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On Fuzzy Regular Volterra Spaces

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Abstract

The aim of this paper is to introduce the concepts of regular G_δ-sets, regular F_σ-sets and regular Volterra spaces in fuzzy setting are introduced and studied. Several characterizations of fuzzy regular Volterra spaces in terms of fuzzy regular F_σ-sets, fuzzy first category sets, fuzzy residual sets and fuzzy σ-nowhere dense sets are also established in this paper.

Key words: Fuzzy open set, fuzzy dense set, fuzzy nowhere dense set, fuzzy σ-nowhere dense set, fuzzy G_δ-set, fuzzy F_σ-set, fuzzy first category set, fuzzy residual set, fuzzy β-open set and fuzzy Volterra spaces.

AMS classification: 54 A 40, 03 E 72

1.Introduction

In 1970, J.Mack [6] introduced the concepts of regular G_δ-sets and regular F_σ-sets in classical topology. K.K.Azad [1] introduced fuzzy regular open and fuzzy regular closed sets in 1981. The concepts of regular G_δ-sets and regular F_σ-sets in fuzzy setting are introduced and studied in this paper. By using fuzzy regular G_δ-sets, the concept of fuzzy regular Volterra spaces is introduced in this paper. Several characterizations of fuzzy regular Volterra spaces in terms of fuzzy regular F_σ-sets, fuzzy first category sets, fuzzy residual sets and fuzzy σ-nowhere dense sets are also established in this paper.

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2.Preliminaries

In 1965, L.A.Zadeh [10] introduced the concept of fuzzy set λ on a base set X as a function from X into the unit interval $I = [0, 1]$. This function is also called a membership function. A membership function is a generalization of a characteristic function.

Definition 2.1 [5] Let λ and μ be fuzzy sets in X. Then for all $x \in X$,

1. $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$,
2. $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$,
3. $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = max\{\lambda(x), \mu(x)\}$,
4. $\delta = \lambda \wedge \mu \Leftrightarrow \delta(x) = min\{\lambda(x), \mu(x)\}$,
5. $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in X, the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$ are defined by $\psi(x) = sup_i\{\lambda_i(x), x \in X\}$, and $\delta(x) = inf_i\{\lambda_i(x), x \in X\}$.

The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.2 [5] A fuzzy topology is a family 'T'of fuzzy sets in X which satisfies the following conditions:

(1) $\Phi, X \in T$,
(2) If $A, B \in T$, then $A \cap B \in T$,
(3) If $A_i \in T$, for each $i \in I$, then $\cup_{i \in I} A_i \in T$.

T is called a fuzzy topology for X and the pair (X, T) is a fuzzy topological space or fts in short. Every member of T is called a T-open fuzzy set. A fuzzy set is T-closed if and only if its complement is T-open. When no confusion is likely to arise, we shall call a T-open (T-closed) fuzzy set simply an open (closed) fuzzy set.

Lemma 2.3 [1] For a family $\mathcal{A} = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X. Then, $\vee\ cl\ \lambda_\alpha \leq cl(\vee \lambda_\alpha)$. In case $\mathcal{A}$ is a finite set, $\vee\ cl\ \lambda_\alpha = cl(\vee \lambda_\alpha)$. Also $\vee\ int\ \lambda_\alpha \leq int(\vee \lambda_\alpha)$.

Definition 2.4 [2] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy F_σ-set in (X, T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1 - \lambda_i \in T$ for $i \in I$.

Definition 2.5 [2] A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy G_δ-set in (X,T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$ for $i \in I$.

Definition 2.6 [7] A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy dense set if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$.

Definition 2.7 [7] Let (X,T) be a fuzzy topological space. A fuzzy set λ in (X,T) is called a fuzzy nowhere dense set if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is, $int\ cl(\lambda) = 0$.

Definition 2.8 [7] Let (X,T) be a fuzzy topological space. A fuzzy set λ in (X,T) is called a fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category.

