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A Study on Fuzzy Subnear Rings Under Homomorphism and Anti-Homomorphism

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Abstract

In this paper, we made an attempt to study the algebraic nature of fuzzy subnear rings of a near ring.

Key words: Fuzzy set, fuzzy subnear ring, fuzzy normal subnear ring, homomorphism, anti-homomorphism.

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1.Introduction

There are many concepts of universal algebras generalizing an associative ring $(R;+,.)$. Some of them in particular, near rings and several kinds of semi rings have been proven very useful. An algebra $(R;+,.)$ is said to be a near ring if $(R,+)$ is a group and $(R,.)$ are semi group satisfying $a.(b+c) = a.b + a.c$, for all a, b and c in R. After the introduction of fuzzy sets by L.A.Zadeh [6], several researchers explored on the generalization of the concept of fuzzy sets. The notion of Fuzzy subnear rings and ideals was introduced by S.Abou Zaid [1]. In this paper, we introduce the some theorems in fuzzy subnear ring of a near ring.

2.Preliminaries

Definition 2.1 Let X be a non-empty set. A fuzzy subset A of X is a function $A : X \to [0,1]$.

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Definition 2.2 Let R be a near ring. An fuzzy subset A of R is said to be a fuzzy subnear ring (FSNR) of R if it satisfies the following conditions:

(i) $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$,
(ii) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in R.

Definition 2.3 Let R be a near ring. An fuzzy subnear ring A of R is said to be a fuzzy normal subnear ring(FNSNR) of R if it satisfies the following conditions:

(i) $\mu_A(x+y) = \mu_A(y+x)$,
(ii) $\mu_A(xy) = \mu_A(yx)$, for all x and y in R.

Definition 2.4 Let A and B be fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by $A \times B$, is defined as $A \times B = \{(x, y), \mu_{A\times B}(x, y) \mid$ for all x in G and y in H $\}$, where $\mu_{A\times B}(x, y) = min\{A(x), B(y)\}$.

Definition 2.5 Let A be a fuzzy subset in a set S, the strongest fuzzy relation on S, that is a fuzzy relation on A is V given by $\mu_V(x, y) = \min\{\mu_A(x), \mu_A(y)\}$, for all x and y in S.

Definition 2.6 If $(R, +, .)$ and $(R', +, .)$ are any two near rings, then the function $f : R \to R'$ is called a homomorphism if $f(x+y) = f(x)+f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R.

Definition 2.7 If $(R, +, .)$ and $(R', +, .)$ are any two near rings, then the function $f : R \to R'$ is called an anti-homomorphism if $f(x + y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R.

Definition 2.8 Let R and R' be any two near rings. Let $f : R \to R'$ be any function and let A be a fuzzy subnear ring in R, V be a fuzzy subnear ring in $f(R) = R'$, defined by $V(y) = A(x)$, for all x in R and y in R'. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

3.Properties of Fuzzy Subnear Ring of a Near Ring R

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Theorem 3.1 Intersection of any two fuzzy subnear ring of a near ring R is a fuzzy subnear ring of R.

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Proof: Let A and B be any two fuzzy subnear rings of a near ring R and let x and y in R. Let $A = \{(x, \mu_A(x)) \mid x \in R\}$ and $B = \{(x, \mu_B(x)) \mid x \in R\}$ and also let $C = A \cap B = \{(x, \mu_C(x)) \mid x \in R\}$, where $\min\{\mu_A(x), \mu_B(x)\} = \mu_C(x)$. Now,

$$\begin{aligned} \mu_C(x-y) &= \min\{\mu_A(x-y), \mu_B(x-y)\} \\ &\geq \min\{\min\{\mu_A(x), \mu_A(y)\}, \min\{\mu_B(x), \mu_B(y)\}\} \\ &= \min\{\min\{\mu_A(x), \mu_B(x)\}, \min\{\mu_A(y), \mu_B(y)\}\} \\ &= \min\{\mu_C(x), \mu_C(y)\}. \end{aligned}$$

Therefore, $\mu_C(x-y) \geq \min\{\mu_C(x), \mu_C(y)\}$, for all x and y in R. And,

$$\begin{aligned} \mu_C(xy) &= \min\{\mu_A(xy), \mu_B(xy)\} \\ &\geq \min\{\min\{\mu_A(x), \mu_A(y)\}, \min\{\mu_B(x), \mu_B(y)\}\} \\ &= \min\{\min\{\mu_A(x), \mu_B(x)\}, \min\{\mu_A(y), \mu_B(y)\}\} \\ &= \min\{\mu_C(x), \mu_C(y)\}. \end{aligned}$$

Therefore, $\mu_C(xy) \geq \min\{\mu_C(x), \mu_C(y)\}$, for all x and y in R. Therefore C is a fuzzy subnear ring of a near ring R. Hence the intersection of any two fuzzy subnear rings of a near ring R is a fuzzy subnear ring of R.

