then, $int_{T_i}int_{T_j}(\mu) \wedge \left(1 - cl_{T_i}cl_{T_j}(\mu)\right) \wedge int_{T_i}int_{T_j}(\lambda) \neq 0$. This will imply that $int_{T_i}int_{T_j}(\mu) \wedge (1-1) \wedge int_{T_i}int_{T_j}(\lambda) \neq 0$ and hence it will be $int_{T_i}int_{T_j}(\mu) \wedge 0 \wedge int_{T_i}int_{T_j}(\lambda) \neq 0$. That is, $0 \neq 0$, a contradiction and hence $\mu \wedge (1-\mu) \wedge \lambda \neq 0$ in (X, T_1, T_2) does not hold and hence it must be that $\mu \wedge (1-\mu) \wedge \lambda = 0$ in (X, T_1, T_2). Hence there are two fuzzy dense sets μ and $1-\mu$ in (X, T_1, T_2) such that $\mu \wedge \lambda \neq 0$, $(1-\mu) \wedge \lambda \neq 0$ and $\mu \wedge (1-\mu) \wedge \lambda = 0$ in (X, T_1, T_2). Therefore, the fuzzy set λ is pairwise fuzzy resolvable relative to X in (X, T_1, T_2).

The following proposition gives a condition for a fuzzy bitopological space to be a pairwise fuzzy resolvable space.

Proposition 4.6 If there exists a fuzzy set λ defined on X such that λ is pairwise

ISSN: 2456-8686, Volume 3, Issue 2, 2019:51-62
DOI : http://doi.org/10.26524/cm54

fuzzy resolvable relative to X and $cl_{T_i}int_{T_i}[int_{T_j}(\lambda)] = 1$ $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy resolvable space.

Proof: Let λ be a fuzzy set defined on X such that $cl_{T_i}int_{T_i}[int_{T_j}(\lambda)] = 1$ $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2). Suppose that λ is pairwise fuzzy resolvable relative to X. Then, there are two pairwise fuzzy dense sets μ_1 and μ_2 in (X, T_1, T_2) such that $\mu_1 \wedge \lambda \neq 0$, $\mu_2 \wedge \lambda \neq 0$ and $\mu_1 \wedge \mu_2 \wedge \lambda = 0$, in (X, T_1, T_2). That is, $cl_{T_i}cl_{T_j}(\mu_1) = 1$ and $cl_{T_i}cl_{T_j}(\mu_2) = 1$ $(i \neq j$ and $i, j = 1, 2)$, in (X, T_1, T_2) such that $\mu_1 \wedge \lambda \neq 0$, $\mu_2 \wedge \lambda \neq 0$ and $\mu_1 \wedge \mu_2 \wedge \lambda = 0$. Now $\mu_1 \wedge \mu_2 \wedge \lambda = 0$, in (X, T_1, T_2) implies that $(\mu_1 \wedge \lambda) \leq (1 - \mu_2)$, in (X, T_1, T_2) and hence $int_{T_i}int_{T_j}(\mu_1 \wedge \lambda) \leq int_{T_i}int_{T_j}(1 - \mu_2)$, in (X, T_1, T_2). Then, $int_{T_i}int_{T_j}(\mu_1 \wedge \lambda) \leq [1 - cl_{T_i}cl_{T_j}(\mu_2)]$, in (X, T_1, T_2) and hence $int_{T_i}int_{T_j}(\mu_1 \wedge \lambda) \leq 1 - 1 = 0$, in (X, T_1, T_2). That is, $int_{T_i}int_{T_j}(\mu_1 \wedge \lambda) = 0$, in (X, T_1, T_2). Then, $int_{T_i}[int_{T_j}(\mu_1 \wedge \lambda)] = 0$, in (X, T_1, T_2) and hence, by theorem 2.3, $int_{T_i}[int_{T_j}(\mu_1) \wedge int_{T_j}(\lambda)] = 0$, in (X, T_1, T_2). Thus, $int_{T_i}int_{T_j}(\mu_1) \wedge int_{T_i}int_{T_j}(\lambda) = 0$, in (X, T_1, T_2). This implies that $int_{T_i}int_{T_j}(\lambda) \leq [1 - int_{T_i}int_{T_j}(\mu_1)]$ and hence $int_{T_i}int_{T_j}(\lambda) \leq cl_{T_i}cl_{T_j}(1 - \mu_1)$, in (X, T_1, T_2). Then, $cl_{T_i}int_{T_i}[int_{T_j}(\lambda)] \leq cl_{T_i}cl_{T_i}cl_{T_j}(1-\mu_1)$ and hence $cl_{T_i}int_{T_i}[int_{T_j}(\lambda)] \leq cl_{T_i}cl_{T_j}(1-\mu_1)$, in (X, T_1, T_2). By hypothesis, $cl_{T_i}int_{T_i}[int_{T_j}(\lambda)] = 1$ in (X, T_1, T_2). This implies that $1 \leq cl_{T_i}cl_{T_j}(1-\mu_1)$, in (X, T_1, T_2). That is, $cl_{T_i}cl_{T_j}(1-\mu_1) = 1$, in (X, T_1, T_2). Hence, there exists a pairwise fuzzy dense set μ_1 in (X, T_1, T_2) such that $cl_{T_i}cl_{T_j}(1-\mu_1) = 1$, in (X, T_1, T_2). Therefore, (X, T_1, T_2) is a pairwise fuzzy resolvable space.