Definition 2.9 [7] Let λ be a fuzzy first category set in a fuzzy topological space (X,T). Then $1-\lambda$ is called a fuzzy residual set in (X,T).

Definition 2.10 [8] Let (X,T) be a fuzzy topological space. A fuzzy set λ in (X,T) is called a fuzzy σ-nowhere dense set if λ is a fuzzy F_σ-set in (X,T) such that $int(\lambda) = 0$.

Definition 2.11 A fuzzy set λ in a fuzzy topological space X is called

(1) fuzzy pre-open if $\lambda \le intcl(\lambda)$ and fuzzy pre-closed if $clint(\lambda) \le \lambda$ [4].
(2) fuzzy semi-open if $\lambda \le clint(\lambda)$ and fuzzy semi-closed if $intcl(\lambda) \le \lambda$ [4].
(3) fuzzy β-open if $\lambda \le clintcl(\lambda)$ and fuzzy β-closed if $intclint(\lambda) \le \lambda$ [3].
(4) fuzzy regular open if $intcl(\lambda) = \lambda$ and fuzzy regular closed if $clint(\lambda) = \lambda$ [1].

Definition 2.12 [9] A fuzzy topological space (X,T) is called a fuzzy Volterra space if $cl\big(\wedge_{i=1}^{N}(\lambda_i)\big) = 1$, where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T).

Theorem 2.13 [1] In a fuzzy topological space (X,T),

(a). The closure of a fuzzy open set is a fuzzy regular closed set
(b). The interior of a fuzzy closed set is a fuzzy regular open set.

3.Fuzzy regular G_δ-sets

Definition 3.1 A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy regular G_δ-set if $\lambda = \wedge_{i=1}^{\infty}\big(int(\lambda_i)\big)$, where $1-\lambda_i \in T$.

Definition 3.2 A fuzzy set μ in a fuzzy topological space (X,T) is called a fuzzy regular F_σ-set if $\mu = \vee_{i=1}^{\infty}\big(cl(\mu_i)\big)$, where $\mu_i \in T$.

Proposition 3.3 If λ is a fuzzy regular G_δ-set in a fuzzy topological space (X,T) if and only if $1-\lambda$ is a fuzzy regular F_σ-set in (X,T).

Proof: Let λ be a fuzzy regular G_δ-set in (X,T). Then $\lambda = \wedge_{i=1}^{\infty}\big(int(\lambda_i)\big)$, where $1-\lambda_i \in T$. Now $1-\lambda = 1-\wedge_{i=1}^{\infty}\big(int(\lambda_i)\big) = \vee_{i=1}^{\infty}\big(1-int(\lambda_i)\big) = \vee_{i=1}^{\infty}\big(cl(1-\lambda_i)\big)$. Let $\mu_i = 1-\lambda_i$. Then $\mu_i \in T$. Hence $1-\lambda = \vee_{i=1}^{\infty}\big(cl(\mu_i)\big)$, $\mu_i \in T$. Therefore $1-\lambda$ is a fuzzy regular F_σ-set in (X,T).

Conversely, let λ be a fuzzy regular F_σ-set in (X,T). Then $\lambda = \vee_{i=1}^{\infty}\big(cl(\mu_i)\big)$, where $\mu_i \in T$. Now $1-\lambda = 1-\vee_{i=1}^{\infty}\big(cl(\mu_i)\big) = \wedge_{i=1}^{\infty}\big(1-cl(\mu_i)\big) = \wedge_{i=1}^{\infty}\big(int(1-\mu_i)\big)$. Let $1-\mu_i = \lambda_i$. Then implies that $\mu_i = 1-\lambda_i$ and $1-\lambda_i \in T$. Hence $1-\lambda = \wedge_{i=1}^{\infty}\big(int(\lambda_i)\big)$, where $1-\lambda_i \in T$. Therefore $1-\lambda$ is a fuzzy regular G_δ-set in (X,T).

Proposition 3.4 Let (X,T) be a fuzzy topological space.

(1). If λ is a fuzzy regular G_δ-set in (X,T), then $\lambda = \wedge_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy regular open sets in (X,T).

