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A study on an optimal replacement policy for a deteriorating system under partial product process

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Abstract

In this paper, we consider an optimal maintenance policy for a reparable deteriorating system subject to random shocks. For a reparable deteriorating system, the repair time by a partial product process and the failure mechanism by a generalized δshock process. Develop an explicit expression of the ling run average cost per unit time under N policy is studied.

Key words: Replacement, Poisson process, Geometric repair process, $\delta-$shock model, Finite search.

1.Introduction

In reliability, the study of maintenance problem is always an important topic. The replacement problem for a reparable system has aroused great attention. Barlow and Proschan (1965) used the replacement policy in which the system will have a preventive replacement as soon as the age of the system reaches T and a failure replacement as soon as it fails, which ever occur earlier and the replacement time is assumed to be negligible. Park (1979) studied the replacement policy in which the system will have a replacement as soon as the number of failures of the system reaches N. The problem is to choose an optimal replacement policy N^* such that the long run average cost per unit time is minimized. Lam (1988) introduced a geometric process and Cheng and Li (2014) introduced a generalization of the α-series process model. Babu, Govindaraju and Rizwan(2018) introduced a partial product process model.

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Shock models were used to model operating time. A system fails due to the shock effect on the system. Li (1984) first introduced the δ shock model to avoid measuring amount of damage. The δ shock model focuses mainly on the frequency of shocks rather than the magnitudes of shocks. The δ shock models if the time interval between two successive shocks is smaller than a thresold value δ, the system fails.

In this paper, we adopt a general δ shock model by the δ be an exponentially distributes a random variable with parameter varying with number of repairs.

2.Basic Definitions and Model assumptions

The preliminary definitions and results about partial product process are given below.

Definition 2.1 For a given two random variables X and Y, X is said to be stochastically larger than Y (or Y is stochastically less than X) if $P(X > \alpha) \geq P(Y > \alpha)$ for all real α.

Definition 2.2 A stochastic process $\{X_n, n = 1, 2, 3, ...\}$ is said to be stochastically increasing (decreasing) if $X_n \leq st(\geq st)X_{n+1}$ for all $n = 1, 2, 3, ...$

Definition 2.3 Let $\{X_n, n = 1, 2, 3, ...\}$ be a sequence of non negative independant random variables and $F(X)$ be the distribution function of X_1. Then $\{X_n, n = 1, 2, 3, ...\}$ is called a partial product process, if the distribution function of X_i is $F(\beta_i X)/, (i = 1, 2, 3, ...)$ where $\beta_i > 0$ are constant and $\beta_i = \beta_0\beta_1\beta_2....\beta_{i-1}$

Lemma 2.4 For real $\beta_i(i = 1, 2, 3, ...)$, $\beta_i = \beta_0^{2^{i-1}}$, then the distribution function of Y_{i+1} is
$F(\beta_0^{2^{i-1}}X)$ for $i = 1, 2, 3, ...$

Lemma 2.5 Given a partial product process $\{X_n, n = 1, 2, 3, ...\}$

(i) if $\beta_0 > 1$, then $\{X_n, n = 1, 2, 3, ..., n\}$ is stochastically decreasing.
(ii) if $0 < \beta_0 < 1$, then $\{X_n, n = 1, 2, 3, ..., n\}$ is stochastically increasing.
(iii) if $\beta_0 = 1$, then $\{X_n, n = 1, 2, 3, ...\}$ is a renewal process.

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Lemma 2.6 Let $E(X_1) = \mu$, $var(Y_1) = p^2$. Then for $i = 1, 2, 3, ...$

$$E(X_{i+1}) = \frac{\mu}{\beta_0^{2^{i-1}}} \qquad \text{and} \qquad var|X_{i+1}| = \frac{\rho^2}{\beta_0^{2^{i-1}}}$$

Definition 2.7 An integer valued random variable N is said to be stopping time for the sequence of independent random variables $X_1, X_2, ...$ if the event $\{N = n\}$ is independent of $X_{n+1}, X_{n+2}, ...$

Theorem 2.8 Wald's equation, If $X_1, X_2, ...$ are independent and identically distributed random variables having finite expectations and if N is the stopping time for $X_1, X_2, ...$ such that $E(N) < \infty$, then

$$E\left[\sum_{n=1}^{N}\right] = E(N)E(X_1)$$

We consider the maintenance model for a deteriorating system and make the following assumptions.

A1 At the beginning a new system is installed. Whenever the system fails, it may be repaired or replaced by a new and identical one.

A2 Shocks arrive according to a Poisson process with rate λ_1 or $EX_i = \frac{1}{\lambda_i}$, where X_i is the i^{th} inter arrival time of two consecutive shocks. Let δ_i be another exponentially distributed random variable associated with X_i. We assumed that the sequence $\{\delta_i, i = 1, 2, ...\}$ form an increasing partial product process with $0 < \beta_0 \leq 1$. Then δ_i has cumulative distribution function $Q(\beta_0^{2^{i-1}} x)$, where $Q(X)$ is the cumulative distribution function of δ_1. (X_i, δ_i) follows a $\delta-$shock model if the system fails at i^{th} shock which satisfies $X_i \leq \delta_i$ and the life time or equivalently the operating time is the sum of all X_i until the one satisfying the above condition. We assume that X_i is independent of δ_i.

