



## Solving Unbalanced Intuitionistic Fuzzy Transportation Problem

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### Abstract

In this paper, we find the optimal solution for an unbalanced intuitionistic fuzzy transportation problem by using monalisha's approximation method. The main aim of this method is to avoid large number of iterations. To illustrate this method a numerical example is given.

**Key words:** Triangular intuitionistic fuzzy number, unbalanced intuitionistic fuzzy transportation problem, accuracy function.

**AMS classification:** 03E72, 03F55, 90B06

### 1. Introduction

The concept of fuzzy set theory introduced by Zadeh [7] was extended to intuitionistic fuzzy sets (IFS) by Atanassov [1]. Intuitionistic fuzzy set is a tool in modelling real life problems like sale analysis, new product marketing, financial services, negotiation process, psychological investigations etc., Intuitionistic fuzzy set has greater influence in solving transportation problem. A Transportation problem is one kind of linear programming problem. The basic transportation problem was originally developed by Hitchcock [2]. The main goal of transportation problem is to minimize the transportation cost of transporting commodities from sources to sinks. If the quantities (transportation cost, supply and demand) are intuitionistic fuzzy then it is a intuitionistic fuzzy transportation problem (IFTP). Nagoor et al. [3, 4] solved IFTP by various methods. By using monalisha's approximation method, find an optimal solution for balanced IFTP has been studied by number of authors [5, 6].

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In this paper, we solve unbalanced intuitionistic fuzzy transportation problems, as it requires least iterations to reach optimality. The procedure for the solution is illustrated with a numerical example.

## 2. Preliminaries

**Definition 2.1** An Intuitionistic fuzzy set (IFS)  $\bar{A}^I$  in  $X$  is given by a set of ordered triples :

$$\bar{A}^I = \langle x, \mu_{\bar{A}^I}(x), \nu_{\bar{A}^I}(x) \rangle / x \in X,$$

where  $\mu_{\bar{A}^I}(x), \nu_{\bar{A}^I}(x) : X \rightarrow [0, 1]$  are functions such that  $0 \leq \mu_{\bar{A}^I}(x) + \nu_{\bar{A}^I}(x) \leq 1$  for all  $x \in X$ . For each  $x$  the numbers  $\mu_{\bar{A}^I}(x)$  and  $\nu_{\bar{A}^I}(x)$  represent the degree of membership and degree of non-membership of the element  $x \in X$  to  $A \subset X$ , respectively.

**Definition 2.2** An intuitionistic fuzzy subset  $\bar{A}^I = \langle x, \mu_{\bar{A}^I}(x), \nu_{\bar{A}^I}(x) \rangle / x \in X$ , of the real line  $R$  is called an Intuitionistic Fuzzy Number (IFN) if the following holds:

(i) There exist  $m \in R$ ,  $\mu_{\bar{A}^I}(m) = 1$  and  $\nu_{\bar{A}^I}(m) = 0$ .

(ii)  $\mu_{\bar{A}^I}$  is a continuous mapping from  $R$  to the closed interval  $[0, 1]$  and  $x \in R$ , the relation  $0 \leq \mu_{\bar{A}^I}(x) + \nu_{\bar{A}^I}(x) \leq 1$  holds.

**Definition 2.3** A Triangular Intuitionistic Fuzzy Number  $\bar{d}^I$  is an intuitionistic fuzzy set in  $R$  with the following membership function  $\mu_{\bar{d}^I}$  and non membership function  $\nu_{\bar{d}^I}$ :

$$\mu_{\bar{d}^I}(x) = \begin{cases} \frac{x-d_1}{d_2-d_1}, & d_1 < x < d_2 \\ \frac{d_3-x}{d_3-d_2}, & d_2 \leq x < d_3 \\ 0, & \text{otherwise} \end{cases}$$

and

$$\nu_{\bar{d}^I}(x) = \begin{cases} \frac{d_2-x}{d_2-d'_1}, & d'_1 < x \leq d_2 \\ \frac{x-d_2}{d'_3-d_2}, & d_2 \leq x < d'_3 \\ 1, & \text{otherwise} \end{cases}$$

Where  $d_1 \leq d_2 < d_3 \leq d'_3$ . Triangular intuitionistic fuzzy number  $\bar{d}^I$  is denoted by  $(d_1, d_2, d_3; d'_1, d_2, d'_3)$ .

**Definition 2.4** We define a accuracy function

$$H(\bar{d}^I) = \frac{(d_1 + 2d_2 + d_3) + (d'_1 + 2d'_2 + d'_3)}{8}$$

to defuzzify a given triangular intuitionistic fuzzy number.