(2). If λ is a fuzzy regular F_σ-set in (X,T), then $\lambda = \vee_{i=1}^{\infty}(\mu_i)$, where (μ_i)'s are fuzzy regular closed sets in (X,T).

Proof: (1). Let λ be a fuzzy regular G_δ-set in (X,T). Then $\lambda = \wedge_{i=1}^{\infty}\big(int(\lambda_i)\big)$, where $1-\lambda_i \in T$. Now $1-\lambda_i \in T$ implies that λ_i is a fuzzy closed set in (X,T). By theorem 2.13, $int(\lambda_i)$ is a fuzzy regular open set in (X,T). Let $\delta_i = int(\lambda_i)$. Then $\lambda = \wedge_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy regular open sets in (X,T).

(2). Let λ be a fuzzy regular F_σ-set in (X,T). Then $\lambda = \vee_{i=1}^{\infty}\big(cl(\mu_i)\big)$, where $\mu_i \in T$. Now $\mu_i \in T$. By theorem 2.13, $cl(\mu_i)$ is a fuzzy regular closed set in (X,T). Let $\eta_i = cl(\mu_i)$. Then $\lambda = \vee_{i=1}^{\infty}(\eta_i)$, where (η_i)'s are fuzzy regular closed sets in (X,T).

Proposition 3.5 If λ is a fuzzy regular G_δ-set in a fuzzy topological space (X,T), then λ is a fuzzy G_δ-set in (X,T).

Proof: Let λ be a fuzzy regular G_δ-set in (X,T). Then by proposition 3.4, $\lambda = \wedge_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy regular open sets in (X,T). Since every fuzzy regular

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open set is a fuzzy open set in (X,T), (δ_i)'s are fuzzy open sets in (X,T). Hence $\lambda = \wedge_{i=1}^{\infty}(\delta_i)$, where $\delta_i \in T$. Therefore λ is a fuzzy G_δ-set in (X,T).

Proposition 3.6 If λ is a fuzzy regular F_σ-set in a fuzzy topological space (X,T), then λ is a fuzzy F_σ-set in (X,T).

Proof: Let λ be a fuzzy regular F_σ-set in (X,T). Then by proposition 3.4, $\lambda = \vee_{i=1}^{\infty}(\eta_i)$, where (η_i)'s are fuzzy regular closed sets in (X,T). Since every fuzzy regular closed set is a fuzzy closed set in (X,T), (η_i)'s are fuzzy closed sets in (X,T). Hence $\lambda = \vee_{i=1}^{\infty}(\eta_i)$, where $1-\eta_i \in T$. Therefore λ is a fuzzy F_σ-set in (X,T).

Proposition 3.7 If $cl(\wedge_{i=1}^{\infty} int(\lambda_i)) = 1$, where (λ_i)'s are fuzzy closed sets in a fuzzy topological space (X,T), then (λ_i)'s are fuzzy β-open sets in (X,T).

Proof: Suppose that $cl(\wedge_{i=1}^{\infty} int(\lambda_i)) = 1$, where (λ_i)'s are fuzzy closed sets in (X,T). But $cl(\wedge_{i=1}^{\infty} int(\lambda_i)) \leq \wedge_{i=1}^{\infty} clint(\lambda_i)$. Then, $1 \leq \wedge_{i=1}^{\infty} clint(\lambda_i)$. That is, $\wedge_{i=1}^{\infty} clint(\lambda_i) = 1$. This implies that $clint(\lambda_i) = 1$......(1). Since (λ_i)'s are fuzzy closed sets in (X,T), $cl(\lambda_i) = \lambda_i$. Then $clintcl(\lambda_i) = 1$. From (1), $\lambda_i \leq clintcl(\lambda_i)$. Therefore, (λ_i)'s are fuzzy β-open sets in (X,T).

Proposition 3.8 If a fuzzy regular G_δ-set λ is a fuzzy dense set in a fuzzy topological space (X,T), then $\lambda = \wedge_{i=1}^{\infty} int(\lambda_i)$, where (λ_i)'s are fuzzy β-open sets in (X,T).