A3 Let T_n be the operating time after the $(n-1)^{th}$ repair $\{T_n, n = 1, 2, ...\}$ is a stochastically decreasing random variable sequence induced by the δ shock model.

A4 Let Y_n be the repair time after the n^{th} failure and forms an increasing partial product process with $0 \leq \gamma_0 \leq 1$. Then Y_n has cumulative distribution function $G(\gamma_0^{2^{i-1}} Y)$, where $G(Y)$ is the cumulative distribution function of Y_1 with $E(Y_1) = \gamma_0$ and $EY_n = \gamma_0^{2^{i-1}} Y$.

A5 T_n and Y_n $n = 1, 2, 3, ...$ are independent sequence.

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A6 Let Z be the replacement time with $E(Z) = \tau$.
A7 The repair cost rate is c, the reward rate is r and the replacement cost is R.

3.The replacement policy N

Definition 3.1 A replacement policy N is a policy in which we replace the system at the N^{th} failure of the system.

By the renewal reward theorem, Ross (1983), the long run average cost per unit time under the replacement policy N is given by

$$C(N) = \frac{\text{The expected cost incurred in a cycle}}{\text{The expected length of a cycle}}$$

Let W be the length of a renewal cycle under N replacement policy. We have

$$W = \sum_{i=1}^{N} T_i + \sum_{i=1}^{N-1} Y_i + Z$$

We first calculate $E(T_n)$ the expected operating time of the system after the $(n-1)^{th}$ failure. Let l_{ni} be the inter arrival time between the $(i-1)^{th}$ and i^{th} shock following the $(n-1)^{th}$ repair, where $i = 1, 2, ...$ Define

$$M_n = \min\{m | l_{n1} > \beta_0^{2^{i-1}}, ...l_{m-1} > \beta_0^{2^{i-1}}\delta_1, \ln_m < \beta_0^{2^{i-1}}\delta_1\}$$

and $T_n = \sum_{i=1}^{M_n} l_{ni}$

Where M_n denotes the number of shocks till the first deadly shock occurs and M_n has a geometric distribution with $P(M_n = k) = q_n^{k-1} p_n, k = 1, 2, ...$

Where p_n is the probability of a shock, following the $(n-1)^{th}$ repair and $q_n = 1 - p_n$. We have $EM_n = \frac{1}{p_n}$. As M_n is a stopping time with respect to the random sequence $\{\ln_i, i = 1, 2, ...\}$ which are independence identically distributed random variables, using Wald equation, we have

$$E(T_n) = E\left(\sum_{i=1}^{M_n} l_{ni}\right) = E(l_{ni})E(M_n) = \frac{E(l_{ni})}{p_n}$$

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Since $F(x)$ and $Q(x)$ are all exponentially distributed, we have

$$F(x) = 1 - e^{-\lambda_1 x}, x \geq 0,$$

and

$$Q(\beta_0^{2^{i-1}} x) = 1 - e^{-\beta_0^{2^{i-1}} \lambda_2 x}, x \geq 0$$

and

$$El_{n1} = \int_0^\infty x\, dF(x) = \int_0^\infty x\, d(1 - e^{-\lambda_1 x}) = \frac{1}{\lambda_1}$$

As l_{ni} and $\delta_i(\beta_0^{2^{i-1}} \delta_1)$ are independent and have the marginal exponential distribution with means of $\frac{1}{\lambda_1}$ and and $\frac{1}{\beta_0^{2^{i-1}}\lambda_2}$ respectively.
We obtain,

$$\begin{aligned} p_n &= p(l_{ni} < \delta_n) = \int_0^\infty e^{-\beta_0^{2^{n-1}} \lambda_2 x} e^{-\lambda_1 x}\, dx \\ &= \lambda_1 \int_0^\infty e^{-(\beta_0^{2^{n-1}} \lambda_2 + \lambda_1) x}\, dx \\ &= \frac{\lambda_1}{\lambda_1 + \beta_0^{2^{n-1}} \lambda_2} \end{aligned}$$

and

$$\zeta_n = E(T_n) = \frac{\lambda_1 + \beta_0^{2^{n-1}} \lambda_2}{\lambda_1^2}$$

Then

$$E\left(\sum_{n=1}^{N} T_n\right) = \sum_{n=1}^{N} E(T_n) = \sum_{n=1}^{N} \frac{\lambda_1 + \beta_0^{2^{n-1}} \lambda_2}{\lambda_1^2}$$

