



A study on an optimal replacement policy for a degenerative system under partial product process

P. Govindaraju^{1*} and P. Ashok Kumar²

¹Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi 635 752, Tirupattur District, Tamil Nadu, India.

²Research Scholar, Department of Mathematics, Islamiah College (Autonomous), Vaniyambadi 635 752, Tirupattur District, Tamil Nadu, India.

Abstract

In this paper, we study a degenerative repairable system with two types of failure states. Any system after repair can not be as good as new. A general monotone process model for a degenerative system under partial product process is used. We use a replacement policy N based on the failure number of the system and to determine an optimal replacement policy N^* such that the average cost rate is minimized.

Key words: Maintenance, Degenerative system, α -series process, Monotone process, Replacement policy N

1.Introduction

In a maintenance degenerative repairable system assumes that a failed system after repair will be as good as new, this is the perfect repair model. For a deteriorating repairable system, its successive working times of the system after repair become shorter and shorter while the consecutive repair times of the degenerative system may become longer and longer. Such a system can not work any longer. Neither can it be repaired or replaced new one. Lam [1988] first introduced a geometric process repair model in which he studied two kind of replacement policy, one based on the working age T of the system and the other based on the number of failures N of the system. Cheng and Li (2014) first introduced a α -series process for an optimal replacement policy for a degenerative system with two types of failure states where as

^{1*}govindrajmaths69@gmail.com,²ashokbhuvana1999@gmail.com

both operating times and repair times are assumed to follow partial product process in Babu, Govindaraju and Rizwan (2018) paper.

In this paper, we assume successive operating time and the consecutive repair time for a degenerative system with two types of failure states under a partial product process.

2. Basic Definitions and Model assumptions

The preliminary definitions and results about partial product process are given below.

Definition 2.1 For a given two random variables X and Y , X is said to be stochastically larger than Y (or Y is stochastically less than X) if $P(X > \alpha) \geq P(Y > \alpha)$ for all real α .

Definition 2.2 A stochastic process $\{X_n, n = 1, 2, 3, \dots\}$ is said to be stochastically increasing (decreasing) if $X_n \leq st(\geq st)X_{n+1}$ for all $n = 1, 2, 3, \dots$

Definition 2.3 Let $\{X_n, n = 1, 2, 3, \dots\}$ be a sequence of non negative independent random variables and $F(X)$ be the distribution function of X_1 . Then $\{X_n, n = 1, 2, 3, \dots\}$ is called a partial product process, if the distribution function of X_i is $F(\beta_i X)$, ($i = 1, 2, 3, \dots$) where $\beta_i > 0$ are constant and $\beta_i = \beta_0 \beta_1 \beta_2 \dots \beta_{i-1}$

Lemma 2.4 For real $\beta_i (i = 1, 2, 3, \dots)$, $\beta_i = \beta_0^{2^{i-1}}$, then the distribution function of Y_{i+1} is $F(\beta_0^{2^{i-1}} X)$ for $i = 1, 2, 3, \dots$

Lemma 2.5 Given a partial product process $\{X_n, n = 1, 2, 3, \dots\}$

- (i) if $\beta_0 > 1$, then $\{X_n, n = 1, 2, 3, \dots, n\}$ is stochastically decreasing.
- (ii) if $0 < \beta_0 < 1$, then $\{X_n, n = 1, 2, 3, \dots, n\}$ is stochastically increasing.
- (iii) if $\beta_0 = 1$, then $\{X_n, n = 1, 2, 3, \dots\}$ is a renewal process.

Lemma 2.6 Let $E(X_1) = \mu$, $var(Y_1) = \rho^2$. Then for $i = 1, 2, 3, \dots$

$$E(X_{i+1}) = \frac{\mu}{\beta_0^{2^{i-1}}} \quad \text{and} \quad var|X_{i+1}| = \frac{\rho^2}{\beta_0^{2^{i-1}}}$$

Definition 2.7 An integer valued random variable N is said to be stopping time for the sequence of independent random variables X_1, X_2, \dots if the event $\{N = n\}$ is independent of X_{n+1}, X_{n+2}, \dots

Theorem 2.8 Wald's equation, If X_1, X_2, \dots are independent and identically distributed random variables having finite expectations and if N is the stopping time for X_1, X_2, \dots such that $E(N) < \infty$, then

$$E \left[\sum_{n=1}^N \right] = E(N)E(X_1)$$

We consider a monotone process model for a degenerative system with two types of failure state by making the following assumptions.

- A1** At the beginning, a new system is installed. Whenever the system fails, it will be repaired or replaced by an identical new one.
- A2** Assume that there are three states in the system, that is, 0, 1 and 2 are respectively denoted as the working state. First type failure state and second type failure state of the system. And assume that the occurrence of a failure state of two types is mutually exclusive and stochastic. If the system fails, then with probability p_1 , the system will be in state i , $i = 1, 2$ and $p_1 + p_2 = 1$.
- A3** Let X_1 be the operating time of the system after installation. Let X_n be the operating time of the system after $n - 1^{th}$ repair. Let Y_n be the repair time after the n^{th} failure. Let us denote the time of the n^{th} failure by t_n . Let $S(t_n)$ denote the type of the n^{th} failure where $S(t_n) \in \{1, 2\}$, $n = 1, 2, \dots$. X_n, Y_n are independent. Assume that

$$p(X_1 \leq t) = U(t)$$

$$p(X_n \leq t | S(t_1) = l_1, \dots, S(t_{n-1}) = l_{n-1}) = U \left(\left(\frac{2}{1} \right)^{\alpha_{l_1}} \cdot \left(\frac{3}{2} \right)^{\alpha_{l_2}} \cdots \left(\frac{n}{n-1} \right)^{\alpha_{l_{n-1}}} t \right)$$

Where $l_i \in \{1, 2\}$, $i = 1, 2, \dots, n - 1$; $\alpha_1 \geq 0$; $\alpha_2 \geq 0$.