Theorem 3.2 If A and B are any two fuzzy subnear rings of the near rings R_1 and R_2 respectively, then $A \times B$ is a fuzzy subnear ring of $R_1 \times R_2$.

Proof: Let A and B be two fuzzy subnear rings of the near rings R_1 and R_2 respectively. Let x_1 and x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) are in $R_1 \times R_2$. Now,

$$\begin{aligned} \mu_{A\times B}[(x_1, y_1) - (x_2, y_2)] &= \mu_{A\times B}(x_1 - x_2, y_1 - y_2) \\ &= \min\{\mu_A(x_1 - x_2), \mu_B(y_1 - y_2)\} \\ &\geq \min\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_B(y_1), \mu_B(y_2)\}\} \\ &= \min\{\min\{\mu_A(x_1), \mu_B(y_1)\}, \min\{\mu_A(x_2), \mu_B(y_2)\}\} \\ &= \min\{\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2)\}. \end{aligned}$$

Therefore, $\mu_{A\times B}[(x_1, y_1) - (x_2, y_2)] \geq \min\{\mu_{A\times B}(x_1, y_1), \mu_{A\times B}(x_2, y_2)\}$.

Also,

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$$
\begin{aligned}
\mu_{A\times B}[(x_1,y_1)(x_2,y_2)] &= \mu_{A\times B}(x_1x_2, y_1y_2) \\
&= \min\{\mu_A(x_1x_2), \mu_B(y_1y_2)\} \\
&\geq \min\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_B(y_1), \mu_B(y_2)\}\} \\
&= \min\{\min\{\mu_A(x_1), \mu_B(y_1)\}, \min\{\mu_A(x_2), \mu_B(y_2)\}\} \\
&= \min\{\mu_{A\times B}(x_1,y_1), \mu_{A\times B}(x_2,y_2)\}.
\end{aligned}
$$

Therefore, $\mu_{A\times B}[(x_1,y_1)(x_2,y_2)] \geq \min\{\mu_{A\times B}(x_1,y_1), \mu_{A\times B}(x_2,y_2)\}$. Hence $A \times B$ is a fuzzy subnear ring of near ring of $R_1 \times R_2$.

Theorem 3.3 Let A be a fuzzy subset of a near ring R and V be the strongest fuzzy relation of R. Then A is a fuzzy subnear ring of R if and only if V is a fuzzy subnear ring of $R \times R$.

Proof: Suppose that A is a fuzzy subnear ring of a near ring R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $R \times R$. We have,

$$
\begin{aligned}
\mu_V(x-y) = \mu_V[(x_1,x_2)-(y_1,y_2)] \\
&= \mu_V(x_1-y_1, x_2-y_2) \\
&= \min\{\mu_A(x_1-y_1), \mu_A(x_2-y_2)\} \\
&\geq \min\{\min\{\mu_A(x_1), \mu_A(y_1)\}, \min\{\mu_A(x_2), \mu_A(y_2)\}\} \\
&= \min\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\}\} \\
&= \min\{\mu_V(x_1,x_2), \mu_V(y_1,y_2)\} \\
&= \min\{\mu_V(x), \mu_V(y)\}.
\end{aligned}
$$

Therefore, $\mu_V(x-y) \geq \min\{\mu_V(x), \mu_V(y)\}$, for all x and y in $R \times R$.
And,

$$
\begin{aligned}
\mu_V(xy) = \mu_V[(x_1,x_2)(y_1,y_2)] \\
&= \mu_V(x_1y_1, x_2y_2) \\
&= \min\{\mu_A(x_1y_1), \mu_A(x_2y_2)\} \\
&\geq \min\{\min\{\mu_A(x_1), \mu_A(y_1)\}, \min\{\mu_A(x_2), \mu_A(y_2)\}\}
\end{aligned}
$$

$$\begin{aligned} &= \min\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\}\} \\ &= \min\{\mu_V(x_1, x_2), \mu_V(y_1, y_2)\} \\ &= \min\{\mu_V(x), \mu_V(y)\}. \end{aligned}$$

Therefore, $\mu_V(xy) \geq \min\{\mu_V(x), \mu_V(y)\}$, for all x and y in $R \times R$. This proves that V is a fuzzy subnear ring of $R \times R$.