Proof: Let λ be a fuzzy regular G_δ-set in (X,T) such that $cl(\lambda) = 1$. Then $\lambda = \wedge_{i=1}^{\infty} int(\lambda_i)$, where (λ_i)'s are fuzzy closed sets in (X,T) and $cl[\wedge_{i=1}^{\infty} int(\lambda_i)] = cl(\lambda) = 1$. Then, by proposition 3.7, (λ_i)'s are fuzzy β-open sets in (X,T). Therefore, $\lambda = \wedge_{i=1}^{\infty} int(\lambda_i)$, where (λ_i)'s are fuzzy β-open sets in (X,T).

Proposition 3.9 If $int(\mu) = 0$, where μ is a fuzzy regular F_σ-set in a fuzzy topological space (X,T), then $\mu = \vee_{i=1}^{\infty} cl(\lambda_i)$, where (λ_i)'s are fuzzy β-closed sets in (X,T).

Proof: Let μ be a fuzzy regular F_σ-set in (X,T) such that $int(\mu) = 0$. Then, by proposition 3.3, $1-\mu$ is a fuzzy regular G_δ-set in (X,T) and $cl(1-\mu) = 1 - int(\mu) = 1 - 0 = 1$. Now, by proposition 3.8, $1-\mu = \wedge_{i=1}^{\infty} int(\mu_i)$, where (μ_i)'s are fuzzy β-open sets in (X,T). Hence $\mu = 1 - \wedge_{i=1}^{\infty} int(\mu_i) = \vee_{i=1}^{\infty}\big(1 - int(\mu_i)\big) =$

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$\vee_{i=1}^{\infty} cl(1-\mu_i)$. Since (μ_i)'s are fuzzy β-open sets, $(1-\mu_i)$'s are fuzzy β-closed sets in (X,T). Let $\lambda_i = 1-\mu_i$. Therefore, $\mu = \vee_{i=1}^{\infty} cl(\lambda_i)$, where (λ_i)'s are fuzzy β-closed sets in (X,T).s

4.Fuzzy regular Volterra spaces

Definition 4.1 A fuzzy topological space (X,T) is called a fuzzy regular Volterra space if $cl\big(\wedge_{i=1}^{N}(\lambda_i)\big) = 1$, where (λ_i)'s are fuzzy dense and fuzzy regular G_δ-sets in (X,T).

Proposition 4.2 If $int\big(\vee_{i=1}^{N}(\mu_i)\big) = 0$ where (μ_i)'s are fuzzy regular F_σ-sets with $int(\mu_i) = 0$ in a fuzzy topological space (X,T), then (X,T) is a fuzzy regular Volterra space.

Proof: Suppose that $int\big(\vee_{i=1}^{N}(\mu_i)\big) = 0$, where (μ_i)'s are fuzzy regular F_σ-sets with $int(\mu_i) = 0$. Now $1 - int\big(\vee_{i=1}^{N}(\mu_i)\big) = 1$. Then, $cl\big(1 - \vee_{i=1}^{N}(\mu_i)\big) = 1$. This implies that $cl\big(\wedge_{i=1}^{N}(1-\mu_i)\big) = 1$. Since (μ_i)'s are fuzzy regular F_σ-sets in (X,T), by proposition 3.3, $(1-\mu_i)$'s are fuzzy regular G_δ-sets in (X,T). Also, $int(\mu_i) = 0$ implies that $1 - int(\mu_i) = 1$. Then, $cl(1-\mu_i) = 1$. Let $\lambda_i = 1-\mu_i$. Then (λ_i)'s are fuzzy dense and fuzzy regular G_δ-sets in (X,T). Hence, $cl\big(\wedge_{i=1}^{N}(\lambda_i)\big) = 1$, where (λ_i)'s are fuzzy dense and fuzzy regular G_δ-sets in (X,T). Therefore (X,T) is a fuzzy regular Volterra space.