**Operations on triangular intuitionistic fuzzy number**

Let  $\bar{d}^I = (d_1, d_2, d_3; d'_1, d'_2, d'_3)$  and  $\bar{e}^I = (e_1, e_2, e_3; e'_1, e'_2, e'_3)$  be two triangular intuitionistic fuzzy numbers the arithmetic operations on  $\bar{d}^I$  and  $\bar{e}^I$  is given below:

**Addition:**  $\bar{d}^I + \bar{e}^I = (d_1 + e_1, d_2 + e_2, d_3 + e_3; d'_1 + e'_1, d'_2 + e'_2, d'_3 + e'_3)$

**Subtraction:**  $\bar{d}^I - \bar{e}^I = (d_1 - e_1, d_2 - e_2, d_3 - e_3; d'_1 - e'_1, d'_2 - e'_2, d'_3 - e'_3)$

**Multiplication:**  $\bar{d}^I \times \bar{e}^I = (d_1e_3, d_2e_2, d_3e_1; d'_1e'_3, d'_2e'_2, d'_3e'_1)$

**Scalar Multiplication:**

(i) If  $l > 0$  then  $l\bar{d}^I = (ld_1, ld_2, ld_3; ld'_1, ld'_2, ld'_3)$

(ii) If  $l < 0$  then  $l\bar{d}^I = (ld_3, ld_2, ld_1; ld'_3, ld'_2, ld'_1)$

**3. Intuitionistic fuzzy transportation problem (IFTP)**

Consider a transportation problem with  $m$  intuitionistic fuzzy (IF) origins and  $n$  IF destination. Let  $C_{ij}(i = 1, 2, \dots, m, j = 1, 2, \dots, n)$  be the cost of transporting one unit of the product from  $i$ th origin to  $j$ th destination. Let  $\bar{d}_i^I(i = 1, 2, \dots, m)$  be the quantity of commodity available at IF origin  $i$ . Let  $\bar{e}_j^I(j = 1, 2, \dots, n)$  be the quantity of commodity needed of IF destination  $j$ . Let  $X_{ij}(i = 1, 2, \dots, m, j = 1, 2, \dots, n)$  is quantity transported from  $i$ th IF origin to  $j$ th IF destination.

Mathematical Model of Intuitionistic Fuzzy Transportation Problem is

$$\text{Minimize } \bar{Z}^I = \sum_{i=1}^m \sum_{j=1}^n \bar{x}_{ij}^I \bar{c}_{ij}^I,$$

subject to

$$\sum_{i=1}^m \bar{x}_{ij}^I = \bar{d}_i^I$$

$$\sum_{j=1}^n \bar{x}_{ij}^I = \bar{e}_j^I$$

$\bar{x}_j^I$  for all  $i$  and  $j$ .

**4. Monalisha’s approximation method**

**Step 1:** Determine the cost table from the given problem. (i) Examine whether total demand equals total demand. If yes, go to step 2. (ii) If not, introduce a dummy

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row/column having all its cost elements as zero and supply/ demand as the (+ve) difference of supply and demand.

**Step 2:** Locate the smallest element in each row of the given cost matrix and then subtract the same from each element of that row.

**Step 3:** In the reduced matrix obtained in step 2, locate the smallest element of each column and then subtract the same from each element of that column.

**Step 4:** For each row of the transportation table identify the smallest and the next - to - smallest costs. Determine the difference between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly compute the differences for each column.

**Step 5:** Identify the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tiebreaking choice. Let the greatest difference correspond to  $i$ th row and let 0 be in the  $i$ th row. Allocate the maximum feasible amount  $x_{ij} = \min(a_i, b_j)$  in the  $(i, j)$ th cell and cross off either the  $i$ th row or the  $j$ th column in the usual manner.

**Step 6:** Recompute the column and row differences for the reduced transportation table and go to step 5. Repeat the procedure until all the rim requirements (the various origin capacities and destination requirements are listed in the right most outer column and the bottom outer row respectively) are satisfied.

## 5. Numerical example

**Example 5.1** Consider the following  $3 \times 3$  intuitionistic fuzzy transportation problem whose quantities are triangular intuitionistic fuzzy numbers.

	$D_1$	$D_2$	$D_3$	Supply
$S_1$	(5, 6, 7; 4, 6, 8)	(7, 8, 9; 6, 8, 10)	(2, 4, 6; 1, 4, 7)	(13, 14, 15; 12, 14, 16)
$S_2$	(2, 4, 6; 1, 4, 7)	(8, 9, 10; 7, 9, 11)	(7, 8, 9; 6, 8, 10)	(11, 12, 13; 10, 12, 14)
$S_3$	(0, 1, 2; -1, 1, 3)	(1, 2, 3; 0, 2, 4)	(5, 6, 7; 4, 6, 8)	(4, 5, 6; 3, 5, 7)
Demand	(4, 6, 8; 2, 6, 10)	(14, 15, 16; 13, 15, 17)	(14, 15, 16; 13, 15, 17)	

**Solution:**

By Defuzzifying the quantities we get,

$$a_{11} = 6, a_{12} = 8, a_{13} = 4, a_{21} = 4, a_{22} = 9, a_{23} = 8, a_{31} = 1, a_{32} = 2 \text{ and } a_{33} = 6.$$

Since  $\sum_{i=1}^3 a_i \neq \sum_{j=1}^3 b_j$ . So, the given problem is the unbalanced intuitionistic fuzzy transportation problem.

