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A note on Spherical Continuity

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Abstract

In this paper, wedefine Spherical fuzzy continuity between Spherical fuzzy topological space and we characterize the concept.

Key words: Spherical fuzzy set, spherical fuzzy continuity, Spherical fuzzy topological space.

1. Introductionand Preliminaries

As a generalization of a crisp set, the concept of fuzzy set was introduced by L.A. Zadeh [1]. The concept of fuzzy topological space was defined and few basic notions as open set, closed and continuity were generalized by Chang [8]. By changing a basic property of topology, another definition was given by Lowen [9].

Subsequently, Coker in 1995 introduced Intuitionistic topological space along with some basic concepts with intuitionistic sets by Attanasov [2].Following it, Smarachande [3] introduced Neutrosophic sets.The picture fuzzy set [4, 5, 6] was introduced byCuong and Kreinovich.In 2019, the concept of spherical fuzzy set was proposed by Gndogdu and Kahraman [7] which has the degrees of truthness, abstinence and falseness in the range [0,1] with condition $0 \leq \alpha^2(a)+\beta^2(a)+\gamma^2(a) \leq 1$. The concept of Spherical topological spaces was introducedby Princy and Mohana [10] and also studiedsomeproperties as closure and interior.

A non-empty set X with τ, a collection of subsets of X satisfying conditions ϕ,$X \in \tau$, arbitrary union of elements of τ is in τ and finite intersection of τ also belong to τ is said to be a Topological space. A non-empty fixed set X with τ, a collection of fuzzy subsets of X sustaining the criteria's $0, 1 \in \tau$, arbitrary union of

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elements of τ is in τ and finite intersection of τ also belong to τ is said to be a Fuzzy topological space [8].

The main crux of this paper is to define and characterize the concept of Spherical fuzzy continuity between Spherical fuzzy topological space.

2.Spherical Fuzzy Topological Space

In this section, we introduce the continuity of a function among Spherical fuzzy topological space.

Definition 2.1 Let $X \neq \phi$ be a set and let τ be a family of Spherical fuzzy subsets of X. If

(1) $1_s, 0_s \in \tau$,
(2) For any $S_1, S_2 \in \tau$, we have $S_1 \cap S_2 \in \tau$,
(3) For any $\{A_i\}_{i \in tau}$, we have $\bigcup_{i\in\tau} A_i \in \tau$ where I is an arbitrary index set then τ is called a Spherical fuzzy topology on X.

The pair $X \in \tau$ is said to be Spherical Fuzzy Topological Space (SFTS) [10]. Each member of $\tau$is called an open spherical fuzzy subset. The complement of an open spherical fuzzy subset is called a closed spherical fuzzy subset. As classical topologies or a fuzzy topological space, the family $\{1_s, 0_s\}$ is called the indiscrete spherical fuzzy topological space and the topology that contains all spherical fuzzy subsets is called the discrete spherical fuzzy topological space. A Spherical fuzzy topology τ, is said to be coarser than a Spherical fuzzy topology τ_2 defined on same set if $\tau_1 \subset \tau_2$.

Example 2.2 Let X=1,2. Consider the family of Spherical fuzzy subsets
$\tau = \{1_s, 0_s, S_1, S_2, S_3, S_4\}$ where

$S_1 = \{\langle 1, 0.5, 0.4, 0.3\rangle, \langle 2, 0.7, 0.5, 0.4\rangle\}$

$S_2 = \{\langle 1, 0.6, 0.5, 0.3\rangle, \langle 2, 0.5, 0.6, 0.3\rangle\}$

$S_3 = \{1\langle, 0.5, 0.5, 0.3\rangle, \langle 2, 0.5, 0.6, 0.4\rangle\}$

$S_4 = \{\langle 1, 0.6, 0.4, 0.3\rangle, \langle 2, 0.7, 0.5, 0.3\rangle\}$.

In this example (X, τ) is a Spherical fuzzy topological space.

Any fuzzy subset or picture fuzzy subset of a set can be considered as Spherical fuzzy subset, we observe that any fuzzy topological space or picture fuzzy topological space is a Spherical fuzzy topological space. But a spherical fuzzy topological space need not be a picture fuzzy topological space.