Remark: In view of the propositions 3.9 and 4.2, one will have the following result : "If $int\big(\vee_{i=1}^{\infty}(\eta_i)\big) = 0$, where (η_i)'s are fuzzy β-closed sets in a fuzzy topological space (X,T), then (X,T) is a fuzzy regular Volterra space".

Proposition 4.3 If a fuzzy topological space (X,T) is a fuzzy Volterra space, then (X,T) is a fuzzy regular Volterra space.

Proof: Let (X,T) be a fuzzy Volterra space. Let $\lambda = cl\big(\wedge_{i=1}^{N}(\lambda_i)\big)$.....(1), where (λ_i)'s are fuzzy dense and fuzzy regular G_δ-sets in (X,T). By proposition 3.5, the fuzzy regular G_δ-sets (λ_i)'s are fuzzy G_δ-sets in (X,T). Since (X,T) is a fuzzy Volterra space, $cl\big(\wedge_{i=1}^{N}(\lambda_i)\big) = 1$....(2), where (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X,T). Hence, from (1) and (2), $\lambda = 1$. Therefore (X,T) is a fuzzy regular Volterra space.

Proposition 4.4 If a fuzzy topological space (X,T) is a fuzzy regular Volterra space, then $int\big(\vee_{i=1}^{N}(\mu_i)\big) = 0$, where (μ_i)'s are fuzzy σ-nowhere dense sets in (X,T).

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Proof: Let (X,T) be a fuzzy regular Volterra space. Then $cl\big(\wedge_{i=1}^{N}(\lambda_i)\big) = 1$, where (λ_i)'s are fuzzy dense and fuzzy regular G_δ-sets in (X,T). Now $1 - cl\big(\wedge_{i=1}^{N}(\lambda_i)\big) = 0$ implies that $int\big(\vee_{i=1}^{N}(1-\lambda_i)\big) = 0$. Since (λ_i)'s are fuzzy regular G_δ-sets, by proposition 3.3, $(1-\lambda_i)$'s are fuzzy regular F_σ-sets in (X,T). By proposition 3.6, $(1-\lambda_i)$'s are fuzzy F_σ-sets in (X,T). Also, $cl(\lambda_i) = 1$ implies that $1 - cl(\lambda_i) = 0$ and hence $int(1-\lambda_i) = 0$. Let $\mu_i = 1 - \lambda_i$. Then (μ_i)'s are fuzzy F_σ-sets with $int(\mu_i) = 0$. Then, by the definition of fuzzy σ-nowhere dense sets, (μ_i)'s are fuzzy σ-nowhere dense sets in (X,T). Hence $int\big(\vee_{i=1}^{N}(\mu_i)\big) = 0$, where (μ_i)'s are fuzzy σ-nowhere dense sets in (X,T).

Proposition 4.5 If $int(\lambda) = 0$ for a fuzzy regular F_σ-set λ in a fuzzy topological space (X,T), then λ is a fuzzy first category set in (X,T).

Proof: Let λ be a fuzzy regular F_σ-set in (X,T). Then $\lambda = \vee_{i=1}^{\infty}\big(cl(\mu_i)\big)$, where $\mu_i \in T$. Now $int(\lambda) = 0$ implies that $int\big(\vee_{i=1}^{\infty}\big(cl(\mu_i)\big)\big) = 0$. But $\vee_{i=1}^{\infty}\big(int\big(cl(\mu_i)\big)\big) \le int\big(\vee_{i=1}^{\infty}\big(cl(\mu_i)\big)\big) = 0$. Then, $\vee_{i=1}^{\infty}\big(int\big(cl(\mu_i)\big)\big) = 0$. This implies that $int\big(cl(\mu_i)\big) = 0$. Hence μ_i is a fuzzy nowhere dense set in (X,T). Also $int\big(cl\big(cl(\mu_i)\big)\big) = int\big(cl(\mu_i)\big) = 0$ implies that $cl(\mu_i)$ is a fuzzy nowhere dense set in (X,T). Hence $\lambda = \vee_{i=1}^{\infty}\big(cl(\mu_i)\big)$, where $\big(cl(\mu_i)\big)$'s are fuzzy nowhere dense sets in (X,T). Therefore λ is a fuzzy first category set in (X,T).