Conversely assume that V is a fuzzy subnear ring of $R \times R$. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $R \times R$, we have

$$\begin{aligned} \min\{\mu_A(x_1 - y_1), \mu_A(x_2 - y_2)\} &= \mu_V(x_1 - y_1, x_2 - y_2) \\ &= \mu_V[(x_1, x_2) - (y_1, y_2)] \\ &= \mu_V(x - y) \\ &\geq \min\{\mu_V(x), \mu_V(y)\} \\ &= \min\{\mu_V(x_1, x_2), \mu_V(y_1, y_2)\} \\ &= \min\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\}\}. \end{aligned}$$

If we put $x_2 = y_2 = 0$, we get, $\mu_A(x_1 - y_1) \geq \min\{\mu_A(x_1), \mu_A(y_1)\}$, for all x_1 and y_1 in R. And,

$$\begin{aligned} \min\{\mu_A(x_1 y_1), \mu_A(x_2 - y_2)\} &= \mu_V(x_1 y_1, x_2 y_2) \\ &= \mu_V[(x_1, x_2)(y_1, y_2)] \\ &= \mu_V(x - y) \\ &\geq \min\{\mu_V(x), \mu_V(y)\} \\ &= \min\{\mu_V(x_1, x_2), \mu_V(y_1, y_2)\} \\ &= \min\{\min\{\mu_A(x_1), \mu_A(x_2)\}, \min\{\mu_A(y_1), \mu_A(y_2)\}\}. \end{aligned}$$

If we put $x_2 = y_2 = 0$, we get, $\mu_A(x_1 y_1) \geq \min\{\mu_A(x_1), \mu_A(y_1)\}$, for all x_1 and y_1 in R. Therefore A is a fuzzy subnear ring of R.

4.Fuzzy Subnear Rings of a Near Ring R Under Homomorphism and Anti-Homomorphism

Theorem 4.1 Let $(R, +, .)$ and $(R', +, .)$ be any two near rings. The homomorphic image of a fuzzy subnear ring of R is a fuzzy subnear ring of R'.

Proof: Let $(R, +, .)$ and $(R', +, .)$ be any two near rings. Let $f : R \to R'$ be a homomorphism. Then,

(i) $f(x + y) = f(x) + f(y)$ and

(ii) $f(xy) = f(x)f(y)$, for all x and y in R.

Let $V = f(A)$, where A is a fuzzy subnear ring of R. We have to prove that V is a fuzzy subnear ring of R'. Now, for $f(x), f(y)$ in R',

$$\begin{aligned} \mu_V(f(x) - f(y)) &= \mu_V(f(x - y)), \text{as f is a homomorphism} \\ &\geq \mu_A(x - y) \\ &\geq \min\{\mu_A(x), \mu_A(y)\} \end{aligned}$$

which implies that $\mu_V(f(x) - f(y)) \geq \min\{\mu_A(x), \mu_A(y)\}$.

Again,

$$\begin{aligned} \mu_V(f(x)f(y)) &= \mu_V(f(x - y)), \text{as f is a homomorphism} \\ &\geq \mu_A(x - y) \\ &\geq \min\{\mu_A(x), \mu_A(y)\} \end{aligned}$$

which implies that $\mu_V(f(x) - f(y)) \geq \min\{\mu_A(x), \mu_A(y)\}$. Hence V is a fuzzy subnear ring of R'.

Theorem 4.2 Let $(R, +, .)$ and $(R', +, .)$ be any two near rings. The homomorphic preimage of a fuzzy subnear ring of R' is a fuzzy subnear ring of R.

Proof: Let $(R, +, .)$ and $(R', +, .)$ be any two near rings. Let $f : R \to R'$ be a homomorphism. Then,

(i) $f(x + y) = f(x) + f(y)$ and

(ii) $f(xy) = f(x)f(y)$, for all x and y in R.

Let $V = f(A)$, where V is a fuzzy subnear ring of R'. We have to prove that A is a fuzzy subnear ring of R. Let x and y in R. Then,

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$$\begin{aligned}
\mu_A(x-y) &= \mu_V(f(x-y)), \text{ since } \mu_V(f(x)) = \mu_A(x) \\
&= \mu_V(f(x)-f(y)), \text{ as } f \text{ is a homomorphism} \\
&\geq \min\{\mu_V(f(x)), \mu_V(f(y))\} \\
&= \min\{\mu_A(x), \mu_A(y)\}, \text{since} \mu_V(f(x)) = \mu_A(x)
\end{aligned}$$

which implies that $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$.