Similarly assume that

$$p(Y_1 \leq t | S(t_1) = m_1) = V \left(\left(\frac{2}{1} \right)^{\beta_{m_1}} t \right)$$

$$p(Y_n \leq t | S(t_1) = m_1, \dots, S(t_n) = m_n) = V \left(\left(\frac{2}{1} \right)^{\beta_{m_1}} \cdot \left(\frac{3}{2} \right)^{\beta_{m_2}} \cdots \left(\frac{n+1}{n} \right)^{\beta_{m_n}} t \right)$$

Where $m_j \in \{1, 2\}$, $j = 1, 2, \dots, n$; $\beta_1 \leq 0$; $\beta_2 \leq 0$.

A4 Let c_w be the reward rate per unit time of the system when it is operating c_r be the repair cost rate per unit time of the system.

A5 T_n and Y_n $n = 1, 2, 3, \dots$ are independent sequence.

A6 The policy N is applied.

Theorem 2.9

$$\begin{aligned} P(X_n \leq t) &= U(\alpha^{2^{n-1}}t) \\ P(Y_1 \leq t) &= V(\beta_0 t) \\ P(Y_n \leq t) &= V(\beta_0^{2^{n-1}}t) \end{aligned}$$

Since $\{X_n, n = 1, 2, \dots\}$ will form an partial product process with exponent α and $X_1 \sim U$; and $\{Y_n; n = 0, 1, \dots\}$ are also an partial product process with exponent β and $Y_0 \sim V$. Using $p_i = P(S(t_n) = i)$, $i = 1, 2$ and the distribution function of X_n for $n = 2, 3, \dots$

$$P(X_n \leq t) = \sum_{l_1 \dots l_{n-1} \in \{1,2\}} U \left(\left(\frac{2}{1} \right)^{\alpha_{l_1}} \cdot \left(\frac{3}{2} \right)^{\alpha_{l_2}} \cdots \left(\frac{n}{n-1} \right)^{\alpha_{l_{n-1}}} t \right) p_{l_1} \cdots p_{l_{n-1}}$$

and

$$P(Y_n \leq t) = \sum_{m_1 \dots m_n \in \{1,2\}} V \left(\left(\frac{2}{1} \right)^{\beta_{m_1}} \cdot \left(\frac{3}{2} \right)^{\beta_{m_2}} \cdots \left(\frac{n+1}{n} \right)^{\beta_{m_n}} t \right) p_{m_1} \cdots p_{m_n}$$

Theorem 2.10 Assume that $E(X_1) = \frac{1}{\lambda}$; $E(Y_0) = \frac{1}{\mu}$ then $E(X_1) = \mu$, $E(Y_1) = \mu$;

$$\begin{aligned} E(X_n) &= \frac{\lambda}{\beta_0^{2^{j-1}}} \prod_{j=2}^{n-1} \left(p_1 \left(\frac{j}{j+1} \right)^{\alpha_1} + p_2 \left(\frac{j}{j+1} \right)^{\alpha_2} \right), \quad n = 2, 3, \dots \\ E(Y_n) &= \frac{\mu}{\beta_0^{2^{j-1}}} \prod_{j=2}^n \left(p_1 \left(\frac{j}{j+1} \right)^{\beta_1} + p_2 \left(\frac{j}{j+1} \right)^{\beta_2} \right), \quad n = 2, 3, \dots \end{aligned}$$

3.The replacement policy N

Definition 3.1 A replacement policy N is a policy in which we replace the system at the N^{th} failure of the system. By the renewal reward theorem, Ross (1983), the lens run average cost per unit time under the replacement policy N is given by

$$\begin{aligned}
 C(N) &= \frac{\text{The expected cost incurred in a cycle}}{\text{The expected length of a cycle}} \\
 C(N) &= \frac{E\left(c_w \sum_{k=2}^{N-1} Y_k + R - c_r \sum_{k=2}^N X_k\right)}{E\left(\sum_{k=1}^N X_k + \sum_{k=1}^{N-1} Y_k + \tau\right)} \\
 &= \frac{c_w \sum_{k=1}^{N-1} E(Y_k) + R - c_r \sum_{k=1}^N E(X_k)}{\sum_{k=1}^N E(X_k) + \sum_{k=1}^N E(Y_n) + \tau} \\
 &= \frac{c_w \left[E(Y_1) + \sum_{k=2}^{N-1} E(Y_k) \right] + R - c_r \left[E(X_1) + \sum_{k=2}^N E(X_k) \right]}{E(X_1) + \sum_{k=1}^N E(X_k) + E(Y_1) + \sum_{k=2}^{N-1} E(Y_k) + \tau} \\
 &= \frac{c_w \left[\mu + \sum_{k=2}^{N-1} \frac{\mu}{\beta_0^{2^{k-1}}} \right] + R - r\lambda - r \left[\sum_{k=2}^N \frac{\lambda}{\alpha^{k-1}} \right]}{\mu + \sum_{k=1}^N \frac{\lambda}{\alpha^{k-1}} + \mu + \sum_{k=2}^{N-1} \frac{\mu}{\beta_0^{2^{k-1}}} + \tau}
 \end{aligned}$$

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