Since $Y_n, n = 1, 2, ...$ is increasing partial product process with ratio $0 < \delta_0 \leq 1$, we have

$$E(Y_n) = \frac{\mu}{\gamma_0^{2^{n-1}}}$$

The long run average cost $C(N)$ of the system under the policy N is given by

$$C(N) = \frac{E\left[c \sum_{n=1}^{N-1} Y_n - r \sum_{n=1}^{N} T_n + R\right]}{E\left[\sum_{n=1}^{N} T_n + \sum_{n=1}^{N-1} Y_n + Z\right]}$$

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$$= \frac{c\mu + c\sum_{n=2}^{N-1} E(Y_n) - r\sum_{n=2}^{N} E(T_n) + E(R) + r\lambda_1}{\sum_{n=2}^{N-1} E(Y_n) + \sum_{n=2}^{N} E(T_n) + E(z) + \mu + \lambda_1}$$

$$= \frac{c\sum_{n=1}^{N-1} \frac{\mu}{\gamma_0^{2^{n-1}}} - r\sum_{n=1}^{N} \frac{\lambda_1 + \beta_0^{2^{n-1}}\lambda_2}{\lambda_1^2} + R + \mu - r\lambda_1}{\sum_{n=1}^{N} \frac{\lambda_1 + \beta_0^{2^{n-1}}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{\gamma_0^{2^{n-1}}} + \tau + \mu - \lambda_1} \quad (1)$$

The optimal replacement policy N^* can be calculated by minimizing $C(N)$

4.The optimal replacement policy N^*

Equation (1) can be re-written as

$$C(N) = \frac{(c+r)\sum_{n=2}^{N-1} \frac{\mu}{\beta_0^{2^{n-1}}} + R + (r_p + r)t + c\mu - r\lambda_1}{\sum_{n=2}^{N} \frac{\lambda_1 + \beta_0^{2^{n-1}}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{\beta_0^{2^{n-1}}} + t + \mu + \lambda_1}$$

Let

$$C(N) = \frac{(c+r)\sum_{n=2}^{N-1} \frac{\mu}{\gamma_0^{2^{n-1}}} + R + (r_p + r)t + c\mu - r\lambda_1}{\sum_{n=2}^{N} \frac{\lambda_1 + \beta_0^{2^{n-1}}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{\gamma_0^{2^{n-1}}} + t + \mu + \lambda_1}$$

The difference between $B(N+1)$ and $B(N)$.
Let

$$\begin{aligned} f(N) &= \sum_{n=2}^{N} \frac{\lambda_1 + \beta_0^{2^{n-1}}\lambda_2}{\lambda_1^2} + \sum_{n=1}^{N-1} \frac{\mu}{\gamma_0^{n-1}} + t + \mu + \lambda_1 \\ B(N+1) - B(N) &= \frac{1}{\gamma_0^{2^{n-1}} f(N+1)f(N)} \left\{ (c+r)\mu \left[\sum_{n=2}^{N} \zeta_n - \zeta_{N+1} \sum_{n=2}^{N-1} \gamma_0^{2^{n-1}} + t \right] \right\} \\ &\quad - \left\{ \left[R + (r_p + r)t(\zeta_{N+1}\gamma_0^{2^{n-1}} + \mu) \right] \right\} \end{aligned}$$

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Define

$$A(N) = \frac{(c+r)\mu \left[\sum_{n=1}^{N} \zeta_n - \zeta_{N+1} \sum_{n=1}^{N-1} \gamma_0^{2^{n-1}} + t \right]}{[R + (r_p + r)t](\zeta_{N+1}\gamma_0^{2^{n-1}} + \mu)}$$

Thus we have

$$\begin{aligned} B(N+1) &\leq B(N) \Leftrightarrow A(N) \leq 1 \\ B(N+1) &\geq B(N) \Leftrightarrow A(N) \geq 1 \end{aligned}$$

Conclusions

In this paper, we have studied the optimal replacement policy for a reparable and deteriorating systems using $\delta-$shock model, we derived the optimal replacement policy N^* by minimizing the average cost rate (CN).

References

[1] Barlow R E and Proschan F, Imperfect repair, J. Appl. Probab. 20, 1983, 851-859.

[2] Block H W, Borges W S and Savits T H, Age-dependent minimal repair, J. Appl. Probab. 22, 1985, 370-385.

[3] Kijima M, Some results for repairable systems with general repair, J. Appl. Probab. 26, 1989, 89-102.

[4] Makis V and Jardine A K S, Optimal replacement policy for a general model with imperfect repair, J. Oper. Res. Soc. 43, 1992, 111-120.

[5] Wang G J and Zhang Y L, A bivariate mixed policy for a simple repairable system based on preventive repair and failure repair, Appl. Math. Modell. 33, 2009, 3354-3359.

[6] Yuan L and Xu J, An optimal replacement policy for a repairable system based on its repairman having vacations, Reliab. Eng. Syst. Saf. 96, 2011, 868-875.

[7] Zhang Y L and Wang G J, An extended replacement policy for a deteriorating system with multi-failure modes, Appl. Math. Comput. 218, 2011, 1820-1830.

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