Proposition 4.7 Let (X, T_1, T_2) be a fuzzy bitopological space. Then, a fuzzy set λ with $int_{T_i}int_{T_j}(\lambda) \neq 0$ $(i \neq j$ and $i, j = 1, 2)$ is pairwise fuzzy irresolvable relative to X if and only if for each pairwise fuzzy dense set μ in (X, T_1, T_2), $int_{T_i}int_{T_j}(\lambda \wedge \mu) \neq 0$, in (X, T_1, T_2).

Proof: Let the fuzzy set λ in (X, T_1, T_2) with $int_{T_i}int_{T_j}(\lambda) \neq 0$ $(i \neq j$ and $i, j = 1, 2)$ be pairwise fuzzy irresolvable relative to X. It has to be proved that $int_{T_i}int_{T_j}(\lambda \wedge \mu) \neq 0$, for each pairwise fuzzy dense set μ in (X, T_1, T_2). Assume the contrary. Suppose that $int_{T_i}int_{T_j}(\lambda \wedge \mu) = 0$, in (X, T_1, T_2), where μ is a pairwise fuzzy dense set in (X, T_1, T_2). Then $\lambda \wedge \mu \neq 0$, in (X, T_1, T_2). [For, otherwise $\lambda \wedge \mu = 0$ will imply that $\lambda \leq (1 - \mu)$ and hence $int_{T_i}int_{T_j}(\lambda) \leq int_{T_i}int_{T_j}(1 - \mu) = 1 - cl_{T_i}cl_{T_j}(\mu) = 1 - 1 = 0$ and this will imply that $int_{T_i}int_{T_j}(\lambda) = 0$, a contradiction].

Now $int_{T_i}int_{T_j}(\lambda \wedge \mu) = 0$, implies that $1 - int_{T_i}int_{T_j}(\lambda \wedge \mu) = 1 - 0 = 1$, in (X, T_1, T_2). Then, $cl_{T_i}cl_{T_j}(1 - (\lambda \wedge \mu)) = 1$, in (X, T_1, T_2). Let $\mu_1 = 1 - (\lambda \wedge$

ISSN: 2456-8686, Volume 3, Issue 2, 2019:51-62
DOI : http://doi.org/10.26524/cm54

μ). Then, μ_1 is a pairwise fuzzy dense set in (X, T_1, T_2). Then $\mu_1 \wedge \lambda \neq 0$. [For otherwise, if $\mu_1 \wedge \lambda = 0$, in (X, T_1, T_2), then $\lambda \leq (1 - \mu_1)$ in (X, T_1, T_2) and hence $int_{T_i} int_{T_j}(\lambda) \leq int_{T_i} int_{T_j}(1 - \mu_1) = 1 - cl_{T_i} cl_{T_j}(\mu_1) = 1 - 1 = 0$. This will imply that $int_{T_i} int_{T_j}(\lambda) = 0$, a contradiction]. Suppose that $\lambda \wedge \mu \wedge \mu_1 \neq 0$ in (X, T_1, T_2). Then, $int_{T_i} int_{T_j}(\lambda \wedge \mu \wedge \mu_1) \neq int_{T_i} int_{T_j}(0) = 0$ in (X, T_1, T_2). Then, it will be $int_{T_i} int_{T_j}(\lambda \wedge \mu) \wedge int_{T_i} int_{T_j}(\mu_1) \neq 0$ and hence $0 \wedge int_{T_i} int_{T_j}(\mu_1) \neq 0$. That is, $0 \neq 0$, a contradiction and hence it must be $\lambda \wedge \mu \wedge \mu_1 = 0$ in (X, T_1, T_2). Hence $\mu \wedge \lambda \neq 0$, $\mu_1 \wedge \lambda \neq 0$ and $\lambda \wedge \mu \wedge \mu_1 = 0$ in (X, T_1, T_2). where μ and μ_1 are pairwise fuzzy dense sets in (X, T_1, T_2). This will imply that λ is pairwise fuzzy resolvable relative to X, a contradiction to λ being pairwise fuzzy irresolvable relative to X. Hence the assumption that $int_{T_i} int_{T_j}(\lambda \wedge \mu) = 0$, for a pairwise fuzzy dense set μ in (X, T_1, T_2), does not hold in (X, T_1, T_2). Therefore, $int_{T_i} int_{T_j}(\lambda \wedge \mu) \neq 0$, for each pairwise fuzzy dense set μ in (X, T_1, T_2).