Remark: In view of the propositions 3.9 and 4.5, one will have the following result : "If $int(\lambda) = 0$, for a fuzzy regular F_σ-set in a fuzzy topological space (X,T), then $\lambda = \vee_{i=1}^{\infty}\big(cl(\lambda_i)\big)$, where (λ_i)'s are fuzzy β-closed sets in (X,T), is a fuzzy first category set in (X,T)".

Proposition 4.6 If a fuzzy regular G_δ-set λ is a fuzzy dense set in a fuzzy topological space (X,T), then λ is a fuzzy residual set in (X,T).

Proof: Let λ be a fuzzy regular G_δ-set with $cl(\lambda) = 1$. Then $1-\lambda$ is a fuzzy regular F_σ-set with $1 - cl(\lambda) = 0$. That is, $1-\lambda$ is a fuzzy regular F_σ-set with $int(1-\lambda) = 0$. Then by proposition 4.5, $1-\lambda$ is a fuzzy first category set in (X,T). Therefore λ is a fuzzy residual set in (X,T).

Proposition 4.7 If a fuzzy topological space (X,T) is a fuzzy regular Volterra space, then $cl\big(\wedge_{i=1}^{N}(\lambda_i)\big) = 1$ where (λ_i)'s are fuzzy residual sets in (X,T).

Proof: Let (X,T) be a fuzzy regular Volterra space. Then $cl\big(\wedge_{i=1}^{N}(\lambda_i)\big) = 1$, where (λ_i)'s are fuzzy dense and fuzzy regular G_δ-sets in (X,T). By proposition 4.6,

(λ_i)'s are fuzzy residual sets in (X, T). Hence $cl\big(\wedge_{i=1}^{N} (\lambda_i)\big) = 1$, where (λ_i)'s are fuzzy residual sets in (X, T).

Proposition 4.8 If a fuzzy topological space (X, T) is a fuzzy regular Volterra space, then $int\big(\vee_{i=1}^{N} (\mu_i)\big) = 0$, where (μ_i)'s are fuzzy first category sets in (X, T).

Proof: Let (X, T) be a fuzzy regular Volterra space. Then $cl\big(\wedge_{i=1}^{N} (\lambda_i)\big) = 1$, where (λ_i)'s are fuzzy dense and fuzzy regular G_δ-sets in (X, T). Now $1 - cl\big(\wedge_{i=1}^{N} (\lambda_i)\big) = 0$ implies that $int\big(1 - \wedge_{i=1}^{N}(\lambda_i)\big) = 0$. Then, $int\big(\vee_{i=1}^{N} (1 - \lambda_i)\big) = 0$.

Now (λ_i)'s are fuzzy regular G_δ-sets in (X, T) implies that $(1 - \lambda_i)$'s are fuzzy regular F_σ-sets in (X, T). Also $cl(\lambda_i) = 1$ implies that $1 - cl(\lambda_i) = 0$. Then $int(1 - \lambda_i) = 0$. Hence, $(1 - \lambda_i)$'s are fuzzy regular F_σ-sets with $int(1 - \lambda_i) = 0$. Therefore by proposition 4.5, $(1 - \lambda_i)$'s are fuzzy first category sets in (X, T). Let $\mu_i = 1 - \lambda_i$. Hence if (X, T) is a fuzzy regular Volterra space, then $int\big(\vee_{i=1}^{N} (\mu_i)\big) = 0$, where (μ_i)'s are fuzzy first category sets in (X, T).

5.Conclusion

In this paper, the concepts of fuzzy regular G_δ-sets, fuzzy regular F_σ-sets and fuzzy regular Volterra spaces have introduced and studied. Several characterizations of fuzzy regular Volterra spaces have established in this paper.

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