Again,

$$\begin{aligned}
\mu_A(xy) &= \mu_V(f(xy)), \text{ since } \mu_V(f(x)) = \mu_A(x) \\
&= \mu_V(f(x)f(y)), \text{ as } f \text{ is a homomorphism} \\
&\geq \min\{\mu_V(f(x)), \mu_V(f(y))\} \\
&= \min\{\mu_A(x), \mu_A(y)\}, \text{since} \mu_V(f(x)) = \mu_A(x)
\end{aligned}$$

which implies that $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$. Hence A is a fuzzy subnear ring of R.

Theorem 4.3 Let $(R, +, .)$ and $(R', +, .)$ be any two near rings. The anti-homomorphic image of a fuzzy subnear ring of R is a fuzzy subnear ring of R'.

Proof: Let $(R, +, .)$ and $(R', +, .)$ be any two near rings. Let $f : R \to R'$ be an anti-homomorphism. Then,

(i) $f(x+y) = f(x) + f(y)$ and

(ii) $f(xy) = f(x)f(y)$, for all x and y in R.

Let $V = f(A)$, where A is a fuzzy subnear ring of R. We have to prove that V is a fuzzy subnear ring of R'. Now, for $f(x), f(y)$ in R',

$$\begin{aligned}
\mu_V(f(x)-f(y)) &= \mu_V(f(y-x)), \text{as } f \text{ is an anti-homomorphism} \\
&\geq \mu_A(y-x) \\
&\geq \min\{\mu_A(y), \mu_A(x)\} \\
&= \min\{\mu_A(x), \mu_A(y)\}
\end{aligned}$$

which implies that $\mu_V(f(x)-f(y)) \geq \min\{\mu_A(x), \mu_A(y)\}$.

Again,

$$\begin{aligned} \mu_V(f(x)f(y)) &= \mu_V(f(yx)), \text{as } f \text{ is an anti-homomorphism} \\ &\geq \mu_A(yx) \\ &\geq \min\{\mu_A(y), \mu_A(x)\} \\ &= \min\{\mu_A(x), \mu_A(y)\} \end{aligned}$$

which implies that $\mu_V(f(x)f(y)) \geq \min\{\mu_A(x), \mu_A(y)\}$. Hence V is a fuzzy subnear ring of R'.

Theorem 4.4 Let $(R, +, .)$ and $(R', +, .)$ be any two near rings. The anti-homomorphic preimage of a fuzzy subnear ring of R' is a fuzzy subnear ring of R.

Proof: Let $(R, +, .)$ and $(R', +, .)$ be any two near rings. Let $f : R \to R'$ be an anti-homomorphism. Then,

(i) $f(x+y) = f(x) + f(y)$ and

(ii) $f(xy) = f(x)f(y)$, for all x and y in R.

Let $V = f(A)$, where V is a fuzzy subnear ring of R'. We have to prove that A is a fuzzy subnear ring of R. Let x and y in R. Then,

$$\begin{aligned} \mu_A(x-y) &= \mu_V(f(x-y)), \text{ since } \mu_V(f(x)) = \mu_A(x) \\ &= \mu_V(f(y)-f(x)), \text{as } f \text{ is an anti-homomorphism} \\ &\geq \min\{\mu_V(f(y)), \mu_V(f(x))\} \\ &= \min\{\mu_V(f(x)), \mu_V(f(y))\} \\ &= \min\{\mu_A(x), \mu_A(y)\}, \text{since} \mu_V(f(x)) = \mu_A(x) \end{aligned}$$

which implies that $\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}$.

Again,

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$$\begin{aligned}
\mu_A(xy) &= \mu_V(f(xy)), \text{ since } \mu_V(f(x)) = \mu_A(x) \\
&= \mu_V(f(y)f(x)), \text{as } f \text{ is an anti-homomorphism} \\
&\geq \min\{\mu_V(f(y)), \mu_V(f(x))\} \\
&= \min\{\mu_V(f(x)), \mu_V(f(y))\} \\
&= \min\{\mu_A(x), \mu_A(y)\}, \text{since} \mu_V(f(x)) = \mu_A(x)
\end{aligned}$$

which implies that $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$. Hence A is a fuzzy subnear ring of R.

Conclusions

In this paper, we have discussed the properties of Fuzzy Sub near Ring also fuzzy sub near ring of Homomorphism and Anti-homomorphism.

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