**Iteration-1:** Using step 1, we have

	$D_1$	$D_2$	$D_3$	Supply
$S_1$	6	8	4	(13, 14, 15; 12, 14, 16)
$S_2$	4	9	8	(11, 12, 13; 10, 12, 14)
$S_3$	1	2	6	(4, 5, 6; 3, 5, 7)
$S_4$	0	0	0	(4, 5, 6; 3, 5, 7)
Demand	(4, 6, 8; 2, 6, 10)	(14, 15, 16; 13, 15, 17)	(14, 15, 16; 13, 15, 17)	

Now,  $\sum_{i=1}^3 a_i = \sum_{j=1}^3 b_j$ . so go to step 2.

**Iteration-2:** Using step 2 and step 3, we get

	$D_1$	$D_2$	$D_3$	Supply
$S_1$	2	4	0	(13, 14, 15; 12, 14, 16)
$S_2$	0	5	4	(11, 12, 13; 10, 12, 14)
$S_3$	0	1	5	(4, 5, 6; 3, 5, 7)
$S_4$	0	0	0	(4, 5, 6; 3, 5, 7)
Demand	(4, 6, 8; 2, 6, 10)	(14, 15, 16; 13, 15, 17)	(14, 15, 16; 13, 15, 17)	

**Iteration-3:**

applying step 4, we arrive at

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	$D_1$	$D_2$	$D_3$	Supply	row penalty
$S_1$	2	4	0	(13, 14, 15; 12, 14, 16)	(2)
$S_2$	0	5	4	(11, 12, 13; 10, 12, 14)	(4)
$S_3$	0	1	5	(4, 5, 6; 3, 5, 7)	(1)
$S_4$	0	0	0	(4, 5, 6; 3, 5, 7)	(0)
Demand	(4, 6, 8; 2, 6, 10)	(14, 15, 16; 13, 15, 17)	(14, 15, 16; 13, 15, 17)		
column penalty	(0)	(1)	(0)		

**Iteration-4:**

applying step 5, we get

	$D_1$	$D_2$	$D_3$	Supply	row penalty
$S_1$	2	4	0	(13, 14, 15; 12, 14, 16)	(2)
$S_2$	0(4, 6, 8; 2,6,10)	5	4	(11, 12, 13; 10, 12, 14)	(4)
$S_3$	0	1	5	(4, 5, 6; 3, 5, 7)	(1)
$S_4$	0	0	0	(4, 5, 6; 3, 5, 7)	(0)
Demand	(4, 6, 8; 2, 6, 10)	(14, 15, 16; 13, 15, 17)	(14, 15, 16; 13, 15, 17)		
column penalty	(0)	(1)	(0)		

Go to step 6, we get the table

	$D_1$	$D_2$	$D_3$	Supply
$S_1$	(5, 6, 7; 4, 6, 8)	(7, 8, 9; 6, 8, 10)	(2, 4, 6; 1, 4, 7) (13,14,15;12,14,16)	(13, 14, 15; 12,14,16)
$S_2$	(2, 4, 6; 1, 4, 7) (4, 6, 8; 2, 6, 10)	(8, 9, 10; 7, 9, 11) (2, 5, 8; -1, 5, 11)	(7, 8, 9; 6, 8, 10) (-1, 6, 3; -3, 1, 5)	(11, 12, 13; 10,12,14)
$S_3$	(0, 1, 2; -1, 1, 3)	(1, 2, 3; 0, 2, 4) (4, 5, 6; 3, 5, 7)	(5, 6, 7; 4, 6, 8)	(4, 5, 6; 3,5,7)
Demand	(4, 6, 8; 2, 6, 10)	(14, 15, 16; 13, 15, 17)	(14, 15, 16; 13, 15, 17)	

$$\begin{aligned}
 \text{Total Minimum Cost} &= (2, 4, 6; 1, 4, 7)(13, 14, 15; 12, 14, 16) + \\
 &\quad (2, 4, 6; 1, 4, 7)(4, 6, 8; 2, 6, 10) + \\
 &\quad (8, 9, 10; 7, 9, 11)(2, 5, 8; -1, 5, 11) + \\
 &\quad (7, 8, 9; 6, 8, 10)(-1, 1, 3; -3, 1, 5) + \\
 &\quad (1, 2, 3; 0, 2, 4)(4, 5, 6; 3, 5, 7) \\
 &= (47, 143, 263; -11, 143, 381) \\
 H(\vec{d}') &= 156.50
 \end{aligned}$$

## 6. Conclusion

The main contribution of this paper is to derive the optimal solution of a triangular intuitionistic fuzzy transportation problem using Monalishas approximation method with fewer steps in comparison to other methods. Using this method we could get the optimum solution directly. This method is very helpful for the decision makers, since the methodology is very simple and takes less number of iterations.

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