Conversely, let λ be a fuzzy set defined on X with $int_{T_i} int_{T_j}(\lambda) \neq 0$ and $int_{T_i} int_{T_j}(\lambda \wedge \mu) \neq 0$, in (X, T_1, T_2), for each pairwise fuzzy dense set μ in (X, T_1, T_2). Let μ_1 and μ_2 be any two pairwise fuzzy dense sets in (X, T_1, T_2). Then, by hypothesis, $int_{T_i} int_{T_j}(\lambda \wedge \mu_1) \neq 0$ and $int_{T_i} int_{T_j}(\lambda \wedge \mu_2) \neq 0$, in (X, T_1, T_2). Now $int_{T_i} int_{T_j}(\lambda \wedge \mu_1) \leq \lambda \wedge \mu_1$ and $int_{T_i} int_{T_j}(\lambda \wedge \mu_2) \leq \lambda \wedge \mu_2$, implies that $\lambda \wedge \mu_1 \neq 0$ and $\lambda \wedge \mu_2 \neq 0$ in (X, T_1, T_2). Suppose that $\lambda \wedge \mu_1 \wedge \mu_2 = 0$ in (X, T_1, T_2). Then, $\mu_1 \leq 1 - (\lambda \wedge \mu_2)$ in (X, T_1, T_2) and hence $cl_{T_i} cl_{T_j}(\mu_1) \leq cl_{T_i} cl_{T_j}[1 - (\lambda \wedge \mu_2)] = 1 - int_{T_i} int_{T_j}(\lambda \wedge \mu_2) \neq 1$ (since $int_{T_i} int_{T_j}(\lambda \wedge \mu_2) \neq 0$). Then $cl_{T_i} cl_{T_j}(\mu_1) < 1$, a contradiction to μ_1 being a pairwise fuzzy dense set in (X, T_1, T_2). Hence $\lambda \wedge \mu_1 \wedge \mu_2 \neq 0$ in (X, T_1, T_2). Thus, for any two pairwise fuzzy dense sets μ_1 and μ_2 in (X, T_1, T_2) such that $\lambda \wedge \mu_1 \neq 0$ and $\lambda \wedge \mu_2 \neq 0$, $\lambda \wedge \mu_1 \wedge \mu_2 \neq 0$ implies that the fuzzy set λ is pairwise fuzzy irresolvable relative to X.

The following proposition gives a condition for a fuzzy bitopological space to be a pairwise fuzzy irresolvable space.

Proposition 4.8 If there exists a fuzzy set λ defined on X with $int_{T_i} int_{T_j}(\lambda) \neq 0$ $(i \neq j$ and $i, j = 1, 2)$ in a fuzzy bitopological space (X, T_1, T_2) such that λ is pairwise fuzzy irresolvable relative to X, then (X, T_1, T_2) is a pairwise fuzzy irresolvable space.

Proof: Let the fuzzy set λ defined on X with $int_{T_i} int_{T_j}(\lambda) \neq 0$ $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2) be pairwise fuzzy irresolvable relative to X. Then, by proposition 4.7, for each pairwise fuzzy dense set μ in (X, T_1, T_2), $int_{T_i} int_{T_j}(\lambda \wedge \mu) \neq 0$. Then, $int_{T_i} int_{T_j}(\lambda) \wedge int_{T_i} int_{T_j}(\mu) \neq 0$ in (X, T_1, T_2). Since $int_{T_i} int_{T_j}(\lambda) \neq 0$, $int_{T_i} int_{T_j}(\mu) \neq 0$ in (X, T_1, T_2). [For otherwise, if $int_{T_i} int_{T_j}(\mu) = 0$ in (X, T_1, T_2), then $int_{T_i}int_{T_j}(\lambda \wedge \mu) = int_{T_i}int_{T_j}(\lambda) \wedge int_{T_i}int_{T_j}(\mu) = int_{T_i}int_{T_j}(\lambda) \wedge 0 = 0$, a contradiction to $int_{T_i}int_{T_j}(\lambda \wedge \mu) \neq 0$ in (X, T_1, T_2)]. Hence $1 - int_{T_i}int_{T_j}(\mu) \neq 1$ in (X, T_1, T_2). This implies that $cl_{T_i}cl_{T_j}(1 - \mu) \neq 1$ in (X, T_1, T_2). Thus, for the pairwise fuzzy dense set μ in (X, T_1, T_2), $cl_{T_i}cl_{T_j}(1 - \mu) \neq 1$ $(i \neq j$ and $i, j = 1, 2)$ in (X, T_1, T_2), implies that (X, T_1, T_2) is a pairwise fuzzy irresolvable space.

5. Conclusion

In this paper, the concept of pairwise fuzzy resolvable spaces have studied extensively. Relativization in pairwise fuzzy resolvability and pairwise fuzzy irresolvability have established in this paper.

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