Instead of a neighbourhood of a fuzzy point, Chang [8] gave the definition of

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a neighbourhood of a fuzzy open set. So, by following this, we define:

Definition 2.3 Let S,T be two spherical fuzzy subsets in a Spherical fuzzy topological space. Then T is said to be a neighbourhood of S if there exists an open spherical fuzzy subset R such that $S \subset R \subset T$.

Proposition 2.4 A spherical fuzzy subset S is open in a Spherical fuzzy topological space if and only if it contains a neighbourhood of its each subset.

Here are few definitions to generalize some ordinary topological results.

Definition 2.5 Let Xand Y be 2 non-empty sets, let $f : X \to Y$ be a function and let $A\&B$ be Spherical fuzzy of X,Y respectively. Then the membership function of truthiness, abstinence & falseness of image of A with respect to f is denoted as f[A] and defined as

$$\mu_{f[A]}(y) = \begin{cases} sup \\ x \in f^{-1}(y)^{\mu_A(X)}; f^{-1}(y) \ is \ \ non-empty \\ 0 \qquad\qquad\qquad ; \quad otherwise \end{cases}$$

$$\gamma_{f[A]}(y) = \begin{cases} inf \\ x \in f^{-1}(y)^{\mu_A(X)}; f^{-1}(y) \ is \ \ non-empty \\ 0 \qquad\qquad\qquad ; \quad otherwise \end{cases}$$

$$\sigma_{f[A]}(y) = \begin{cases} inf \\ x \in f^{-1}(y)^{\mu_A(X)}; f^{-1}(y) \ is \ \ non-empty \\ 0 \qquad\qquad\qquad ; \quad otherwise \end{cases}$$

Respectively, The truthiness, abstinence and falseness function of pre-image of B with respect to f is denoted by $f^{-1}[B]$ are defined by
$\mu_{f^{-1}[B]}(z) = \mu_B(f(z))$,
$\gamma_{f^{-1})[B]}(z) = \gamma_B(f(z))$,$\sigma_{f^{-1}[B]}(z) = \sigma_B(f(z))$ respectively.

The image and pre-image of A,B respectively are spherical fuzzy subset. Since $mu_A\gamma_A$ and σ_A are non-negative,

$$\begin{aligned} &\mu^2_{f(A)}(y) + \gamma^2_{f(A)}(y) + \sigma^2_{f(A)}(y) \\ &\qquad = \left(sup_{X\in f^{-1}(y)^{\mu_A(x)}}\right)^2 + \left(inf_{X\in f^{-1}(y)^{\gamma_A(x)}}\right)^2 + \left(inf_{X\in f^{-1}(y)^{\sigma_A(x)}}\right)^2 \\ &\qquad = sup_{X\in f^{-1}(y)^{\mu^2_A(x)}} + inf_{X\in f^{-1}(y)^{\gamma^2_A(x)}} + inf_{X\in f^{-1}(y)^{\sigma^2_A(x)}} \end{aligned}$$

if $f^{-1}(y)$ is non-empty otherwise, if $f^{-1}(y) \neq \phi$, then

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$\mu^2_{f(A)}(y) + \gamma^2_{f(A)}(y) + \sigma^2_{f(A)}(y) = 1$

Similarly, for $f^{-1}[B]$,

$\mu^2_{f^{-1}[B]}(X) + \gamma^2_{f^{-1}[B]}(X) + \sigma^2_{f^{-1}[B]}(X) = \mu^2_B(f(X)) + \gamma^2_B(f(X)) + \sigma^2_B(f(X)) \leq 1$

(Since A,B is spherical fuzzy subset).

Thus $f^{-1}[B]$ is also a spherical fuzzy subset.

Proposition 2.6 Let X and Y be 2 non-empty sets and let $f : X \to Y$ be a function. Then

(1) $f^{-1}[B^c] = f^{-1}[B]^c$ for any spherical fuzzy subset B of Y.
(2) $f^{-1}[A^c] \subset f^{-1}[A]^c$ for any spherical fuzzy subset A of X.
(3) If $B_1 \subset B_2$ then $f^{-1}[B_1] \subset f^{-1}[B_2]$ where B_1 and B_2 are spherical fuzzy subsets of Y.
(4) If $A_1 \subset A_2$ then $f[A_1] \subset f[A_2]$ where A_1 and A_2 are spherical fuzzy subsets of X.
(5) $f \subset f^{-1}[B] \subset B$ for any spherical fuzzy subset B of Y.
(6) $A \subset f^{-1}(f[A])$ for any spherical fuzzy subset A of X.

Definition 2.7 Let (X, τ) and (Y, τ) be two SFTS and let $f : X \to Y$ be a function. Then f is named as Spherical fuzzy continuous (SFCN) if for any SFS of X and for any neighbourhood V of f[A] there exists a neighbourhood U of A such that $f[V] \subset U$.

Theorem 2.8 Let (X, τ) and $(Y, \tau^{'})$ be two SFTS and let $f : X \to Y$ be a function. Then the following statement are equivalent.

(1) f is SPCN
(2) For any spherical fuzzy subset A of X and for any neighbourhood V of f[A], there exists a neighbourhood U of A such that for any $B \subset U, f[B] \subset V$.
(3) For any spherical fuzzy subset A of X and for any neighbourhood V of f[A], there exists a neighbourhood U of A such that $U \subset f^{-1}(V)$.
(4) For any spherical fuzzy subset A of X and for any neighbourhood V of f[A],$f^{-1}[V]$ is a neighbourhood of A.

Proof:

(1) $\Rightarrow$ (2) Let us consider f is SFCN, let A be a spherical fuzzy subset of X & V be a neighbourhood of f[A]. By definition, there exists a neighbourhood U of A such that $f[U] \subset V$. But if $B \subset U$, then $f(B) \subset f(U) \subset V$ thus $f(B) \subset V$.

(2) $\Rightarrow$ (3) Let us assume (2) holds, A be a spherical fuzzy subset of X and let V be

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a neighbourhood of f[A]. By (2), we get $f(B) \subset V$, where $B \subset U$.
Then $B \subset f^{-1}(f(B)) \subset f^{-1}(V)$. Since B is arbitrary, $U \subset f^{-1}(V)$.

(3) $\Rightarrow$ (4) Assume (3) and let A be SFS of X and let V be a neighbourhood of f[A]. Problem (3), there exists a neighbourhood U of A such that $U \subset f^{-1}(V)$. Since U is a neighbourhood, by definition we have an open spherical fuzzy subset D of X, such that $A \subset D \subset U.U \subset f^{-1}(V)$, thus $A \subset D \subset f^{-1}(V)$. Therefore $f^{-1}(V)$ is a neighbourhood of A.

(4) $\Rightarrow$ (1) By assuming (4), Let A be a spherical fuzzy subset of X and V, a neighbourhood of f(A). Terefore by (4) there exists an open spherical fuzzy subset D such that
$A \subset D \subset f^{-1}(V) \to f(D) \subset f^{-1}(f(V)) \subset V$.
$\Rightarrow f(D) \subset V$. Hence f is SFCN.
Following is a characterization of spherical fuzzy continuity, can also be used as the other definition of spherical fuzzy continuous function.

Theorem 2.9 Let $(X, \tau) \& (Y, \tau')$ be two SFTS. A function $f : X \to Y$ is SFCN iff for each open SFS B of Y we have $f^{-1}(B)$ is open SFS of X.
Proof:

Assume f is continuous. Let B be an open SFS of Y. And $A \subset f^{-1}[B]$, then $f(A) \subset B$. Since B is open by Proposition (1), there exists a neighbourhood V of $f(A) \to V \subset B$.f is SFCN and by them (1), (4), we have $f^{-1}(V)$ is a neighbourhood of A. By (3) of Proposition (2), $f^{-1}(V) \subset f^{-1}(B)$. Therefore $f^{-1}(B)$ is a neighbourhood of A. Since A is arbitrary SFS of $f^{-1}(B)$, by Proposition 1, $f^{-1}(B)$ is open.

Conversely, let A be SFS of $X \& V$ be a neighbourhood of f(A). Then, there exists an DSFS P of X such that $f(A) \subset P \subset V$. By hypothesis $f^{-1}(B)$ is open. $A \subset f(f^{-1}(A)) \subset f^{-1}(P) \subset f^{-1}(V)$. Therefore, $f^{-1}(V)$ is a neighbourhood of A implies the Spherical fuzzy Continuity of f.

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